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# SUSY see-saw and NMSO(10)GUT inflation after BICEP2

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**Abstract.** Supersymmetric see-saw slow roll inflection point inflation occurs along a MSSM $D$-flat direction associated with gauge invariant combination of Higgs, slepton and right-handed sneutrino at a scale set by the right-handed neutrino mass $M_{\nu^c} \sim 10^6$–$10^{13}$ GeV. The tensor to scalar perturbation ratio $r \sim 10^{-3}$ can be achieved in this scenario. However, this scenario faced difficulty in being embedded in the realistic new minimal supersymmetric $SO(10)$ grand unified theory (NMSO(10)GUT). The recent discovery of $B$-mode polarization by BICEP2, changes the prospects of NMSO(10)GUT inflation. Inflection point models become strongly disfavoured, as the trilinear coupling of SUSY see-saw inflation potential gets suppressed relative to the mass parameter favoured by BICEP2. Large values of $r \approx 0.2$ can be achieved with super-Planck scale inflaton values and mass scales of inflaton $\geq 10^{13}$ GeV. In NMSO(10)GUT, this can be made possible with an admixture of heavy Higgs doublet fields, i.e., other than MSSM Higgs field, which are present and have masses of order GUT scale.

**Keywords.** Unified theories; early Universe; SUSY.

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## 1. Introduction

Inflationary cosmology has become one of the cornerstones of modern cosmology. Inflation not only solves the puzzles of Big Bang (horizon, flatness problem etc.), but also make predictions on the large-scale structure [1–3] of our Universe on the basis of quantum fluctuations which grew via gravitational instability and are the seeds for the large-scale structure of the present-day Universe. Recently, BICEP2, a telescope mounted at the south pole for background imaging of cosmic extragalatic polarization claimed the detection of the signal of primordial gravitational waves. The gravitational waves active during the inflationary epoch produce polarization in cosmic microwave background (CMB) radiation. They leave their imprint on the CMB radiation in terms of a curl or rotation which is known as the primordial $B$-mode polarization or the tensor component. Scalar density fluctuations also produce polarization in CMB radiation known as $E$-mode polarization or scalar component. The difference in two modes cannot be detected simply by looking at the variation of CMB temperature. Moreover, the *B*-mode is weaker than *E*-mode and thus difficult to detect. However, certain angles in polarization can be measured which allow one to distinguish between the two modes. The BICEP2 experiment claimed to measure this difference and has given their results in terms of tensor-to-scalar component ratio $r = 0.2^{+0.07}_{-0.05}$ at $3\sigma$ [4] level. They also discarded the possibility of $r = 0$ at $7\sigma$ level. The central value (0.2) of $r$ is quite large as compared to the upper limits found by WMAP [5] and PLANCK [6]. If confirmed, then it would have been direct evidence of primordial inflation and quantum gravitational effects [7] because the large *B*-mode polarization they claim to have detected could only be due to primordial gravitational waves which were produced during inflation via quantum effects. However, doubts were raised regarding their treatment of contamination of polarization data by foreground dust and as of writing this article, the consensus is that this contamination has annulled the BICEP2 claim [8–10]. The BICEP2 claim has led to the realization among model builders that large values of $r$ are definitely possible but at the same time it is stressed that to confirm the results of $r$ detection, great scrutiny is required. If BICEP2 results are later confirmed, or even a somewhat smaller value of $r$ in the range $10^{-3}$–$10^{-1}$ is detected, it will be a big blow to a large number of inflationary models and start a new era of modern cosmology.

In [11], we presented the conditions on NMSGUT superpotential parameters consistent with supersymmetric see-saw inflation (SSI) and results on inflation parameters compatible with fermion fits. Although we failed to achieve the required number of *e*-folds together with a viable fermion fit, the results in [11] did not include high-scale threshold corrections [12] which are crucial for determining the $SO(10)$ Yukawas and hence the inflation parameters. For a non-zero value of $r$, from [13] we have

$$V_0^{1/4} = \left(\frac{r}{0.1}\right)^{1/4} \times 2 \times 10^{16}\ \text{GeV}. \tag{1}$$

Thus, the BICEP2 claim favours single-field inflation models at precisely the MSSM gauge coupling unification scale! Also the inflation takes place over a field interval $\delta\phi \sim 5M_{\text{Pl}}$ [13,14]. Previously, almost all inflation models were designed to avoid such a large value of $r$ as the results of the PLANCK Collaboration [6] seem compatible with $r \ll 0.1$. A large value of $r$ changes the whole scenario of inflection point inflation models. For such a large-field digression one needs a steep potential rather than a flat potential. The condition found on the trilinear mass term tuning parameter ($\Delta \approx 10^{-28}\ \text{M}^2\ \text{GeV}^{-2}$) in [11] stops being a fine-tuning condition for $M > 10^{13}$ GeV and the assumptions of our previous analysis break down. In this work, we derive new conditions for slow roll inflation with generic renormalizable potential and BICEP2 level large tensor scalar ratio. Then we revisit SSI in NMSGUT without any fine tuning of inflation potential and without making any assumption on the family index of matter fields in the inflaton LHN condensate.

## 2. Generic renormalizable inflation potential without fine tuning

GUT scale inflation does not require fine-tuned flat potential and for inflaton mass $\geq 10^{13}$ GeV, the analysis of [11] does not hold. The general renormalizable inflationary potential is

$$V(\phi) = M^2\frac{\phi^2}{2} - \frac{Ah}{6\sqrt{3}}\phi^3 + h^4\frac{\phi^4}{12}. \tag{2}$$

Here $M$, $A$, $h$ are real without loss of generality. Its dimensionless form

$$\tilde{V} = \frac{V}{V_0} = \frac{x^2}{2} - \frac{\tilde{A}x^3}{6\sqrt{3}} + \frac{x^4}{12}, \tag{3}$$

where $x = \phi/\phi_0$, $V_0 = M^4/h^2$, $\tilde{A} = A/M$ and $\phi_0 = M/h$ is inflaton VEV, is more convenient for numerical work.

If $\omega = M/hM_{\rm pl}$, then for a given potential the slow roll parameters $\epsilon$ and $\eta$ are given by

$$\epsilon = \frac{\tilde{V}_x^2}{2\omega^2\tilde{V}}; \quad \eta = \frac{\tilde{V}_{xx}}{\omega^2\tilde{V}}, \tag{4}$$

where $M_{\rm Pl} = 2.43 \times 10^{18}$ GeV. The slow roll power spectrum $P_R$, spectral index $n_s$ and tensor to scalar ratio $r$ are given by

$$P_R = \frac{\omega^4 h^2 \tilde{V}}{24\pi^2\epsilon}, \tag{5}$$

$$n_s = 1 + 2\eta - 6\epsilon; \quad r = 16\epsilon. \tag{6}$$

The values given by PLANCK [6] are

$$P_R = (2.1977 \pm 0.103) \times 10^{-9}; \quad n_s = 0.958 \pm 0.008 \tag{7}$$

and BICEP2 value [4] for tensor-to-scalar ratio, $r$

$$r = 0.2^{+0.07}_{-0.05}. \tag{8}$$

The number of $e$-folds of inflation remaining when pivot scale crosses the horizon can be found from the equation of motion of inflaton field

$$N = \omega^2 \int_{x_{\rm CMB}}^{x_{\rm end}} \frac{\tilde{V}}{\tilde{V}_x} {\rm d}x. \tag{9}$$

Integrating this equation gives

$$N = \frac{\omega^2}{128}\left( -3(\tilde{A}^2 - 16)\log(-3\tilde{A}x + 4x^2 + 12) - \frac{2\tilde{A}(3\tilde{A}^2 - 80)\tan^{-1}\left(\frac{3\tilde{A}-8x}{\sqrt{192-9\tilde{A}^2}}\right)}{\sqrt{\frac{64}{3} - \tilde{A}^2}} + 8x(2x - \tilde{A})\right)\Bigg|_{x_{\rm CMB}}^{x_{\rm end}}. \tag{10}$$

Now we need to find the viable parameter values of $M$, $h$ and $\tilde{A}$ to achieve values given in eq. (7) and $N_{\text{e-folds}} \approx 50$. The procedure we follow is:

(1) Take some value of $r$ and $n_s$ in the allowed range given by eq. (7).
(2) Calculate the value of $\epsilon$ and $\eta$ by eqs (6).

(3) Then define a ratio $\theta = \eta/\epsilon$ and solve $x$. It gives one real root for $x$ which we take as $x_{\text{CMB}}$ (note that it does not give real root for the whole range of values given in (7)).
(4) Then the field value ($x_{\text{end}}$) is calculated from $\eta \approx 0.8$.
(5) Throwing the values of $M$, $h$, $A$, we calculate the values of $P_R$, $V_0^{1/4}$ and $N_{\text{e-folds}}$.

In figures 1–4, we have shown contour plots between quartic coupling $h$ and number of e-folds with contour lines bearing constant value of $\omega$. We have kept the value of $r = 0.2$ fixed and three values of spectral index $n_s = 0.954, 0.958, 0.962$. So each figure contains three contour plots. For each figure, we vary the trilinear term parameter $\tilde{A}$ over the values (0.1, 0.01, 0.001, 0). We accepted only those values of $M$ and $h$ for which the value of $P_R$ falls in the range $(1.0947–1.3007) \times 10^{-9}$ and the value of $V_0^{1/4}$ as given by eq. (1). By comparing figures 1 and 2, we see that parameter $\tilde{A}$ has only a small effect: when we change $\tilde{A}$ from 0.1 to 0.01, the plots shift a little bit towards higher values of $N_{\text{CMB}}$. However, reducing $\tilde{A}$ further does not make any difference. For $n_s = 0.962$, we need $h \sim 10^{-6.1}$ and $\omega \approx 9$ to have $N_{\text{e-folds}} \approx 50$. This corresponds to an inflaton mass of order $10^{13.1}$ GeV. For $n_s = 0.958$, to achieve $N_{\text{e-folds}} \approx 50$, we need smaller $h$ ($h \sim 10^{-6.35}$) and $\omega \approx 15$. For $n_s = 0.954$, the required quartic coupling is less than $10^{-6.4}$ and $\omega$ more than 20. These estimates thus provide a revised rule of

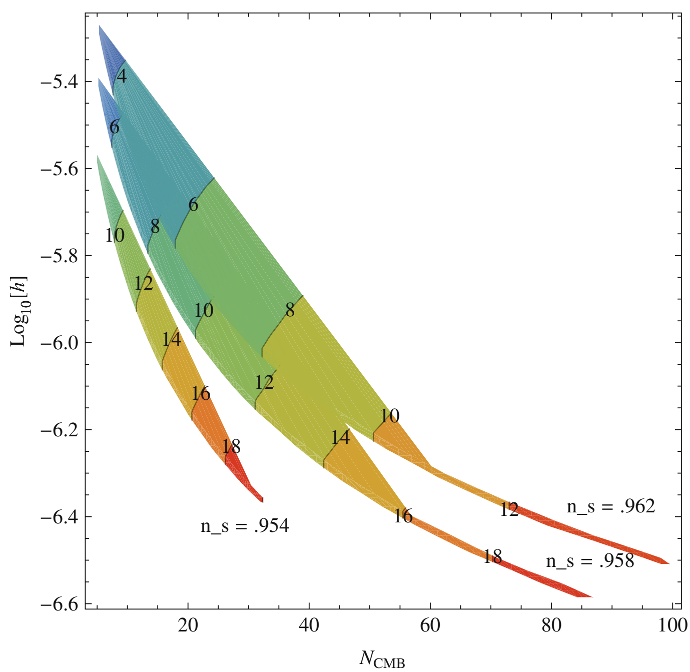


**Figure 1.** Plot of quartic coupling $h$ and number of e-folds for different values of $\omega$ at the trilinear dimensionless parameter $\tilde{A} = 0.1$. The contour line bears the constant value of $\omega$ and different colours of the contour represent a range for $\omega$. As we move towards higher values of $\omega$ the number of e-folds increases and quartic coupling decreases.

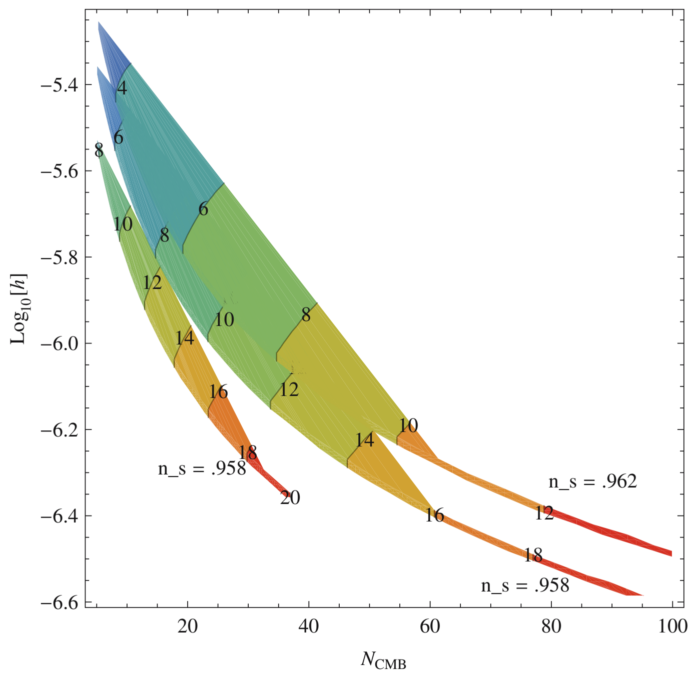


**Figure 2.** Same as figure 1 with $\tilde{A} = 0.01$.

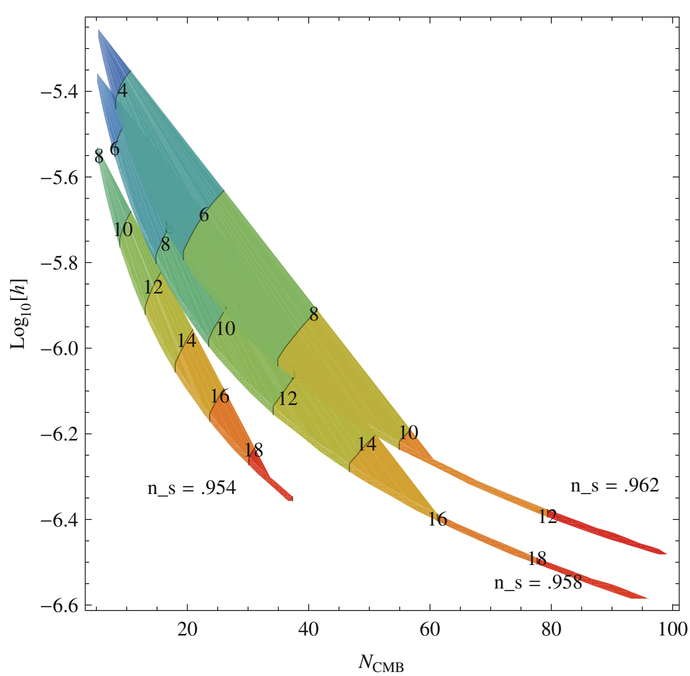


**Figure 3.** Same as figure 1 with $\tilde{A} = 0.001$.

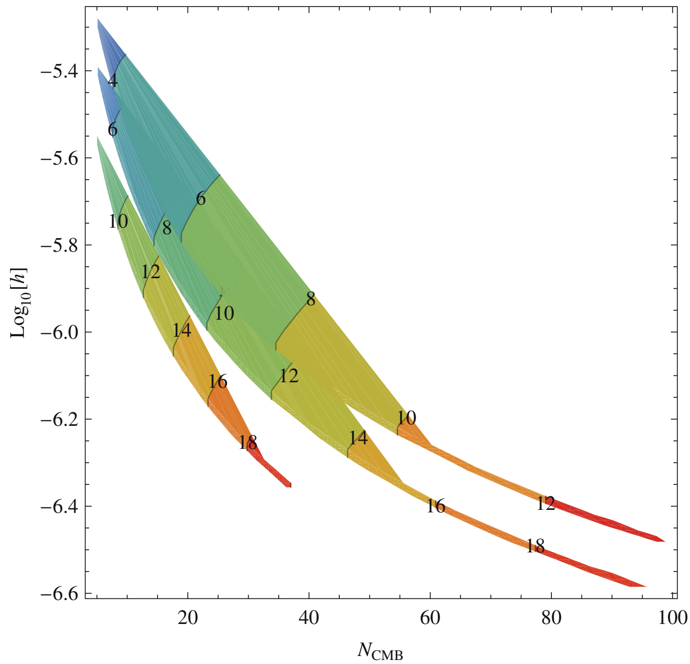


**Figure 4.** Same as figure 1 with $\tilde{A} = 0$.

thumb replacing the rule $h^2 \sim 10^{-25}$ (M/GeV) derived earlier in the fine-tuned case [11]. Note that $\omega$ is controlled by $M/h$ so that for $\omega \approx 20$, $h \approx 10^{-6.35}$ corresponds to $M \approx 10^{13.1}$ GeV and $h^2/M \sim 10^{-26}\,\mathrm{GeV}^{-1}$, which is about an order of magnitude smaller than for $M \ll 10^{13}$ GeV.

## 3. NMSO(10)GUT inflation after BICEP2

In [11], while embedding the SSI in NMSO(10)GUT [12,15], we assumed that only the light Higgs will contribute to inflaton flat direction so as to keep $M \leq 10^{13}$ GeV. With $M$ allowed to be large, the five heavy Higgs pair with GUT scale masses in the NMSGUT can also be allowed to contribute i.e., the strong condition that only the light Higgs corresponding to MSSM contributes to the inflaton condensate can be relaxed. So in this case, the heavy Higgs having masses $O(10^{16}$ GEV) will control the mass of the inflaton. Also, no assumption on the family index for $s$-fermion components of inflaton is made. Our new ansatz for inflaton is

$$\tilde{\bar{\nu}}_A = b_A \frac{\phi}{\sqrt{3}}; \quad \tilde{\nu}_A = c_A \frac{\phi}{\sqrt{3}}; \quad h_l = U_{lm} H_m = U_{lm} a_m \frac{\phi}{\sqrt{3}}. \tag{11}$$

Here $A = 1, 2, 3$ (family index) and $l, m = 1$–$6$ (types of different Higgs). The parameters $b_A$, $c_A$ and $a_m$ are complex numbers which decide the fraction each field contributes to the inflaton. From $D$-flatness condition we have

$$\sum_A |c_A|^2 = \sum_l |a_l|^2 = \sum_A |b_A|^2 + \sum_l 2|U_{4l} a_l|^2. \tag{12}$$

To achieve canonically normalized kinetic term for inflaton field we need

$$\sum_A |b_A|^2 + \sum_A |c_A|^2 + \sum_l |a_l|^2 = 1 \tag{13}$$

and this can be achieved in this scenario in the following way:

$$\sum_A |b_A|^2 = \sum_A |c_A|^2 = \sum_l |a_m|^2 = \frac{1}{3}. \tag{14}$$

With $\sum_l U_{4l} a_l = 0$, now $y^\nu$ is replaced by this new matrix given by

$$Z_{AB} = \tilde{h}_{AB} V_{1l} a_l - 2\sqrt{3} \tilde{f}_{AB} V_{2l} a_l - \tilde{g}_{AB}(V_{5l} a_l + i\sqrt{3} V_{6l} a_l). \tag{15}$$

The inflation parameters are given by

$$h^2 = \frac{4}{3}(|b_A \tilde{h}_{AB} c_B|^2 + 12|b_A \tilde{f}_{AB} c_B|^2 + 4|b_A \tilde{g}_{AB} c_B|^2 + |b_A Z_{AB}|^2 + 4|b_A \tilde{f}_{AB} b_B|^2 + |Z_{AB} c_B|^2) \tag{16}$$

$$M^2 = \frac{32}{3} |\tilde{f}_{AB} b_B|^2 |\bar{\sigma}|^2 + \frac{2}{3} |a_l|^2 m_{H_l}^2 \tag{17}$$

$$A = \frac{16}{h} |c_A^* Z_{AB}^\dagger \tilde{f}_{BC} b_C \bar{\sigma}|. \tag{18}$$

Now we throw the $24 = 6(b_A) + 6(c_A) + 12(a_m)$ real numbers randomly along with 38 parameters of NMSGUT superpotential at $M_X$ while fitting. The dimensionality of space in which our search engine will look for the solution is thus 62. The basic idea is that

**Table 1.** Inflation parameters along with parameters $b_A, c_A, a_m$. Note that $|b_1|, |c_1|, |a_1| \approx 1$, and the rest are very small.

| Inflation parameter | Value |
|---|---|
| $M$ | $7.8826 \times 10^{13}$ GeV |
| $h$ | $1.7736 \times 10^{-5}$ |
| $N_{\text{e-folds}}$ | 0.73 |
| $\epsilon$ | $1.25 \times 10^{-2}$ |
| $\eta$ | $1.85 \times 10^{-2}$ |
| $n_s$ | 0.962 |
| $P_R$ | $2.1977 \times 10^{-9}$ |
| $r$ | 0.20 |
| $A_0$ | $1.277 \times 10^{-4}$ |
| $V_0^{1/4}$ | $2.1825 \times 10^{16}$ GeV |
| $\omega$ | 1.82 |
| $\|a_m\|$ | $0.999, 4.295 \times 10^{-2}, 6.287 \times 10^{-4}, 3.049 \times 10^{-4}, 2.752 \times 10^{-4}, 5.439 \times 10^{-5}$ |
| $\|b_A\|$ | $0.999, 3.601 \times 10^{-2}, 4.119 \times 10^{-3}$ |
| $\|c_A\|$ | $0.999, 4.325 \times 10^{-2}, 3.209 \times 10^{-3}$ |

there can be an interplay between $b_A, c_A, a_m$ and $SO(10)$ Yukawas $h_{AB}, f_{AB}, g_{AB}$ such that they can give us small quartic coupling $h \sim 10^{-6.1}$ or less required for 50 e-folds along with fermion fitting. Also, the mass of inflaton of $O(10^{13}$ GeV) can be achieved with heavy Higgs fields of GUT scale masses. An important point worth mentioning is that GUT scale threshold corrections [12] which lower the required $SO(10)$ Yukawas so much that the proton decay rate $\Gamma_{\mathrm{d}=5}^{\Delta B \neq 0}$ is suppressed to less than $10^{-34}$ $\mathrm{y}^{-1}$ will also control the quartic coupling $h \sim 10^{-6.3}$. In this way NMSGUT may connect the baryon stability with the primordial inflation. In table 1, we quote the inflation parameters which are compatible with fermion fit. We are able to achieve the quartic coupling value $h = 1.77 \times 10^{-5}$ and approximately just one e-fold. Note that this is an improvement over [11] by a factor of $10^4$. The quartic coupling as well as the mass of inflaton are somewhat larger than required for successful inflation. The value of $\omega$ ($\approx$2) achieved is still smaller than required. The rest of the parameters are in an acceptable range.

## 4. Conclusion and discussions

Our reconsideration of the embedding of SSI in the NMSGUT to favour the possibility of a GUT scale inflaton energy density during inflation, as claimed by the BICEP2, shows that although tuning between trilinear and mass terms is no longer required, the SSI rule of thumb $h^2\,\mathrm{GeV}/M \sim 10^{-25}$ is only slightly modified to $h^2\,\mathrm{GeV}/M \sim 10^{-26}$. Super-Planckian VEV defined in terms of parameter $\omega = \phi_0/M_{\mathrm{pl}} \sim 15$ is required to achieve sufficient e-folds. Achieving enough $e$-folds during inflation requires $h \sim 10^{-6.35}$ and thus $M \sim 10^{13.3}$. In NMSGUT fits, we achieve $M = 10^{13.0-13.2}$ GeV but $h^2 \sim 10^{-12.2}$–$10^{-12.7}$ seems harder to achieve but not impossible. To search in such a high-dimensional parameter space, one needs patience for running of code which can take months to yield an acceptable answer. However, improved numerical methods may improve the situation in the future. For such a large value of inflaton mass, the trilinear term comes out very small and plays a minor role. Thus, one can even drop that term from inflation potential. The BICEP2 results favour inflation at GUT scale and if BICEP2 results stand scrutiny then NMSGUT can be a desirable candidate to explain inflation. If (as seems likely at the time of going to press in 2015) the BICEP2 results fall by the wayside and $r \ll 0.1$, then a smaller value of $M$ will be required: which is also achievable in the NMSGUT. Thus, the NMSGUT may well be compatible with a high inflaton mass in the range $10^{12.5}$–$10^{13.5}$ GeV. Further detailed studies with improved numerical methods on the theoretical side and of course better determinations of the tensor-to-scalar ratio are required.

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