PRAMANA — journal of physics

Vol. 86, No. 2
February 2016
pp. 307–314

Hawking radiation from quasilocal dynamical horizons

AYAN CHATTERJEE
Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Kangra 176 215, India
E-mail: ayan.theory@gmail.com

DOI: 10.1007/s12043-015-1150-1; ***e*Publication:** 6 January 2016

Abstract. In completely local settings, we establish that a dynamically evolving spherically symmetric black hole horizon can be assigned a Hawking temperature and with the emission of flux, radius of the horizon shrinks.

Keywords. Black hole evolution; Hawking radiation.

PACS Nos **04.70.Dy; 04.60.−m; 04.62.+v**

A black hole is usually described to be a region of space–time where the gravitational attraction is so high that even light cannot escape to infinity. This notion, that light cannot escape to future null infinity, is an appropriate characterization of a black hole in asymptotically flat space–time. In asymptotically flat space–times, an observer, placed far away from the black hole, perceives a flat Minkowski space–time in the neighbourhood. The light-like information that this observer receives in future can be traced back and it lies outside the black hole region. In more mathematical terms, one says that the black hole region lies outside the causal past of the future null infinity $\mathscr{I}^+$. The boundary of such a region is called the event horizon $\mathscr{H}$ [1,2]. To be more precise, consider a strongly asymptotically predictable space–time $(\mathcal{M}, g_{ab})$. The space–time is said to contain a black hole if $\mathcal{M}$ is not contained in $J^-(\mathscr{I}^+)$. The black hole region is denoted by $\mathscr{B} = \mathcal{M} - J^-(\mathscr{I}^+)$ and the event horizon is the boundary of $\mathscr{B}$ [2]. The definition of event horizon thus requires that we are able to construct the future null infinity $\mathscr{I}^+$ and hence, the entire future of the space–time has to be determined to prove the existence of an event horizon. Thus, $\mathscr{H}$ is a global concept and it becomes difficult to proceed much further. However, the notions simplify for stationary space–times. In equilibrium, these space–times admit Killing symmetries and thus exhibit a variety of interesting features. Indeed, the strong rigidity theorem implies that the event horizon of a stationary black hole is a Killing horizon [3]. However, not all Killing horizons are event horizons; stationary event horizon requires a globally defined time-like Killing vector field whereas, Killing horizon only requires a Killing vector field in the neighbourhood of the horizon. The Killing vector is null on the horizon.

Using the fact that the event horizon of a stationary black hole is a Killing horizon, the laws of mechanics for event horizons was proved [4]. It was shown that in general relativity, the surface gravity $\kappa_{\rm H}$ of a stationary black hole must be a constant over the event horizon. This is called the zeroth law of black hole mechanics. The first law of black hole mechanics refers to stationary space–times admitting an event horizon and small perturbations about them. This law states that the differences in mass M, area A and angular momentum J to two nearby stationary black hole solutions are related through $\delta M = \kappa_{\rm H}\delta A/8\pi + \Omega_{\rm H}\delta J$. One gets additional terms like charge if matter fields are present. It was also shown that black holes also admit a second law which states that the area of a black hole can never decrease in a classical process [3].

The laws of black hole mechanics in general relativity are remarkably analogous to the laws of thermodynamics [4]. Based on these grounds, it was argued that the entropy of the black holes must be proportional to its area [5,6]. However, such an argument had a flaw because, if black holes have entropy, they must be assigned a temperature and hence, black holes must radiate. But, classically, nothing can come out of black holes. As it turned out, this analogy, between black holes and thermal objects, is exact when quantum effects are taken into account. Indeed, Hawking's semiclassical analysis establishes that quantum mechanically, a stationary black hole with surface gravity κ radiates particles to infinity with a perfect black body spectrum at temperature $\kappa/2\pi$ [7,8]. Consequently, asymptotic observers perceive a thermal state and assign a physical temperature to the black hole. The precise match to thermodynamics is complete when the thermodynamic entropy of the black hole is identified with a quarter of its area [5]. Hawking's derivation of these quantum effects are independent of the gravitational field equations. One only needs to study the quantum fields in a geometry describing a stationary black hole formed out of gravitational collapse. During the last two decades, several more techniques have been developed to study the quantum behaviour and for obtaining Hawking temperature for more general space–times. Out of these, the Hartle–Hawking proposal [9] and the Euclidean approach [10] have been studied most extensively. Based on these formalisms, one can argue that it is possible to associate thermal states to space–times with bifurcate Killing horizons. In fact, it has been established that in any globally hyperbolic space–time with bifurcate Killing horizon, there can exist a vacuum thermal state at temperature $\kappa/2\pi$ which remains invariant under the isometries generating the horizon [11].

These proofs are however restrictive, and do not immediately generalize to space–times with superradiance [11]. Furthermore, in these formulations, one does not show explicitly that the thermal state that arises is a result of some flux emitted by the black hole. Neither does one establish that the black hole area shrinks in the process. More precisely, these formulations do not indicate how a thermal state may arise as a result of some physical process. In addition, it seems to be a reasonable physical expectation that even with a local definition of black hole horizon one should be able to establish the analogy to thermodynamics. This expectation is based on the fact that the laws of black hole mechanics apply equally well to local black hole horizons. For these local horizons, the laws of mechanics can be proved using only local geometrical properties of the null surfaces, without any assumptions on the global development of the space–time in which the horizon is embedded [12–15]. These horizons can be assigned an entropy proportional to the area of the local horizon [16,17]. More precisely, such horizons should have a temperature of $\kappa/2\pi$. Incidentally, this question has been investigated in a semiclassical approach which treats Hawking radiation as a quantum tunnelling phenomenon [18,19], but it is not clear how the horizon loses area due to emission of a flux of radiation.

In this paper, a formalism is developed to establish two basic issues (see [20] for details): (1) that one can associate a temperature to local dynamical horizons and (2) that there exists a precise relation between the radiation emitted by the horizon and area loss, i.e., flux of the outgoing radiation through the horizon in between two partial Cauchy slices exactly equals the difference of radii of the sphere that foliates the horizon at those two instances.

We shall use the formalism of future outward trapping horizon (FOTH) [12,13] which is closely related to dynamical horizons (DH) [21,22]. As the horizon that we describe is partially time-like and partially null, the $2+2$ framework of FOTH is ideal. We present our arguments as follows: First, to calculate temperature for local dynamical horizons, we shall begin by considering the Kodama vector field [23]. For spherical horizon, the Kodama vector (K^a) is orthogonal to the round spheres and goes along the lines of constant r in the orthogonal complement of the spheres, $K^a\nabla_a r = 0$. This vector field plays a role analogous to Killing vector fields. In fact, for spherical dynamical horizons, it provides a preferred time-like direction, becomes null on the FOTH, and is parallel to the Killing vector at spatial infinity which we assume to be flat. More specifically, the Kodama vector is a Killing vector if the Ricci tensor vanishes. We construct positive frequency field modes on both sides of the horizon by considering the Kodama vector field but the outgoing modes exhibit logarithmic singularities on the horizon under some approximation. However, if considered as distribution valued, these modes can be interpreted as horizon crossing and the probability current for these modes remain well defined. The Hawking temperature is obtained by equating the conditional probability, that modes incident on one side is emitted to the other side, to the Boltzmann factor [24,25].

The plan of the paper is as follows: We discuss the geometrical set-up based on future outer trapping horizon (FOTH) and then show how the Hawking temperature is proportional to the dynamical surface gravity associated with the Kodama vector. Finally, we calculate the flux of the energy radiated in a dynamical process. The conventions are those of ref. [13].

Let us consider a four-dimensional space–time $\mathcal{M}$ with signature $(-,+,+,+)$. A three-dimensional submanifold Δ in $\mathcal{M}$ is said to be a future outer trapping horizon (FOTH) if: (1) It is foliated by a preferred family of topological two-spheres such that, on each leaf S, the expansion θ_+ of a null normal l_+^a vanishes and the expansion θ_- of the other null normal l_-^a is negative definite and (2) the directional derivative of θ_+ along the null normal l_-^a (i.e., $\mathcal{L}_{l_-}\theta_+$) is negative definite. Thus, Δ is foliated by marginally trapped two-spheres. We choose a spherically symmetric background metric

$$\mathrm{d}s^2 = -2\mathrm{e}^{-f}\mathrm{d}x^+\mathrm{d}x^- + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2), \tag{1}$$

where both f and r are smooth functions of $x^\pm$. The expansions of the two null normals are $\theta_\pm = (2/r)\,\partial_\pm r$ respectively, where $\partial_\pm = \partial/\partial x^\pm$. In this coordinate system, the requirement for FOTH translates to $\partial_-\theta_+ < 0$ on Δ.

Let the vector field $t^a = l_+^a + h\, l_-^a$ be tangential to the FOTH for some smooth function h. Then the Raychaudhuri equation for l_+^a and the Einstein equation implies that

$$\partial_+\theta_+ = -h\partial_-\theta_+ = -8\pi T_{++}, \tag{2}$$

where $T_{++} = T_{ab}\, l_+^a l_+^b$ and T_{ab} is the energy–momentum tensor. As $t^2 = -2h\, \mathrm{e}^{-f}$, a FOTH becomes space-like if and only if $T_{++} > 0$ and is time-like if and only if $T_{++} < 0$. For a time-like FOTH, several consequences follow. Here, $\mathcal{L}_t r < 0$, and hence, Δ is time-like if and only if the area A and the Misner–Sharp energy E decrease along the horizon. This is also expected on general grounds as the horizon receives an incoming flux of negative energy, $T_{++} < 0$.

In the dynamical space–time (1), the Kodama vector field plays the analog role of the Killing vector [23]. For this space–time, it is given by

$$K^a = \mathrm{e}^f\, (\partial_- r)\, \partial_+^a - \mathrm{e}^f\, (\partial_+ r)\, \partial_-^a. \tag{3}$$

The surface gravity is defined through the equation $K^a \nabla_{[b} K_{a]} = \kappa K_b$ and its value is obtained to be $\kappa = -\mathrm{e}^f\, \partial_-\partial_+ r$. The FOTH condition $\partial_-\theta_+ < 0$ implies $\kappa > 0$.

We determine the positive frequency modes of the Kodama vector. Any smooth function of r is a zero mode of the Kodama vector. The proof goes as follows: Consider any smooth function $F(r)$, $\partial_\pm F = \partial_\pm r F'(r)$. Then, the result follows from eq. (3). Once, a zero mode is obtained, other positive frequency eigenmodes are evaluated using the eigenfunctions corresponding to the Kodama vector given in (3):

$$iKZ_\omega = \omega Z_\omega. \tag{4}$$

The quantity Z_ω are the eigenfunctions corresponding to the positive frequency ω. For simplification, we introduce the coordinates, $y = x^-$ and r and two new functions, $\bar{Z}_\omega(y, r) = Z_\omega(x^+, x^-)$ and $G(y, r) = \mathrm{e}^f\, (\partial_+ r)$. As a result, the eigenvalue eq. (4) reduces to

$$G\partial_y \bar{Z}_\omega = i\omega \bar{Z}_\omega. \tag{5}$$

Integrating and transforming back to old coordinates, the above equation gives

$$Z_\omega = F(r) \exp\left(i\omega \int_r \frac{\mathrm{d}x^-}{\mathrm{e}^f\, \partial_+ r} \right), \tag{6}$$

where $F(r)$ is an arbitrary smooth function of r and the lower limit r of the integral sign denotes that while evaluating the integral, r is to be kept fixed. To determine the integral in (6), we multiply the numerator and the denominator by $(\partial_-\theta_+)$ and use the fact that for any fixed r surface, $\mathrm{e}^f\, (\partial_-\theta_+) = -2\kappa/r$ (although the strict interpretation of κ as the surface gravity holds only for surfaces with $\theta_+ = 0$, it exists as a function in any neighbourhood of the horizon). In some neighbourhood of the horizon we get

$$\int_r \frac{\mathrm{d}x^-\, \partial_-\theta_+}{\mathrm{e}^f\, \partial_+ r \partial_-\theta_+} = -\int_r \frac{\mathrm{d}\theta_+}{\kappa\theta_+}, \tag{7}$$

where r indicates that the integral is to be evaluated on a constant r surface. Let the dynamical evolution of κ be a slowly varying function in some small neighbourhood of the horizon. Our assumption implies that the horizon is slowly radiating and thus, the geometry is quasistationary in a small neighbourhood of the horizon. This can also be understood as follows: since the black hole has a mass larger than the Planck mass (as we are using a semiclassical approach, this assumption is inherent), the back-reaction is extemerely small and the geometry is locally, approximately that of a Schwarzschild (with mass decreasing slowly with time). This assumption gives a closed form of the eigenmodes and hence, a simple identification of $\kappa/2\pi$ as the 'temperature' of the quasistationary horizon. In situations where this assumption fails, one has to evaluate the full integral and the identification of $\kappa/2\pi$ as the horizon temperature becomes unclear (this is not completely unexpected though, because κ is the bare surface gravity and one may have to use dressed surface gravity for rapidly radiating horizons). This gives

$$Z_\omega = F(r)\begin{cases} \theta_+^{-(i\omega/\kappa)} & \text{for } \theta_+ > 0 \\ (|\theta_+|)^{-(i\omega/\kappa)} & \text{for } \theta_+ < 0 \end{cases}, \tag{8}$$

where the spheres are not trapped 'outside the trapping horizon' ($\theta_+ > 0$) and fully trapped 'inside' ($\theta_+ < 0$). These are precisely the modes which are defined outside and inside the dynamical horizon respectively but not on the horizon. Now we have to keep in mind that the modes (8) are not ordinary functions, but are distribution-valued. Using the standard results [26], we find for $\epsilon \to 0^+$

$$(\theta_+ + i\epsilon)^\lambda = \begin{cases} \theta_+^\lambda & \text{for } \theta_+ > 0 \\ |\theta_+|^\lambda \mathrm{e}^{i\lambda\pi} & \text{for } \theta_+ < 0 \end{cases} \tag{9}$$

for $\lambda = -i\omega/\kappa$. For spherically symmetric static case, see [25]. The distribution (9) is well-defined for all values of θ_+ and λ, and it is differentiable to all orders. The modes Z_ω^* are given by the complex conjugate distribution.

We calculate the probability density in a single-particle Hilbert space for positive frequency solutions across the dynamical horizon and is given by, apart from a positive function of r,

$$\begin{aligned} \varrho(\omega) &= -\frac{i}{2}\left[Z_\omega^* K Z_\omega - K Z_\omega^* Z_\omega\right] = \omega Z_\omega^* Z_\omega \\ &= \omega(\theta_+ + i\epsilon)^{-(i\omega/\kappa)}(\theta_+ - i\epsilon)^{i\omega/\kappa}. \\ &= \begin{cases} \omega & \text{for } \theta_+ > 0 \\ \omega e^{(2\pi\omega/\kappa)} & \text{for } \theta_+ < 0. \end{cases} \end{aligned} \tag{10}$$

The conditional probability that a particle emits when it is incident on the horizon from inside is,

$$P_{(\text{emission}|\text{incident})} = \mathrm{e}^{-(2\pi\omega/\kappa)}. \tag{11}$$

This gives the correct Boltzmann weight with the temperature $\kappa/2\pi$, which is the desired value.

We now show that as the horizon evolves, the radius of the 2-sphere foliating the horizon shrinks in precise accordance with the amount of flux radiated by the horizon. To study the flux equation, consider the new coordinates, $(x^+, x^-) \mapsto (\theta_+, \tilde{x}^-)$ where

$\tilde{x}^- = x^-$. On FOTH, $(\partial_-\theta_+)/(\partial_+\theta_+)$ is equal to $-(\partial_-\partial_+ r)/(4\pi r T_{++})$ and negative definite. As a result, the derivatives are related to each other by

$$\tilde{\partial}_- = \partial_- + \left(\frac{\partial_-\partial_+ r}{4\pi r T_{++}}\right)\partial_+. \tag{12}$$

It is not difficult to show that $\tilde{\partial}_-$ is proportional to the tangent vector t^a to the FOTH. Observe that the normal one-form to Δ must be proportional to $(\mathrm{d}r - 2\,\mathrm{d}E)$ (because the horizon is defined to be the surface $r = 2E$), which on the horizon is equal to the one-form

$$8\pi \mathrm{e}^f r^2\, T_{++}\partial_- r \mathrm{d}x^+ - 2r\mathrm{e}^f \partial_-\partial_+ r\partial_- r \mathrm{d}x^-. \tag{13}$$

In arriving at the above identity, we have made use of two Einstein's equations [13]

$$r\,\partial_-\partial_+ r + \partial_+ r\,\partial_- r + \tfrac{1}{2}\mathrm{e}^{-f} = 4\pi r^2\, T_{-+},$$

$$\partial_+^2 r + \partial_+ f\,\partial_+ r = -4\pi r\, T_{++}, \tag{14}$$

and energy equations

$$\partial_\pm E = 2\pi \mathrm{e}^f r^3 (T_{-+}\,\theta_\pm - T_{\pm\pm}\,\theta_\mp). \tag{15}$$

As a result, the normal vector n^a is proportional to

$$\partial_+ - \left(\frac{4\pi r T_{++}}{\partial_-\partial_+ r}\right)\partial_- = \partial_+ - h\partial_-, \tag{16}$$

so that the tangent vector $t^a = \partial_+^a + h\partial_-^a$, which is clearly proportional to (12).

So $\tilde{x}^-, \theta, \phi$ are natural coordinates on FOTH. The line element (1) induces a line element on Δ

$$\mathrm{d}s^2 = -2\mathrm{e}^{-f} h^{-1}(\mathrm{d}\tilde{x}^-)^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2). \tag{17}$$

Consequently, the volume element on the FOTH is given by $\mathrm{d}\mu = \sqrt{2\mathrm{e}^{-f}h^{-1}}r^2 \sin\theta\, \mathrm{d}\tilde{x}^-\, \mathrm{d}\theta\mathrm{d}\phi$. We can now calculate the flux of the matter energy that crosses the dynamical horizon – it is an integral on a slice of horizon bounded by two spherical sections S_1 and S_2

$$\mathcal{F} = \int \mathrm{d}\mu\; T_{ab}\hat{n}^a K^b, \tag{18}$$

where $\hat{n}^a$ is the unit normal vector

$$\hat{n}^a = \frac{1}{\sqrt{2h\mathrm{e}^{-f}}}(\partial_+^a - h\partial_-^a) \tag{19}$$

and K^a is the Kodama vector. Using spherical symmetry and the Einstein equations, we get (see [20] for details)

$$\mathcal{F} = -\frac{1}{2}(r_2 - r_1), \tag{20}$$

where r_1, r_2 are respectively the two radii of S_1, S_2. As the area decreases along the horizon, $r_2 < r_1$, where S_2 lies in the future of S_1. As a result, the outgoing flux of the matter energy radiated by the dynamical horizon is positive definite (and the ingoing flux of the matter energy is negative definite). The flux formula (20) differs from the one given in [22] in an important way: as the Kodama vector field provides a time-like direction and is null on the horizon, it is appropriate to use K^a in the flux formula for the dynamical horizon.

Our derivation of Hawking temperature and the flux law depends on two assumptions. The first is the existence of the Kodama vector field and the Misner–Sharp energy. For spherically symmetric space–times, the Kodama vector field exists unambiguously and the Misner–Sharp energy is well defined. For more general space–times, a Kodama-like vector field is not known. However, one can still define some mass for such cases that reduces to the Misner–Sharp energy in the spherical limit. The second assumption is the slow variation of the dynamical surface gravity κ during evolution. For large black holes, the horizon evolves slowly enough so that the surface gravity function should vary slowly in some small neighbourhood of the horizon. In other words, this derivation implies that the Hawking temperature for a dynamically evolving large black hole is $\kappa/2\pi$ if the dynamical surface gravity is slowly varying in the vicinity of the horizon. Also, this derivation implies that back reaction effects cause the black hole to loose mass and hence shrink in size. If the mass of the black hole is very large compared to the Planck mass, the back reaction effects are quite small and the black hole looses mass very slowly. After some time, roughly proportional to the cube of the initial mass, the black hole would evaporate completely. However, when mass of the black hole reaches the Planck mass, this calculation cannot be trusted and must be superseded by quantum gravity calculations. Recent results indicate that if quantum gravity results are taken into account, one may be able to address this issue [27]. These results also indicate that the loss of quantum coherence due to black hole evaporation, where a pure quantum state evolves to a mixed state, may be better addressed if quantum gravity is taken into account.

It is also interesting to speculate on the extension of the present method for other diffeomorphism invariant theories of gravity. While the zeroth and the first laws hold for any arbitrary diffeomorphism invariant theory, the second law has only been proved for a class of such theories [28]. If the present formalism can be extended to other theories of gravity, it will lend support to the existence of the area increase theorem for such theories. While more interesting and deeper issues can only be understood in a full quantum theory of gravity, the present framework can provide a better understanding of the Hawking radiation process.

Acknowledgement

This article is based on a work done by the author in collaboration with A Ghosh and B Chatterjee. The author is partially supported through the UGC BSR start-up grant vide their letter No. F. 20-1(30)/2013(BSR)/3082.

References

[1] S W Hawking and G F R Ellis, *The large scale structure of space-time* (Cambridge University Press, Cambridge, 1973)

[2] R M Wald, *General relativity* (Chicago Univ Press, Chicago, USA, 1984) p. 491
[3] S W Hawking, *Commun. Math. Phys.* **25**, 152 (1972)
[4] J M Bardeen, B Carter and S W Hawking, *Commun. Math. Phys.* **31**, 161 (1973)
[5] J D Bekenstein, *Phys. Rev. D* **7**, 2333 (1973)
[6] J D Bekenstein, *Phys. Rev. D* **9**, 3292 (1974)
[7] S W Hawking, *Commun. Math. Phys.* **43**, 199 (1975)
[8] R M Wald, *General relativity* (University Press, Chicago, USA, 1994) p. 205
[9] J B Hartle and S W Hawking, *Phys. Rev. D* **13**, 2188 (1976)
[10] G W Gibbons and M J Perry, *Proc. R. Soc. London A* **358**, 467 (1978)
[11] B S Kay and R M Wald, *Phys. Rep.* **207**, 49 (1991)
[12] S A Hayward, *Phys. Rev. D* **49**, 6467 (1994)
[13] S A Hayward, *Class. Quant. Grav.* **15**, 3147 (1998), arXiv:gr-qc/9710089
[14] A Ashtekar, C Beetle, O Dreyer, S Fairhurst, B Krishnan, J Lewandowski and J Wisniewski, *Phys. Rev. Lett.* **85**, 3564 (2000), arXiv:gr-qc/0006006
[15] I Booth and S Fairhurst, *Phys. Rev. Lett.* **92**, 011102 (2004), arXiv:gr-qc/0307087
[16] A Ashtekar, J Baez, A Corichi and K Krasnov, *Phys. Rev. Lett.* **80**, 904 (1998), arXiv:gr-qc/9710007
[17] A Ghosh and A Perez, *Phys. Rev. Lett.* **107**, 241301 (2011), arXiv:1107.1320 [gr-qc]
[18] M K Parikh and F Wilczek, *Phys. Rev. Lett.* **85**, 5042–5 (2000)
[19] L Vanzo, G Acquaviva and R Di Criscienzo, *Class. Quant. Grav.* **28**, 183001 (2011)
[20] A Chatterjee, B Chatterjee and A Ghosh, *Phys. Rev. D* **87**, 084051 (2012)
[21] A Ashtekar and B Krishnan, *Living Rev. Rel.* **7**, 10 (2004)
[22] A Ashtekar and B Krishnan, *Phys. Rev. Lett.* **89**, 261101 (2002), arXiv:gr-qc/0207080; *Phys. Rev. D* **68**, 104030 (2003), arXiv:gr-qc/0308033
[23] H Kodama, *Prog. Theor. Phys.* **63**, 1217 (1980)
[24] T Damour and R Ruffini, *Phys. Rev. D* **14**, 332 (1976)
[25] B Chatterjee and A Ghosh, arXiv:1201.4017 [gr-qc]
[26] I M Gel'fand and G E Shilov, *Generalized functions* (Academic Press, 1964) Vol. 1
[27] A Ashtekar and M Bojowald, *Class. Quant. Grav.* **22**, 3349 (2005), arXiv:gr-qc/0504029
A Ashtekar, V Taveras and M Varadarajan, *Phys. Rev. Lett.* **100**, 211302 (2008), arXiv:0801.1811 [gr-qc]
[28] A Chatterjee and S Sarkar, *Phys. Rev. Lett.* **108**, 091301 (2012), arXiv:1111.3021 [gr-qc]
S Kolekar, T Padmanabhan and S Sarkar, arXiv:1201.2947 [gr-qc]