PRAMANA
— journal of
physics

Vol. 86, No. 2
February 2016
pp. 281–294

Twin-unified $SU(5) \times SU(5)'$ GUT and phenomenology

ZURAB TAVARTKILADZE
Center for Elementary Particle Physics, ITP, Ilia State University, 0162 Tbilisi, Georgia
E-mail: zurab.tavartkiladze@gmail.com

DOI: 10.1007/s12043-015-1148-8; ***e*Publication:** 24 January 2016

Abstract. In this article, after a short introduction, grand unified $SU(5) \times SU(5)'$ model augmented by D_2 parity has been discussed. The latter turns out to be important for phenomenology. Specific pattern of the GUT symmetry breaking causes new strong dynamics at low energies. Consequently, the Standard Model leptons, along with right-handed/sterile neutrinos, come out as composite states. Issues of the gauge coupling unification, generation of the charged fermion and neutrino masses will be presented. Also, various phenomenological implications and constraints will be discussed.

Keywords. Unification; fermion masses; compositeness; proton decay.

PACS Nos 12.10.Dm; 11.30.Fs; 12.15.Ff; 12.60.Rc

1. Introduction

Thanks to the Higgs boson discovery [1] at CERN's Large Hadron Collider, the triumph of the celebrated Standard Model (SM) of electroweak interactions occurred. In spite of this success, several phenomenological and theoretical issues motivate one to think of some physics beyond the SM. With the Higgs mass $\simeq$126 GeV, due to renormalization group (RG) running, the self-coupling of the SM Higgs boson becomes negative at scale near $\sim 10^{10}$ GeV [2]. This causes vacuum instability. Moreover, the SM fails to accommodate atmospheric and solar neutrino data. The renormalizable part of the SM renders neutrinos to be massless, while Planck scale suppressed $d = 5$ lepton number violating operators do not give sufficiently large neutrino masses. These are already strong motivations to think about the existence of some new physics between electroweak (EW) and Planck scales.

The grand unification theory (GUT) [3,4] is a leading candidate among various extensions of the SM. In spite of many salient futures, GUT model building encounters numerous problems and phenomenological difficulties. With single-scale breaking, grand unified theories (such as minimal $SU(5)$ and $SO(10)$) do not lead to successful gauge coupling unification. Besides this, constructing GUT with desirable GUT symmetry breaking pattern, with realistic fermion sector and adequately stabile nucleon, still remain challenging.

Motivated by these issues, we consider $SU(5) \times SU(5)'$ GUT (which we call twinification) augmented with D_2 parity (exchange symmetry). The proposed model does not suffer from the problems mentioned above and together with a potential of avoiding numerous difficulties, has interesting features.

Now we discuss this model, its features and various implications and constraints. This contribution is based on the work of Tavartkiladze [5].

2. The model

Let us consider the theory based on $SU(5) \times SU(5)'$ gauge symmetry. Besides this symmetry, we postulate discrete parity D_2, which exchanges two $SU(5)$s. Therefore, the symmetry of the model is

$$G_{\rm GUT} = SU(5) \times SU(5)' \times D_2. \tag{1}$$

As noted, the action of D_2 interchanges the gauge fields (in adjoint representations) of $SU(5)$ and $SU(5)'$. Thanks to D_2, at and above the GUT scale M_G, we have single gauge coupling

$$\alpha_5 = \alpha_{5'}. \tag{2}$$

In our case, as we show below, the EW part (i.e., $SU(2)_w \times U(1)_Y$) of the SM gauge symmetry belongs to the diagonal subgroup of $SU(5) \times SU(5)'$.

2.1 *Symmetry breaking and gauge coupling unification*

For a desirable $G_{\rm GUT}$ symmetry breaking we introduce the states

$$\begin{aligned} &H \sim (5,1)\,, \quad \Sigma \sim (24,1)\,, \quad H' \sim (1,5)\,, \\ &\Sigma' \sim (1,24)\,, \quad \Phi \sim (5,\bar{5})\,, \end{aligned} \tag{3}$$

where transformation properties under $SU(5) \times SU(5)'$ symmetry are indicated within brackets. H includes SM Higgs doublet h. Inclusion of H' is required by D_2 symmetry. By the same reason, two adjoints Σ and Σ' are introduced. The bifundamental state Φ also serves for the symmetry breaking.

The D_2 parity acts as $D_2: \quad H_a \rightleftarrows H'_{a'}$, $\Sigma^a_b \rightleftarrows \Sigma'^{a'}_{b'}$ and $\Phi^{b'}_a \rightleftarrows (\Phi^\dagger)^b_{a'}$, where we have made explicit the indices of $SU(5)$ and $SU(5)'$. With these, the kinetic part of the scalar field Lagrangian is invariant. The scalar potential, invariant under $G_{\rm GUT}$ symmetry (of eq. (1)) is

$$V = V_{H\Sigma} + V_{H'\Sigma'} + V^{(1)}_{\rm mix} + V_\Phi + V^{(2)}_{\rm mix}\,, \tag{4}$$

with

$$
\begin{aligned}
V_{H\Sigma} &= -M_\Sigma^2\, \mathrm{tr}\Sigma^2 + \lambda_1(\,\mathrm{tr}\Sigma^2)^2 + \lambda_2\, \mathrm{tr}\Sigma^4 \\
&\quad + H^\dagger\left(M_H^2 - h_1\Sigma^2 + h_2 \mathrm{tr}\Sigma^2\right)H + \lambda_H(H^\dagger H)^2\,, \\
V_{H'\Sigma'} &= -M_\Sigma^2 \mathrm{tr}\Sigma'^2 + \lambda_1(\mathrm{tr}\Sigma'^2)^2 + \lambda_2 \mathrm{tr}\Sigma'^4 + H'^\dagger\left(M_H^2 - h_1\Sigma'^2 + h_2\, \mathrm{tr}\Sigma'^2\right)H' \\
&\quad + \lambda_H(H'^\dagger H')^2, \\
V_{\mathrm{mix}}^{(1)} &= \lambda(\,\mathrm{tr}\Sigma^2)(\,\mathrm{tr}\Sigma'^2) + \tilde{h}\left(H^\dagger H\, \mathrm{tr}\Sigma'^2 + H'^\dagger H'\, \mathrm{tr}\Sigma^2\right) \\
&\quad + \hat{h}(H^\dagger H)(H'^\dagger H'), \\
V_\Phi &= -M_\Phi^2 \Phi^\dagger\Phi + \lambda_{1\Phi}\left(\Phi^\dagger\Phi\right)^2 + \lambda_{2\Phi}\Phi^\dagger\Phi\Phi^\dagger\Phi, \\
V_{\mathrm{mix}}^{(2)} &= \mu(H^\dagger\Phi H' + H\Phi^\dagger H'^\dagger) + \frac{\lambda_{1H\Phi}}{\sqrt{25}}(\Phi^\dagger\Phi)[(H^\dagger H) + (H'^\dagger H')] \\
&\quad + \frac{\lambda_{2H\Phi}}{\sqrt{10}}(H^\dagger\Phi\Phi^\dagger H + H'^\dagger\Phi^\dagger\Phi H') \\
&\quad + \lambda_{1\Sigma\Phi}(\Phi^\dagger\Phi)(\,\mathrm{tr}\Sigma^2 + \,\mathrm{tr}\Sigma'^2) - \lambda_{2\Sigma\Phi}(\Phi^\dagger\Sigma^2\Phi + \Phi\Sigma'^2\Phi^\dagger)\,. \qquad (5)
\end{aligned}
$$

The couplings in eqs (4) and (5) allow us to have a desirable and self-consistent pattern of symmetry breaking. First, we shall sketch the symmetry breaking pattern. At the first step, Σ develops the vacuum expectation value (VEV) $\sim M_G$ with $\langle\Sigma\rangle = v_\Sigma \mathrm{Diag}(2, 2, 2, -3, -3)$, $v_\Sigma \sim M_G$. This causes the symmetry breaking

$$SU(5) \xrightarrow{\langle\Sigma\rangle} SU(3) \times SU(2) \times U(1) \equiv G_{321}. \qquad (6)$$

With $\langle\Sigma'\rangle = v_{\Sigma'}\mathrm{Diag}\,(2, 2, 2, -3, -3)$, the breaking

$$SU(5)' \xrightarrow{\langle\Sigma'\rangle} SU(3)' \times SU(2)' \times U(1)' \equiv G'_{321} \qquad (7)$$

is achieved. The last stage of the GUT breaking is done by $\langle\Phi\rangle$ with a direction $\langle\Phi\rangle = v_\Phi \cdot \mathrm{Diag}(0,\ 0,\ 0,\ 1,\ 1)$. This configuration of $\langle\Phi\rangle$ breaks symmetries $SU(2) \times U(1)$ (subgroup of $SU(5)$) and $SU(2)' \times U(1)'$ (subgroup of $SU(5)'$) to the diagonal symmetry group

$$SU(2) \times U(1) \times SU(2)' \times U(1)' \xrightarrow{\langle\Phi\rangle} [SU(2) \times U(1)]_{\mathrm{diag}}\,. \qquad (8)$$

As we see, all VEVs preserve $SU(3)$ and $SU(3)'$ groups arising from $SU(5)$ and $SU(5)'$ respectively. However, unbroken $SU(2)_{\mathrm{diag}}$ is coming (as superposition) partly from $SU(2) \subset SU(5)$ and partly from $SU(2)' \subset SU(5)'$. Same condition applies to $U(1)_{\mathrm{diag}}$; i.e., it is superposition of two Abelian factors: $U(1) \subset SU(5)$ and $U(1)' \subset SU(5)'$. With the identifications $SU(3) \equiv SU(3)_c$, $SU(2)_{\mathrm{diag}} \equiv SU(2)_w$, $U(1)_{\mathrm{diag}} \equiv U(1)_Y$ and taking into account eqs (6)–(8), we see that GUT symmetry is broken as

$$G_{\mathrm{GUT}} \to SU(3)_c \times SU(2)_w \times U(1)_Y \times SU(3)' = G_{\mathrm{SM}} \times SU(3)'\,, \qquad (9)$$

where $G_{\mathrm{SM}} = SU(3)_c \times SU(2)_w \times U(1)_Y$ denotes the SM gauge symmetry.

While $\langle\Sigma\rangle \sim M_G$, the VEVs $\langle\Phi\rangle$ and Σ' are at intermediate scales M_I and M'_I, respectively, $v_\Phi \sim M_I$, $v_{\Sigma'} \sim M'_I$, with the hierarchical pattern $M_I \ll {M_I}' \ll M_G$. Detailed analysis of the whole potential shows that there is true minimum with considered VEV configuration and $\langle H\rangle = \langle H'\rangle = 0$.

2.1.1 *The spectrum.* At the first stage of symmetry breaking, the (X, Y) gauge bosons (of $SU(5)$) obtain GUT scale masses. They absorb appropriate states from the adjoint scalar Σ. The remaining physical fragments $(\Sigma_8, \Sigma_3, \Sigma_1)$ (the $SU(3)$ octet, $SU(2)$ triplet, and a singlet, respectively) receive GUT scale masses.

The mass of the $SU(3)'$ octet (from Σ') is denoted by $M_{\Sigma'_{8'}}$.

The triplet $\Sigma'_{3'}$ mixes with a real (CP even) $SU(2)_w$ triplet Φ_3 (from Φ). (Both these states are real adjoints of $SU(2)_w$.) The CP-odd real $SU(2)_w$ triplet from Φ is absorbed by appropriate gauge fields after $SU(2) \times SU(2)' \to SU(2)_w$ breaking and becomes genuine Goldstone modes.

By the VEVs v_Σ and $v_{\Sigma'}$, the symmetry $SU(5) \times SU(5)' \times D_2$ is broken down to $G_{321} \times G'_{321}$ (see eqs (6) and (7)). Thus, between the scales M_I and M'_I, we have this symmetry, and $\Phi(5, \bar{5})$ splits into fragments

$$\Phi(5, \bar{5}) = \Phi_{DD'} \oplus \Phi_{DT'} \oplus \Phi_{TT'} \oplus \Phi_{TD'} \tag{10}$$

with nontrivial transformation properties under $G_{321} \times G'_{321}$ gauge symmetry. The masses of these fragments will be denoted by $M_{DD'}$, $M_{DT'}$, $M_{TT'}$ and $M_{TD'}$, respectively. Since the breaking $G_{321} \times G'_{321} \to G_{SM} \times SU(3)'$ is realized by the VEV of the fragment $\Phi_{DD'}$ at scale M_I, we take $M_{DD'} \simeq M_I$. The masses of the remaining states (with obvious transformation properties) will be denoted by $M_{DT'}$, $M_{TT'}$ and $M_{TD'}$.

The states H and H' are split as $H \to (D_H, T_H)$ and $H' \to (D_{H'}, T_{H'})$, where $D_H, D_{H'}$ are doublets, while T_H and $T_{H'}$ are $SU(3)_c$ and $SU(3)'$ triplets, respectively. Masses of these triplets are M_{T_H} and $M_{T_{H'}}$. Both states D_H and $D_{H'}$, under $G_{\rm SM}$, have quantum numbers of the SM Higgs doublet. They mix by the VEV $\langle\Phi\rangle$ and thus we shall get two physical states h and D'. We identify h with the SM Higgs doublet and set its mass square (by fine-tuning) $M_h \sim 100$ GeV. We assume the second doublet D' to be heavy $M_{D'} \gg M_h$. For the mixing angle θ_h (between these two doublets), we also assume $\theta_h \ll 1$. Therefore, the SM Higgs mainly resides in D_H.

Detailed analysis [5] show that, enough parameters are involved, and one can always consider symmetry breaking pattern and desirable spectrum. One example, with the masses and scale selection, is given in table 1. This, together with other implications, give successful gauge coupling unification depicted in figure 1.

Table 1. Particle spectroscopy.

M_a	GeV	M_a	GeV	M_a	GeV	M_a	GeV	M_a	GeV
$M^{(1)}_{\hat{l}l}$	$7.54 \cdot 10^4$	$M^{(2)}_{e^c\hat{e}^c}$	$7.54 \cdot 10^4$	$M_{D'}$	$4.16 \cdot 10^6$	$M_{TD'}$	$3.92 \cdot 10^6$	$M_{X'}$	$2.08 \cdot 10^6$
$M^{(2)}_{\hat{l}l}$	$7.54 \cdot 10^4$	$M^{(3)}_{e^c\hat{e}^c}$	$1.2 \cdot 10^5$	$M_{TT'}$	1874.7	$M_{\Sigma'_{8'}}$	9277	M_{T_H}	$5 \cdot 10^{11}$
$M^{(3)}_{\hat{l}l}$	$1.2 \cdot 10^5$	Λ'	1851	$M_{DD'}$	$8.25 \cdot 10^4$	$M_{\Sigma'_{3'}}$	$2M_{\Sigma'_{8'}}$	M_X	$4.95 \cdot 10^{11}$
$M^{(1)}_{e^c\hat{e}^c}$	$7.54 \cdot 10^4$	$M_{T_{H'}}$	1851	$M_{DT'}$	8250	$M_{\Sigma'_{1'}}$	$4.16 \cdot 10^6$	M_Σ	$5 \cdot 10^{11}$

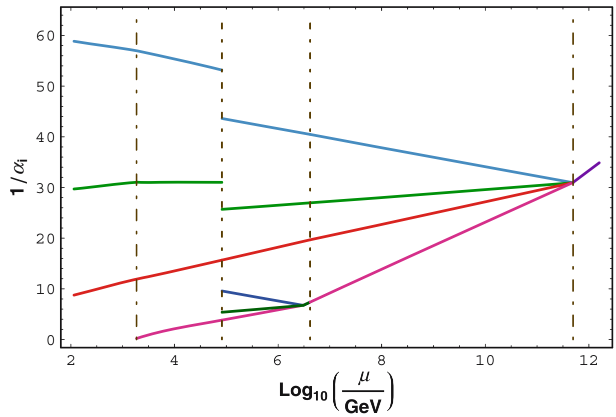

Figure 1. Gauge coupling unification. $\alpha_G(M_G) \simeq 1/31$ and $\{\Lambda', M_I, M_I', M_G\} \simeq \{1800, 8.25 \cdot 10^4, 4.16 \cdot 10^6, 4.95 \cdot 10^{11}\}$ GeV.

2.2 *Fermion sector: Composite leptons*

Fermion states are introduced as

$$3 \times \left[\Psi(10, 1) + F(\bar{5}, 1)\right], \quad 3 \times \left[\Psi'(1, \bar{10}) + F'(1, 5)\right], \tag{11}$$

(in brackets we indicate the transformation properties under $SU(5) \times SU(5)'$). Here, each fermionic state is a two-component Weyl spinor, in $(\frac{1}{2}, 0)$ representation of the Lorentz group. On these fields D_2 parity acts as D_2: $\Psi \rightleftarrows \bar{\Psi}' \equiv (\Psi')^\dagger$, $\mathbf{F} \rightleftarrows \bar{F}' \equiv (F')^\dagger$. With these, the invariant Yukawa Lagrangian is: $\mathcal{L}_Y + \mathcal{L}_{Y'} + \mathcal{L}_Y^{\rm mix}$ with

$$\mathcal{L}_Y = \sum_{n=0} C_{\Psi\Psi}^{(n)} \left(\frac{\Sigma}{M_*}\right)^n \Psi\Psi H + \sum_{n=0} C_{\Psi F}^{(n)} \left(\frac{\Sigma}{M_*}\right)^n \Psi\mathbf{F}H^\dagger + \text{h.c.} \tag{12}$$

$$\mathcal{L}_{Y'} = \sum_{n=0} C_{\Psi\Psi}^{(n)*} \left(\frac{\Sigma'}{M_*}\right)^n \Psi'\Psi'H'^\dagger + \sum_{n=0} C_{\Psi F}^{(n)*} \left(\frac{\Sigma'}{M_*}\right)^n \Psi'\mathbf{F}'H' + \text{h.c.} \tag{13}$$

$$\mathcal{L}_Y^{\rm mix} = \lambda_{FF'} F\Phi F' + \lambda_{FF'} \bar{F}'\Phi^\dagger \bar{F} + \frac{\lambda_{\Psi\Psi'}}{M} \Psi(\Phi^\dagger)^2\Psi' + \frac{\lambda_{\Psi\Psi'}}{M} \bar{\Psi}'\Phi^2\bar{\Psi}, \tag{14}$$

where M_*, M are some cut-off scales. The coupling matrices $\lambda_{FF'}$ and $\lambda_{\Psi\Psi'}$ are Hermitian due to the D_2 symmetry. The last two higher-order operators in eq. (14) are important for phenomenology. (They can be generated by integrating out some heavy states with mass at or above the GUT scale. See discussion in [5].)

For the components from Ψ, F, Ψ', F' states, we shall use the following notations:

$$\Psi = \{q, u^c, e^c\}, \quad F = \{l, d^c\}, \quad \Psi' = \{\hat{q}, \hat{u}^c, \hat{e}^c\}, \quad F' = \{\hat{l}, \hat{d}^c\}\,. \tag{15}$$

Substituting in eqs (12)–(14), the VEVs $\langle\Sigma\rangle, \langle\Sigma'\rangle$ and $\langle\Phi\rangle$, the relevant couplings obtained are

$$\mathcal{L}_Y \to q^T Y_U u^c h + q^T Y_D d^c h^\dagger + e^{cT} Y_{e^c l} l h^\dagger$$
$$+(C_{qq} qq + C_{u^c e^c} u^c e^c) T_H + (C_{ql} ql + C_{u^c d^c} u^c d^c) T_H^\dagger + \text{h.c.} \qquad (16)$$

$$\mathcal{L}_{Y'} \to C_{\Psi\Psi}^{(0)*} \left(\frac{1}{2}\hat{q}\hat{q} + \hat{u}^c \hat{e}^c\right) T_{H'}^\dagger + C_{\Psi F}^{(0)*} (\hat{q}\hat{l} + \hat{u}^c \hat{d}^c) T_{H'} + \text{h.c.} + \cdots \qquad (17)$$

$$\mathcal{L}_Y^{\text{mix}} \to \hat{l}^T M_{\hat{l}l} l + e^{cT} M_{e^c \hat{e}^c} \hat{e}^c + \text{h.c.} \qquad (18)$$

In eq. (17) we have dropped out the couplings with the Higgs doublet because, as we have assumed, $D_{H'}$ includes the SM Higgs doublet with very suppressed weight. Also, we have ignored powers of $\langle\Sigma'\rangle/M_*$ in comparison with $\langle\Sigma\rangle/M_*$'s exponents. As we shall see, the couplings of h in (16) and terms shown in eqs (17) and (18) are responsible for fermion masses and mixings and lead to realistic phenomenology.

2.2.1 *Quark masses and mixings.* Transformation properties of the quark states q, u^c, d^c coincide with those of the SM. Therefore, for quark masses and CKM mixings, the first two couplings of eq. (16) are relevant. As in $Y_{U,D}$ and $Y_{e^c l}$ contribute also higher-dimensional operators, Y_U is not symmetric and $Y_D \neq Y_{e^c l}$ [5a]. Thus, quark Yukawa matrices can be diagonalized by biunitary transformations $L_u^\dagger Y_U R_u = Y_U^{\text{Diag}}$ and $L_d^\dagger Y_D R_d = Y_D^{\text{Diag}}$. With these, the CKM matrix (in standard parametrization) is $V_{\text{CKM}} = P_1 L_u^T L_d^* P_2$, where P_1 and P_2 are some phase matrices.

2.2.2 *Composite leptons and their mass generation.* Note that $\hat{l}$ and $\hat{e}^c$ have opposite/conjugate transformation properties with respect to l and e^c, respectively. From couplings in eq. (18), we see that these vector-like states acquire masses $M_{\hat{l}l}$ and $M_{e^c\hat{e}^c}$ and decouple. However, within this scenario, composite leptons emerge. $SU(3)'$ becomes strongly coupled (discussed above) and confines at scale $\Lambda' \sim$ TeV. Because of confinement, $SU(3)'$ singlet composite states – baryons (B') and/or mesons (M') – can emerge. Within our scenario, lepton states turn out to be composite. As was shown in ref. [5] (see detailed discussion in this work), three families of l_0, e_0^c, ν_0^c are composite states

$$(\hat{q}\hat{q})\hat{q} \sim l_{0\alpha} = \begin{pmatrix} \nu_0 \\ e_0 \end{pmatrix}_\alpha ,$$
$$(\hat{q}^c\hat{q}^c)\hat{q}^c = \left((\hat{u}^c\hat{d}^c)\hat{d}^c,\ (\hat{u}^c\hat{d}^c)\hat{u}^c\right) \sim l_{0\alpha}^c \equiv (\nu_0^c,\ e_0^c)_\alpha, \qquad (19)$$

emerge, where $\alpha = 1, 2, 3$. Here, combinations $(\hat{q}\hat{q})\hat{q}$ and $(\hat{q}^c\hat{q}^c)\hat{q}^c$ stand for the spin-1/2 states with suppressed gauge and/or flavour indices. Because of the proper match of all quantum numbers, the states l_0 and e_0^c will be identified as three families of SM leptons. Besides these, we get three families of composite SM singlets fermions – ν_0^c. The latter will be treated as composite right-handed/sterile neutrinos. Note that, with this composition, as was expected, the gauge anomalies also vanish (together with the chiral anomaly matching (for details, see [5]).

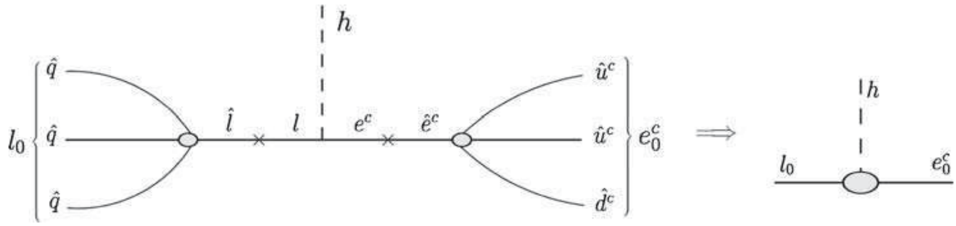

Figure 2. Diagram corresponding to the generation of charged lepton effective Yukawa matrix.

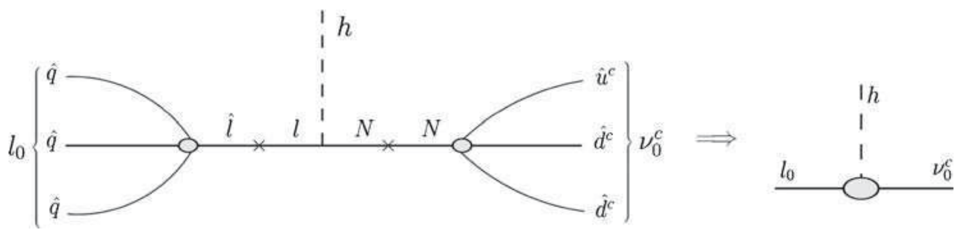

Figure 3. Diagram corresponding to the generation of effective Dirac Yukawa matrix for the neutrinos.

2.2.3 *Charged lepton masses.* Now, we turn to the masses of the charged leptons, which are composite within our scenario. As it turns out, their mass generation does not require additional extension. It happens via integration of the states that are present in the model. With the integration of $SU(3)'$ triplet scalar $T_{H'}$ with mass $M_{T_{H'}}$ and vector-like states $\hat{l}, l$ and e^c, $\hat{e}^c$ (with masses $M_{\hat{l}l}$ and $M_{e^c\hat{e}^c}$ respectively) through the coupling in eqs (16), (18) and (19), the effective Yukawa couplings are generated. The diagram corresponding to the generation of this effective Yukawa operator is shown in figure 2. This mechanism is novel and differs from those suggested earlier for the mass generation of composite fermions [7].

2.2.4 *Neutrino masses.* Within our model, among the composite fermions, we have SM singlets ν_0^c. Here, we stick to the possibility of the Dirac-type neutrino masses. Because of compositeness, there is no direct Dirac couplings Y_ν of ν_0^c's with lepton doublets l_0 and we need to generate this coupling. For this purpose, we introduce the $SU(5) \times SU(5)'$ singlet (two-component) fermionic states N with the D_2-parity transformations $N \rightleftarrows \bar{N}$. Thus, the relevant couplings will be $\mathcal{L}_N = C_{FN} F N H + C^*_{FN} F' N H'^{\dagger} - \frac{1}{2} N^T M_N N +$ h.c. with $M_N = M_N^*$. These give the following interaction terms: $\mathcal{L}_N \to C_{FN} l N h + C^*_{FN} \hat{d}^c N T^{\dagger}_{H'} - \frac{1}{2} N^T M_N N +$ h.c. With these, by integrating out the heavy $\hat{l}, l$ and N states, we get Dirac-type Yukawa coupling for the neutrinos. The relevant diagram is given in figure 3. By proper selection of appropriate coupling we get right neutrino mass scales to explain neutrino anomalies.

3. Various phenomenological implications and constraints

In this section, we discuss and summarize some phenomenological implications of our model, and constraints needed to be satisfied in order to be consistent with experiments. Also, we list issues opening the prospects for further investigations.

3.1 *Nucleon stability*

Although the given model has relatively low ($\simeq 5\cdot 10^{11}$ GeV) GUT scale, lepton compositeness plays a crucial role for achieving nucleon stability. Within our model, the baryon number violating $d=6$ operators, induced by integrating out of the X, Y bosons, are

$$\frac{g_X^2}{M_X^2}\times\{\tilde{\mathcal{C}}^{(e^c)}_{\alpha\beta}(\bar{u^c}\gamma_\mu u)(\bar{e^c_\alpha}\gamma^\mu d_\beta),\;\; \mathcal{C}^{(e)}_{\alpha\beta}(\bar{u^c}\gamma_\mu u)(\bar{d^c}_\beta\gamma^\mu e_\alpha),$$
$$\mathcal{C}^{(\nu)}_{\alpha\beta\gamma}(\bar{u^c}\gamma_\mu d_\alpha)(\bar{d^c}_\beta\gamma^\mu \nu_\gamma)\} \tag{20}$$

with

$$\begin{aligned}\mathcal{C}^{(e^c)}_{\alpha\beta} &= (R_u^\dagger L_u^*)_{11}\left(R_e^\dagger\tilde{\mu}^*\frac{1}{M^*_{e^c\hat{e}^c}}L_u^*P_1^*V_{CKM}\right)_{\alpha\beta}\\ &\quad+(R_u^\dagger L_u^*P_1^*V_{CKM})_{1\beta}\left(R_e^\dagger\tilde{\mu}^*\frac{1}{M^*_{e^c\hat{e}^c}}L_u^*\right)_{\alpha 1}\\ \mathcal{C}^{(e)}_{\alpha\beta} &= (R_u^\dagger L_u^*)_{11}\left(R_d^\dagger\frac{1}{M_{\hat{l}l}}\hat{\mu}L_e^*\right)_{\beta\alpha},\\ \mathcal{C}^{(\nu)}_{\alpha\beta\gamma} &= (R_u^\dagger L_u^*P_1^*V_{CKM})_{1\alpha}\left(R_d^\dagger\frac{1}{M_{\hat{l}l}}\hat{\mu}L_e^*\right)_{\beta\gamma}.\end{aligned} \tag{21}$$

With proper selection of appropriate parameters, i.e., $\tilde{\mu}(1/M_{e^c\hat{e}^c})$, $(1/M_{\hat{l}l})\hat{\mu}$ (see ref. [5] for definitions), and/or the corresponding entries in some of the unitary matrices, we can adequately suppress nucleon decays [7a]. With the notations $R_u^\dagger L_u^*\equiv\mathcal{U}$, $R_d^\dagger(1/M_{\hat{l}l})\hat{\mu}L_e^*\equiv\mathcal{L}$, $R_e^\dagger\tilde{\mu}^*(1/M^*_{e^c\hat{e}^c})L_u^*\equiv\mathcal{R}$, and selection

$$\mathcal{U}_{11}=0,\quad \mathcal{L}=\begin{pmatrix}\epsilon_1 & \epsilon_2 & \epsilon_3\\ \times & \times & \times\\ \times & \times & \times\end{pmatrix},\quad \mathcal{R}=\begin{pmatrix}0 & \times & \times\\ 0 & \times & \times\\ \times & \times & \times\end{pmatrix}, \tag{22}$$

($\times$ stands for some nonzero entry), we have $\mathcal{C}^{(e^c)}_{\alpha\beta}=\mathcal{C}^{(e)}_{\alpha\beta}=0$ (for $\alpha,\beta=1,2$), and therefore nucleon decays with the emission of charged leptons do not take place. Moreover, by proper selection of $\mathcal{U}_{12}$ and $\mathcal{U}_{13}$ we can achieve $(\mathcal{U}P_1^*V_{CKM})_{11}=0$. The latter leads to $\mathcal{C}^{(\nu)}_{12\gamma}=\mathcal{C}^{(\nu)}_{11\gamma}=0$. Thus, the decays $p\to\bar{\nu}\pi^+$, $n\to\bar{\nu}\pi^0$, $n\to\bar{\nu}\eta$ also do not take place. Thus, we remain with $p\to\bar{\nu}K^+$ and $n\to\bar{\nu}K^0$ decays, with the corresponding widths given by

$$\Gamma(p\to\bar{\nu}K^+)\simeq\Gamma(n\to\bar{\nu}K^0)=5.9\cdot 10^{33}\text{ y}\times\left(\sum_{\gamma=1}^{3}|\epsilon_\gamma|^2/(2.3\cdot 10^{-11})\right). \tag{23}$$

Note that the selection $\sqrt{|\epsilon_1|^2+|\epsilon_3|^2+|\epsilon_3|^2} \lesssim 4.8\cdot 10^{-6}$ (needed for the nucleon stability) is fully consistent with the fermion Yukawa sector. However, as it turns out, it would be more natural to have $\sqrt{|\epsilon_1|^2+|\epsilon_3|^2+|\epsilon_3|^2} \gtrsim \sqrt{3}\cdot 10^{-6}$. This dictates an upper bound for the proton lifetime $\tau_p = \tau(p\to\bar{\nu}K^+) \lesssim 5\cdot 10^{34}$ y and will allow us to test the model in the future [10].

3.2 *Higgs vacuum stability*

Within our model, above the Λ' scale, new states appear, and so the problem with the Higgs vacuum stability can be avoided. The coupling $\lambda_H(H^\dagger H)^2$ gives the self-interaction term $\lambda_h(h^\dagger h)^2$ (with $\lambda_h \approx \lambda_H$ at the GUT scale). The running of λ_h will be given by

$$16\pi^2\frac{\mathrm{d}}{\mathrm{d}t}\lambda_h = \beta^{\mathrm{SM}}_{\lambda_h} + \Delta\beta_{\lambda_h},$$

where $\beta^{\mathrm{SM}}_{\lambda_h}$ corresponds to the SM part, while $\Delta\beta_{\lambda_h}$ accounts for new contributions:

$$\begin{aligned}\Delta\beta_{\lambda_h} \approx{}& \frac{(\lambda_{1H\Phi})^2}{25}\,[9\theta(\mu-M_{TT'}) \\ &+ 6\theta(\mu-M_{DT'}) + 6\theta(\mu-M_{TD'}) + 4\theta(\mu-M_{DD'})] \\ &\times\frac{(\lambda_{2H\Phi})^2}{10}\,[3\theta(\mu-M_{DT'}) + 2\theta(\mu-M_{DD'})] \\ &+ 3\hat{h}^2\theta(\mu-M_{T_{H'}}) + \cdots .\end{aligned} \tag{24}$$

Detailed analysis requires numerical studies by solving the system of coupled RG equations (involving multiple couplings). While this is beyond the scope of this work, we see that due to positive contributions into the β function, there is potential to prevent λ_h becoming negative all the way up to the Planck scale.

3.3 *Anomalous magnetic moments and LFV rare decays*

Lepton compositeness give additional contributions to the leptons' anomalous magnetic moments [11]: $\delta a_\alpha \sim \left(m_{e_\alpha}/\Lambda'\right)^2$. Current experimental measurements [12] of the muon anomalous magnetic moment give $\Delta a_\mu^{\exp} \approx 6\cdot 10^{-10}$. This, with a possible range $\sim(1/5-1)$ of an undetermined prefactor, constrains the scale: $\Lambda' \gtrsim (1.8–4.3)$ TeV. The selected value of Λ', within our model ($\Lambda' = 1851$ GeV), fits well with this bound, and has the potential of resolving a_μ's 3–4σ discrepancy between the theory and the experiment [12,13]. The value of δa_e is more suppressed (for $\Lambda' \simeq 1.8$ TeV, we get $\delta a_e \sim 10^{-13}$) and is compatible with experiments ($\Delta a_e^{\exp} \approx 2.7\cdot 10^{-13}$). Planned measurements [14] with reduced uncertainties will provide severe constraints and test the viability of the proposed scenario.

Similarly, having flavour-violating couplings at the level of constituents (i.e., in the sector of $SU(3)'$ fermions $\hat{q}, \hat{u}^c, \hat{d}^c$), the new contribution in LFV $e_\alpha \to e_\beta\gamma$ rare decay processes will emerge. For instance, the contribution in the $\mu \to e\gamma$ transition amplitude will be $\sim\lambda_{12}(m_\mu/(\Lambda')^2)$, where λ_{12} is the (unknown) flavour-violating coupling coming from the Yukawa sector of $\hat{q}, \hat{u}^c, \hat{d}^c$. This gives $\mathrm{Br}(\mu \to e\gamma) \sim \lambda_{12}^2(M_W/\Lambda')^4$, and for $\Lambda' \simeq 1.8$ TeV the constraint $\lambda_{12} \lesssim 4 \cdot 10^{-4}$ should be satisfied in order to be consistent with the latest experimental limit $\mathrm{Br}^{\mathrm{exp}}(\mu \to e\gamma) < 5.7 \cdot 10^{-13}$ [15].

3.4 *EW precision parameters (T, S, U)*

New states around and above the $\Lambda' \approx 1.8$ TeV scale, will give additional corrections to the EW precision parameters T, S, U etc. Symmetry arguments provide a good estimate of the additional corrections ΔT, ΔS, etc. The isospin breaking effects are suppressed in the sector of additional states. Therefore, the mass splittings between the doublet components of the additional states will be suppressed (i.e. $\Delta M \ll M$) and pieces ΔT_f, ΔT_s of $\Delta T = \Delta T_f + \Delta T_s$ will be given as [16]

$$\Delta T_f \simeq \frac{N_f}{12\pi s_W^2}\left(\frac{\Delta M_f}{m_W}\right)^2, \qquad \Delta T_s \simeq \frac{N_s}{24\pi s_W^2}\left(\frac{\Delta M_s}{m_W}\right)^2, \tag{25}$$

where subscripts f and s stand for fermions and scalars, respectively and N_f, N_s account for the multiplicity of the corresponding doublet state. Within our model, in the sector of extra vector-like $(\hat{l} + l)_\alpha$ states, the mass splitting between the doublet components is suppressed as $\Delta M_{\hat{l}l}^{(\alpha)} \lesssim (v_h^2/M_{\hat{l}l}^{(\alpha)})$. This, according to eq. (25) and table 1, gives a negligible contribution: $\Delta T_{\hat{l}l} \lesssim (2 \cdot 2/12\pi s_W^2)v_h^4/(m_W M_{\hat{l}l}^{(1)})^2 \sim 10^{-5}$. Within the fragments of the scalar Φ, the lightest is $\Phi_{DT'}$ with mass $M_{DT'} \simeq 8.3$ TeV. Splitting between the doublet components comes from the potential term $(\lambda_{2H\Phi}/\sqrt{10})H^\dagger\Phi\Phi^\dagger H$, giving $\Delta M_{DT'} \simeq \lambda_{2H\Phi}v_h^2/(4\sqrt{10}M_{DT'})$. This causes enough suppression: $\Delta T_{DT'} \lesssim 2 \cdot 10^{-5}$ (for $\lambda_{2H\Phi} \lesssim 1.5$). Besides the fundamental Higgs doublet (h), which dominantly includes SM Higgs, there is a composite doublet (π' – similar to technicolour models) with suppressed VEV – $F_{\pi'}$. Contribution of this extra doublet, into the T parameter, is estimated to be

$$\Delta T_{\pi'} \approx \frac{1}{24\pi s_W^2}\left(\frac{\Delta M_{\pi'}}{m_W}\right)^2 - \frac{c_W^2}{4\pi}c_{\pi'}^2 \ln\frac{M_{\pi'}^2}{m_Z^2}\,, \tag{26}$$

where the first term is due to the mass splitting $\Delta M_{\pi'}$ $(\sim v_h^2/(4M_{\pi'}))$ between the doublet components of π', while second term emerges due to the VEV $\langle\pi'\rangle = F_{\pi'}$ with $c_{\pi'} \approx 2m_Z^2F_{\pi'}/(M_{\pi'}^2v_h)$ (where $F_{\pi'} \lesssim 0.2v_h$). Moreover, the source of the isospin breaking in the strong $SU(3)'$ sector is $F_{\pi'} \lesssim 0.2v_h$, causing mass splitting between composite 'technihadrons' (denoted collectively as $\{\rho'\}$) of $\Delta M_{\rho'} \sim F_{\pi'}^2/M_{\rho'}$. This, for $M_{\rho'} \sim \Lambda'$, would give the correction $\Delta T_{\rho'} \lesssim 10^{-5}$. Thus, we conclude that within the considered scenario, extra corrections to the T parameter are under control.

Contributions to the S parameter from the additional vector-like $(\hat{l}+l)_\alpha$, $(\hat{e}^c+e^c)_\alpha$ states decouple [17] and are estimated to be

$$\Delta S_{\hat{l}l} \sim \Delta S_{\hat{e}^c e^c} \lesssim \frac{1}{4\pi}\frac{v_h^2}{(M_{\hat{l}l}^{(1)})^2}\ln\frac{M_{\hat{l}l}^{(1)}}{m_\tau} \sim 10^{-5}.$$

Also, as it turns out, the contribution from the scalar $\Phi_{DT'}$ is $\Delta S_{DT'} \lesssim 2\cdot 10^{-5}$ which also is suppressed. The contribution of extra (heavy π') composite doublet is

$$\Delta S_{\pi'} \approx \frac{1}{6\pi}\frac{\Delta M_{\pi'}}{M_{\pi'}} + \frac{1}{6\pi}c_{\pi'}^2 \ln\frac{M_{\pi'}}{m_h}\ . \tag{27}$$

With $\Delta M_{\pi'} \sim v_h^2/(4M_{\pi'})$ and $M_{\pi'} \gtrsim 1$ TeV, eq. (27) gives $\Delta S_{\pi'} \lesssim 10^{-3}$. Similarly, suppressed contributions would arise from the techni-ρ' hadrons, are estimated to be: $\Delta S_{\rho'} \lesssim 4\cdot 10^{-5}$ (for $M_{\rho'} \sim \Lambda'$).

As far as the contribution from the matter states $\hat{q}$, $\hat{u}^c$, $\hat{d}^c$ are concerned, as their masses are too suppressed, in the chiral limit $(m_f/m_Z) \to 0$ we have $\Delta S_f \to 0$. Moreover, all new contributions to the U parameter are more suppressed [5]. This is understandable as U is related to the effective operator with a dimension higher than those of S and T. Thus, we conclude that new contributions to the EW precision parameters are well below the current experimental bounds [18].

3.5 *Other constraints and implications*

(i) As discussed in [5], the matter sector of $SU(3)'$ symmetry (ignoring EW and Yukawa interactions) possesses $G_f^{(6)}$ chiral symmetry with sextets $6_L \sim \hat{q}_\alpha$ and $6_R \sim \hat{q}_\alpha^c$. The breaking of this chiral symmetry proceeds by several steps. At the first stage, at scale $\Lambda' \approx 1.8$ TeV, the condensates $\langle 6_L 6_L T_{H'}^\dagger\rangle \sim \langle 6_R 6_R T_{H'}\rangle \sim \Lambda'$ break $G_f^{(6)}$. However, these condensates preserve SM gauge symmetry. At the next stage (of chiral symmetry breaking), the condensate $\langle 6_L 6_R\rangle \equiv F_{\pi'}$, together with the Higgs VEV $\langle h\rangle \equiv v_h$, contributes to the EW symmetry breaking. $F_{\pi'}$ denotes the decay constant of the (techni) π' meson and should satisfy $v_h^2 + F_{\pi'}^2 = (246.2\ \text{GeV})^2$. With the light (very SM-like) Higgs boson mainly residing in h and with $F_{\pi'} \lesssim 0.2 v_h$, the h's signal will be very compatible with LHC data. As the low-energy potential involves VEVs $\langle 6_L 6_L T_{H'}^\dagger\rangle$, $\langle 6_R 6_R T_{H'}\rangle$, $F_{\pi'}$ and v_h, obtaining mild hierarchy $(F_{\pi'}/\Lambda') \lesssim 1/40$ will be possible by proper selection (not by severe fine-tunings) of parameters from perturbative and nonperturbative (effective) potentials. Our approach is rather phenomenological and assume $F_{\pi'}/v_h \lesssim 0.2$ and h being the Higgs boson (with mass $\approx$ 126 GeV). So, there is an allowed window for a heavier π' state and the model is compatible with current experiments. Models with partially composite Higgs, in which the light Higgs doublet has some ed-mixture of a composite (technipion π') state, with various interesting implications (including necessary constraints, limits and compatibility with LHC data), were studied in ref. [19].

In addition, it is rather generic that the model with composite leptons will be accompanied with excited massive leptons (lepton resonances). Current experiments have placed low bounds on masses of such states to be heavier than ∼1.8 TeV. This scale is close to the value of Λ' we have chosen within our model, and will allow us to test the lepton substructure [20] hopefully in the not-far future.

(ii) As the condensate $\langle 6_L 6_R \rangle = F_{\pi'}$, by some amount, can contribute to the chiral (of the $SU(3)'$ strong sector) and EW symmetry breaking, the scenario shares some properties of hybrid technicolour models with fundamental Higgs states. Moreover, together with technipion π', near the Λ' scale, there will be technimeson states ρ_T, ω_T, etc., with peculiar signatures [21,22], which can be probed by collider experiments.

(iii) Within the proposed model, spontaneous breaking of two non-Abelian groups $SU(5) \times SU(5)'$ and discrete D_2 parity will give monopole and domain wall solutions, respectively. As the symmetry breaking scales are relatively low ($\lesssim 5 \cdot 10^{11}$ GeV), the inflation would not dilute the number densities of these topological defects in a straightforward way. Thus, one can think of alternative solutions. For instance, as shown in ref. [23], within models with a certain field content and couplings, it is possible that symmetry restoration cannot happen for arbitrary high temperatures. This would avoid phase transitions (which usually cause the formation of topological defects). Moreover, by proper selection of the model parameters, it is possible to suppress the thermal production rates of the topological defects (for detailed discussions, see the last two works of ref. [23]). From this viewpoint, our model with a multiscalar sector and various couplings has the potential to avoid domain wall and monopole problems. Thus, it is interesting to investigate the parameter space and see how desirable ranges are compatible with those needed values appearing in eq. (24) (for improving the running of λ_h).

To cure problems related with topological defects, other different non-inflationary solutions have also been proposed [24], and one (if not all) of them could be invoked as well.

Certainly, these and other cosmological implications of the presented scenario, deserve separate investigations.

Acknowledgements

The author thanks Charanjit Aulakh and other organizers of UNICOS 2014 for their kind invitation and for providing a possibility to give the webcam presentation. This research is partially supported by Shota Rustaveli National Science Foundation (Contracts No. 31/89 and No. DI/12/6-200/13).

References

[1] ATLAS Collaboration: G Aad *et al*, *Phys. Lett. B* **716**, 1 (2012)
CMS Collaboration: S Chatrchyan *et al*, *Phys. Lett. B* **716**, 30 (2012)

[2] D Buttazzo, G Degrassi, P P Giardino, G F Giudice, F Sala, A Salvio and A Strumia, *J. High Energy Phys.* **1312**, 089 (2013)

[3] J C Pati and A Salam, *Phys. Rev. Lett.* **31**, 661 (1973); *Phys. Rev. D* **10**, 275 (1974)

[4] H Georgi and S L Glashow, *Phys. Rev. Lett.* **32**, 438 (1974)
H Georgi, H R Quinn and S Weinberg, *Phys. Rev. Lett.* **33**, 451 (1974)

[5] Z Tavartkiladze, *Phys. Rev. D* **90**(1), 015022 (2014)

[5a] Thus degeneracies between quark and lepton masses are avoided without invoking additional mechanisms [6]

[6] E Witten, *Phys. Lett. B* **91**, 81 (1980)
J R Ellis and M K Gaillard, *Phys. Lett. B* **88**, 315 (1979)
H Georgi and C Jarlskog, *Phys. Lett. B* **86**, 297 (1979)
For recent discussion and references, see K S Babu, B Bajc and Z Tavartkiladze, *Phys. Rev. D* **86**, 075005 (2012)

[7] For reviews, see M E Peskin, *Compositeness of quarks and leptons,* eConf C **810824**, 880 (1981)
S Raby, *Light composite fermions: An overview,* LA-UR-82-1416
L Lyons, *Prog. Part. Nucl. Phys.* **10**, 227 (1983)
R N Mohapatra, *Unification and supersymmetry. The frontiers of quark–lepton physics* (Springer, New York, 2003) p. 421

[7a] The importance of flavour dependence in $d = 6$ nucleon decay was discussed in refs [9] and [8]. As was shown [8], in specific circumstances, within GUTs one can suppress or even completely rotate away the $d = 6$ nucleon decays

[8] C Jarlskog, *Phys. Lett. B* **82**, 401 (1979)
R N Mohapatra, *Phys. Rev. Lett.* **43**, 893 (1979)
S Nandi, A Stern and E C G Sudarshan, *Phys. Lett. B* **113**, 165 (1982)
Q Shafi and Z Tavartkiladze, *Phys. Lett. B* **451**, 129 (1999); *Nucl. Phys. B* **573**, 40 (2000)
I Dorsner and P Fileviez Perez, *Phys. Lett. B* **605**, 391 (2005); *Phys. Lett. B* **606**, 367 (2005)
Z Tavartkiladze, *Phys. Rev. D* **76**, 055012 (2007)
K-S Choi, *Phys. Lett. B* **668**, 392 (2008)

[9] A De Rujula, H Georgi and S L Glashow, *Phys. Rev. Lett.* **45**, 413 (1980)
M Yoshimura, *Prog. Theor. Phys.* **64**, 1756 (1980)
D B Reiss and S Rudaz, *Phys. Rev. D* **30**, 118 (1984)

[10] K S Babu *et al*, arXiv:1311.5285 [hep-ph]
S Raby *et al*, arXiv:0810.4551 [hep-ph]

[11] R Barbieri, L Maiani and R Petronzio, *Phys. Lett. B* **96**, 63 (1980)
S J Brodsky and S D Drell, *Phys. Rev. D* **22**, 2236 (1980)

[12] Particle Data Group Collaboration: J Beringer *et al*, *Phys. Rev. D* **86**, 010001 (2012)

[13] M Passera, W J Marciano and A Sirlin, *Chin. Phys. C* **34**, 735 (2010), arXiv:1001.4528 [hep-ph]; *AIP Conf. Proc.* **1078**, 378 (2009), arXiv:0809.4062 [hep-ph]

[14] Fermilab P989 Collaboration: B Lee Roberts, *Nucl. Phys. Proc. Suppl.* **218**, 237 (2011)
J-PARC g-2/EDM Collaboration: N Saito, *AIP Conf. Proc.* **1467**, 45 (2012)

[15] MEG Collaboration: J Adam *et al*, *Phys. Rev. Lett.* **110**, 201801 (2013)

[16] M J G Veltman, *Nucl. Phys. B* **123**, 89 (1977)
M B Einhorn, D R T Jones and M J G Veltman, *Nucl. Phys. B* **191**, 146 (1981)
M E Peskin and T Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990)
H-J He, N Polonsky and S-f Su, *Phys. Rev. D* **64**, 053004 (2001)

[17] L Lavoura and J P Silva, *Phys. Rev. D* **47**, 2046 (1993)
G Cynolter and E Lendvai, *Eur. Phys. J. C* **58**, 463 (2008)
S Dawson and E Furlan, *Phys. Rev. D* **86**, 015021 (2012)

[18] M Baak, M Goebel, J Haller, A Hoecker, D Ludwig, K Moenig, M Schott and J Stelzer, *Eur. Phys. J. C* **72**, 2003 (2012)

[19] C D Carone and M Golden, *Phys. Rev. D* **49**, 6211 (1994)
C D Carone, *Phys. Rev. D* **86**, 055011 (2012)
S Bar-Shalom, arXiv:1310.2942 [hep-ph]
T Abe and R Kitano, *Phys. Rev. D* **88**, 015019 (2013)

[20] E J Eichten, K D Lane and M E Peskin, *Phys. Rev. Lett.* **50**, 811 (1983)
[21] ATLAS Collaboration: N T Hod, *EPJ Web Conf.* **49**, 15004 (2013)
ATLAS Collaboration: G Aad *et al*, *J. High Energy Phys.* **1211**, 138 (2012)
CMS Collaboration: S Chatrchyan *et al*, *Phys. Rev. Lett.* **109**, 141801 (2012); and references therein
[22] E Eichten, K Lane, A Martin and E Pilon, *Phys. Rev. D* **86**, 074015 (2012); arXiv:1201.4396 [hep-ph]
[23] P Salomonson, B S Skagerstam and A Stern, *Phys. Lett. B* **151**, 243 (1985)
G R Dvali and G Senjanovic, *Phys. Rev. Lett.* **74**, 5178 (1995)
G R Dvali, A Melfo and G Senjanovic, *Phys. Rev. Lett.* **75**, 4559 (1995)
[24] P Langacker and S-Y Pi, *Phys. Rev. Lett.* **45**, 1 (1980)
V V Dixit and M Sher, *Phys. Rev. Lett.* **68**, 560 (1992)
T H Farris, T W Kephart, T J Weiler and T C Yuan, *Phys. Rev. Lett.* **68**, 564 (1992)
G R Dvali, H Liu and T Vachaspati, *Phys. Rev. Lett.* **80**, 2281 (1998)
B Bajc, A Riotto and G Senjanovic, *Phys. Rev. Lett.* **81**, 1355 (1998) and references therein