



Dynamical generation of flavour

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Abstract. We propose the generation of Standard Model fermion hierarchy by the extension of renormalizable $SO(10)$ GUT with $O(N_g)$ family gauge symmetry. In this scenario, Higgs representations of $SO(10)$ also carry family indices and are called Yukawons. Vacuum expectation values of these Yukawon fields break GUT and family symmetry and generate MSSM Yukawa couplings dynamically. We have demonstrated this idea using $\mathbf{10} \oplus \mathbf{210} \oplus \mathbf{126} \oplus \mathbf{126}$ Higgs irrep, ignoring the contribution of $\mathbf{120}$ -plet which is, however, required for complete fitting of fermion mass-mixing data. The effective MSSM matter fermion couplings to the light Higgs pair are determined by the null eigenvectors of the MSSM-type Higgs doublet superfield mass matrix \mathcal{H} . A consistency condition on the doublet $([1, 2, \pm 1])$ mass matrix ($\text{Det}(\mathcal{H}) = 0$) is required to keep one pair of Higgs doublets light in the effective MSSM. We show that the Yukawa structure generated by null eigenvectors of \mathcal{H} are of generic kind required by the MSSM. A hidden sector with a pair of $(S_{ab}; \phi_{ab})$ fields breaks supersymmetry and facilitates $D_{O(N_g)} = 0$. SUSY breaking is communicated via supergravity. In this scenario, matter fermion Yukawa couplings are reduced from 15 to just 3 parameters in MSGUT with three generations.

Keywords. Supersymmetry; grand unified theory; Yukawon.

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1. Introduction

The Standard Model (SM) fermion mass-mixing data pose several questions. Fermion masses vary from milli-eV for neutrinos to between 0.5 MeV and 174 GeV for charged fermions. Leptonic mixing is large as compared to quark sector mixing. Why do we have three fermion generations? Do they follow some flavour symmetry? The mass hierarchy is different for up-type quarks, down-type quarks and for leptons. All these questions constitute the flavour puzzle of the SM and neutrino oscillations data. Introduction of family symmetry and generation of flavour structure by Yukawa couplings arising as vacuum expectation values (VEVs) of ‘spurion’ fields offers an attractive alternative prospect for understanding flavour structure [1]. Model builders have considered various

possibilities like discrete (tetrahedral group A_4 and permutation group S_3), Abelian/non-Abelian (global or local) symmetries. The establishment of the lepton mixing pattern triggered great interest in the discrete family symmetry approach (for reviews, see [2–4]). In the so-called Yukawa-on models [5], a different symmetry is considered for each type of fermion. The dimension-1 Yukawa-on field (\mathcal{Y}) makes the Higgs vertex non-renormalizable ($\mathcal{L} = f^c \mathcal{Y} f H / \Lambda_{\mathcal{Y}} + \dots$) and Yukawa-on dynamics is controlled by a high-scale $\Lambda_{\mathcal{Y}}$.

The presence of the right-handed neutrino in the **16**-plet makes $SO(10)$ GUTs the natural home for neutrino masses through see-saw mechanism. The model based on the **10** \oplus **210** \oplus **126** \oplus **126** Higgs irreducible representations (irreps) is called minimal supersymmetric $SO(10)$ GUT (MSGUT) [6–9]. The **210** \oplus **126** \oplus **126** Higgs irreps break symmetry from $SO(10)$ to MSSM in steps or at once. In MSGUT, MSSM Higgs pair emerges from the multiple MSSM-type doublets of UV theory and fermion hierarchy is generated by the $SO(10)$ matter Yukawa couplings. Realistic neutrino and charged fermion mass-mixing data require the addition of **120**-plet in the MSGUT and this completed model is known as NMSGUT [9]. $SO(10)$ GUTs along with gauge unification also unify third-generation Yukawa at large $\tan \beta$. In our view, this is the strongest motivation and hint for the flavour symmetry. It indicates that the GUT gauge symmetry breaking may generate the fermion hierarchy. Combining this hint with the successful fitting of the fermion data in the NMSGUT motivated us to extend the minimal renormalizable supersymmetric $SO(10)$ GUT with $O(N_g)$ family group [10]. Higgs multiplets of $SO(10)$ become symmetric representation of family group (Yukawons) and their VEVs generate Yukawa couplings of SM fermion and neutrino. In our study, Yukawons also carry representation of the gauge (SM/GUT) dynamics. NMSGUT can generate realistic fermion mass mixing data and experiment compatible B -decay rates after the inclusion of superheavy thresholds [11]. Therefore, from our viewpoint of combined family and GUT unification, it is the logical base for a dynamical theory of flavour. To start with, we study the extension of MSGUT. We shall comment on the minor changes required to include the **120**-plet, which may ultimately be necessary.

2. Yukawon ultraminimal GUTs

Yukawon ultraminimal GUTs are extensions of minimal supersymmetric $SO(10)$ model by $O(N_g)$ family gauge group. The **10**(**H**) \oplus **210**(**Φ**) \oplus **126**(**$\bar{\Sigma}$**) \oplus **126**(**Σ**) Higgs irreps become symmetric representations of $O(N_g)$ family group. Matter fermions are present in the form of three copies of **16**(**Ψ**)-plet. Superpotential of the model has the same form as that of the MSGUT (with sum over flavour indices):

$$\begin{aligned}
 W_{\text{GUT}} &= \text{Tr}(m\Phi^2 + \lambda\Phi^3 + M\bar{\Sigma} \cdot \Sigma + \eta\Phi \cdot \bar{\Sigma} \cdot \Sigma + \Phi \cdot H \cdot (\gamma\Sigma + \bar{\gamma} \cdot \bar{\Sigma}) \\
 &\quad + M_H H \cdot H) \\
 W_{\text{F}} &= \Psi_A \cdot (hH_{AB} + f\Sigma_{AB} + g\Theta_{AB})\Psi_B.
 \end{aligned}
 \tag{1}$$

Here A and B are the family indices. Now $SO(10)$ Yukawa couplings h, f, g are complex numbers because flavour indices are carried by MSGUT Higgs irreps themselves. Here

we have included $\mathbf{120}(\odot)$ -plet in W_F but for simplicity we study only MSGUTs. However, the addition of $\mathbf{120}$ -plet does not effect GUT SSB as it does not contain any MSSM singlet. Notice that $\mathbf{120}$ -plet carries antisymmetric representation of the family group. In this scenario, matter fermion Yukawa couplings are reduced from 15(21) to just 3(5) parameters in MSGUT (NMSGUT) with three generations and so we call it Yukawon ultraminimal grand unified theory (YUMGUTs). Each Higgs irrep contains one MSSM Higgs-type multiplet ($[1, 2, \pm 1]$). Mass matrix is given as

$$\mathcal{H} = \begin{pmatrix} -M_H & \bar{\gamma}\sqrt{3}\Omega(\omega - a) & -\gamma\sqrt{3}\Omega(\omega + a) & -\bar{\gamma}\Omega(\bar{\sigma}) \\ \gamma\sqrt{3}\Omega(\omega - a) & -(2M + 4\eta\Omega(a - \omega)) & \emptyset_d & -2\eta\sqrt{3}\Omega(\bar{\sigma}) \\ -\bar{\gamma}\sqrt{3}\Omega(\omega + a) & \emptyset_d & -(2M + 4\eta\Omega(\omega + a)) & \emptyset_d \\ -\gamma\Omega(\sigma) & -2\eta\sqrt{3}\Omega(\sigma) & \emptyset_d & 6\lambda\Omega(\omega - a) - 2m \end{pmatrix}. \quad (2)$$

The rows are labelled by the $N_g(N_g + 1)/2$ -tuples (ordered and normalized, for a symmetric ϕ_{AB} , $A, B = 1 \dots N_g$, as $\{\phi_{11}, \phi_{22}, \dots, \phi_{N_g N_g}, \sqrt{2}\phi_{12}, \sqrt{2}\phi_{13}, \dots, \sqrt{2}\phi_{N_g-1, N_g}\}$) containing MSSM-type $\bar{H}[1, 2, -1]$ doublets from $\mathbf{10}$, $\mathbf{126}$, $\mathbf{126}$, $\mathbf{210}$. The columns represent $H[1, 2, 1]$ doublets in the order $\mathbf{10}$, $\mathbf{126}$, $\mathbf{126}$, $\mathbf{210}$. \emptyset_d is the d -dimensional null square matrix. The matrix function Ω ($N_g(N_g + 1)/2$ dimensional) is introduced to write \mathcal{H} in compact notations and its form is determined by symmetric invariant $\phi_{AB}\phi_{BC}\phi_{CA}$ (here one field can have VEV and the other two should contain H and \bar{H}). For $N_g = 2$ it is

$$\Omega[V] = \begin{pmatrix} V_{11} & 0 & V_{12}/\sqrt{2} \\ 0 & V_{22} & V_{12}/\sqrt{2} \\ V_{12}/\sqrt{2} & V_{12}/\sqrt{2} & (V_{11} + V_{22})/2 \end{pmatrix}, \quad (3)$$

with labels $\{\bar{H}_{11}, \bar{H}_{22}, \sqrt{2}\bar{H}_{12}\} \oplus \{H_{11}, H_{22}, \sqrt{2}H_{12}\}$. Higgs mass matrix is now $2N_g(N_g + 1)$ -dimensional which would become $N_g(3N_g + 1)$ -dimensional if we include $\mathbf{120}$ -plet. MSSM being an effective theory requires one light Higgs pair out of these large number of Higgs multiplets. Consistency condition of the light Higgs pair assumption (fine tuning $\text{Det } \mathcal{H} = 0$) ensures this. From left (\hat{W}) and right (\hat{V}) null eigenvectors, we can determine MSSM Yukawa couplings. For $N_g = 2$ Yukawas of up- and down-type quarks, we get

$$Y_u = \begin{pmatrix} \hat{h}\hat{V}_1 + \hat{f}\hat{V}_4 & (\hat{h}\hat{V}_3 + \hat{f}\hat{V}_6)/\sqrt{2} \\ (\hat{h}\hat{V}_3 + \hat{f}\hat{V}_6)/\sqrt{2} & \hat{h}\hat{V}_2 + \hat{f}\hat{V}_5 \end{pmatrix}; \quad \hat{h} = 2\sqrt{2}h, \\ Y_d = \begin{pmatrix} \hat{h}\hat{W}_1 + \hat{f}\hat{W}_7 & (\hat{h}\hat{W}_3 + \hat{f}\hat{W}_9)/\sqrt{2} \\ (\hat{h}\hat{W}_3 + \hat{f}\hat{W}_9)/\sqrt{2} & \hat{h}\hat{W}_2 + \hat{f}\hat{W}_8 \end{pmatrix}; \quad \hat{f} = -4i\sqrt{\frac{2}{3}}f. \quad (4)$$

By replacing $\hat{f} \rightarrow -3\hat{f}$ in Y_u, Y_d we get Y_v, Y_l . Clearly for $f \sim h$, one can get Y_v, Y_l different from Y_u, Y_d , while $f \ll h$ implies $Y_u \approx Y_v$ and $Y_d \approx Y_l$. The MSSM-type Higgs mass matrix (and consequently \hat{V}, \hat{W}) is determined in terms of symmetry breaking VEVs ($p, a, \omega, \sigma, \bar{\sigma}$). The next step is to calculate these VEVs.

3. Spontaneous symmetry breaking

The multiplets **210**, $\overline{126}$, **126** break the GUT and flavour symmetry to MSSM. YUMGUT superpotential written in terms of VEVs $((p, a, \omega) \in 210, \sigma \in 126, \bar{\sigma} \in \overline{126})$ of SM singlets:

$$W = \text{Tr}[m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2)] + \text{Tr} \left[M\bar{\sigma}\sigma + \eta(p + 3a - 6\omega) \frac{(\bar{\sigma}\sigma + \sigma\bar{\sigma})}{2} \right]. \quad (5)$$

SUSY vacuum is determined by the vanishing of F and D terms. The F -term vanishing equations can be written as

$$2m(p - a) - 2\lambda a^2 + 2\lambda\omega^2 = 0, \quad (6)$$

$$2m(p + \omega) + \lambda(p + 2a + 3\omega)\omega + \lambda\omega(p + 2a + 3\omega) = 0, \quad (7)$$

$$M\sigma + \eta(\chi\sigma + \sigma\chi)/2 = 0, \quad (8)$$

$$M\bar{\sigma} + \eta(\chi\bar{\sigma} + \bar{\sigma}\chi)/2 = 0, \quad (9)$$

$$\bar{\sigma}\sigma + \sigma\bar{\sigma} = -\frac{4}{\eta}(mp + 3\lambda\omega^2) \equiv F, \quad (10)$$

where $\chi \equiv (p + 3a - 6\omega)$. $SO(10)$ has only one non-trivial D -term condition namely $\langle D_{B-L} \rangle = 0$. We satisfy this by the condition $|\sigma_{AB}| - |\bar{\sigma}_{AB}| = 0$. The set of homogeneous eqs (8) and (9) can be written in a more transparent form as

$$\Xi \cdot \hat{\Sigma} = \Xi \cdot \hat{\bar{\Sigma}} = 0, \quad (11)$$

where $\hat{\Sigma}, \hat{\bar{\Sigma}}$ are $(N_g(N_g + 1)/2)$ -plet of $\sigma, \bar{\sigma}$ VEVs. Non-trivial solutions of eqs (8) and (9) for $\sigma, \bar{\sigma}$ exist only if $\text{Det}[\Xi] = 0$. In the MSGUT ($N_g = 1$), the linear condition ($\chi = -M/\eta$) supplements eqs (6) and (7) and allows determination of p, a, ω via a cubic equation for ω . After solving F -term conditions (actual procedure will be discussed in the next section), the D -term conditions ($D_{B-L} = 0$ from $SO(10)$ and $D^A = 0$ from $O(N_g)$) need to be solved. When $N_g = 1$

$$D_{B-L} = |\sigma|^2 - |\bar{\sigma}|^2 = 0. \quad (12)$$

As $\text{Arg}[\sigma] - \text{Arg}[\bar{\sigma}]$ can be removed by $U(1)_{B-L}$ transformations, we choose $\sigma = \bar{\sigma}$. For $N_g > 1$ also, we only consider the cases corresponding to $\sigma_{AB} = \bar{\sigma}_{AB}$, so that D_{B-L} is automatically zero. The D -terms of the family group vanish automatically only for trivial solutions of the F -term conditions. We are interested in non-trivial solutions because only these can generate generation mixing. One needs to introduce additional fields to cancel GUT sector contribution to the family D -terms. F -terms corresponding to extra fields should not interfere with the GUT F -terms so as not to disturb the MSGUT SSB. The best possible choice is to locate these fields in the hidden sector responsible for breaking supersymmetry. In [12] it has been shown that Bajc–Melfo (BM) two-field (S_s, ϕ_s) superpotential is an appropriate candidate. This superpotential reads as

$$W_H = S_s(\mu_B\phi_s + \lambda_B\phi_s^2). \quad (13)$$

It has a SUSY-preserving global minima at $S_s = \phi_s = 0$ and $S_s = 0, \phi_s = -\mu_B/\lambda_B$ and SUSY-breaking local minima at $\langle \phi_s \rangle = -\mu_B/2\lambda_B$, where S_s remains undetermined with a condition $|\langle S_s \rangle| \geq |\langle \phi_s \rangle/\sqrt{2}|$. $\langle S \rangle$ can be fixed either by radiative corrections [13] or by couplings to $N = 1$ supergravity [14,15]. In [12], $\langle S \rangle$ is determined by coupling S, ϕ fields to $N = 1$ supergravity. To enable YUMGUT proposal, two fields are taken as symmetric multiplets of $O(N_g)$ where singlet (S_s) breaks SUSY and traceless part \hat{S} is fixed against visible sector fields contribution to $O(N_g)$ D -terms and thus facilitates YUMGUTs. BM-type hidden sector necessarily implies a number of light SM singlet scalars ($O(m_{3/2})$) and even lighter fermions that get mass only from radiative effects. Note that these light modes may provide light DM candidates (<50 GeV) as indicated by the DAMA/LIBRA [16] experiments.

4. Analytical and numerical analysis

Writing VEVs $\{p, \omega, a, \sigma, \tilde{\sigma}, \chi\} = (m/\lambda)\{P, W, A, \tilde{\sigma}, \tilde{\tilde{\sigma}}, \tilde{\chi}\}$, $\tilde{\chi}_A = \tilde{\chi}_{AA} + \xi$ in units of m/λ , we can eliminate all the parameters in F -term equations (eqs (6)–(10)) except two ratios $\xi = \lambda M/\eta m$ and λ/η :

$$2\left(\frac{m}{\lambda}\right)^2 (P - A - A^2 + W^2) = 0, \quad (14)$$

$$\left(\frac{m}{\lambda}\right)^2 (P + W + (P + 2A + 3W)W + W(P + 2A + 3W)) = 0, \quad (15)$$

$$\left(\frac{m}{\lambda}\right)^2 (\xi\tilde{\sigma} + (\tilde{\chi}\tilde{\sigma} + \tilde{\sigma}\tilde{\chi})/2) = 0, \quad (16)$$

$$\left(\frac{m}{\lambda}\right)^2 (\xi\tilde{\tilde{\sigma}} + (\tilde{\chi}\tilde{\tilde{\sigma}} + \tilde{\tilde{\sigma}}\tilde{\chi})/2) = 0, \quad (17)$$

$$\tilde{\tilde{\sigma}}\tilde{\sigma} + \tilde{\sigma}\tilde{\tilde{\sigma}} = -\frac{4\lambda}{\eta}(P + 3W^2) = \frac{\lambda^2 F}{m^2} = \tilde{F}. \quad (18)$$

It is convenient to use dimensionless form of equations for SSB analysis because we can get most of the VEVs independent of model parameters. Before analysing realistic SM case ($N_g = 3$), we shall study the simplest toy model ($N_g = 2$).

4.1 Toy model ($N_g = 2$)

$\hat{\Sigma} = \{\tilde{\sigma}_{11}, \tilde{\sigma}_{22}, \tilde{\sigma}_{12}\}$, the matrix Ξ involves the combinations $\tilde{\chi}_A = \tilde{\chi}_{AA} + \xi$:

$$\Xi = \begin{pmatrix} \tilde{\chi}_1 & 0 & \tilde{\chi}_{12} \\ 0 & \tilde{\chi}_2 & \tilde{\chi}_{12} \\ \tilde{\chi}_{12} & \tilde{\chi}_{12} & \tilde{\chi}_1 + \tilde{\chi}_2 \end{pmatrix}, \quad (19)$$

$$\begin{aligned} \text{Det}[\Xi] &= (\tilde{\chi}_1 + \tilde{\chi}_2)(\tilde{\chi}_{12}^2 - \tilde{\chi}_1\tilde{\chi}_2) = 0 \\ \Rightarrow \tilde{\chi}_1 &= -\tilde{\chi}_2 \quad \text{or} \quad \tilde{\chi}_{12} = \pm\sqrt{\tilde{\chi}_1\tilde{\chi}_2}. \end{aligned} \quad (20)$$

$\text{Det}[\Xi]$ should vanish for non-trivial solutions. Null 2×2 minors provide $\tilde{\chi}_1 = \tilde{\chi}_2 = \tilde{\chi}_{12} = 0$ ($\Xi \equiv 0$). Thus, $\text{Rank}[\Xi] < 2$ implies $\text{Rank}[\Xi] = 0$, so that all the six $\sigma, \tilde{\sigma}$ remain

undetermined. However, we find that $\text{Rank}[\Xi] = 0$ is a degenerate case implying large coloured and charged pseudo-Goldstone multiplets. So we consider only $\text{Rank}[\Xi] = 2$ case. For a non-trivial solution, one out of the two factors $(\tilde{\chi}_1 + \tilde{\chi}_2)$ and $(\tilde{\chi}_{12}^2 - \tilde{\chi}_1\tilde{\chi}_2)$ of $\text{Det}[\Xi]$ should vanish. One can calculate $\tilde{\sigma}_{11}$ and $\tilde{\sigma}_{22}$ in terms of $\tilde{\sigma}_{12}$ from $\Xi \cdot \tilde{\Sigma} = 0$ as

$$\tilde{\sigma}_{11} = -\frac{\tilde{\chi}_{12}}{\tilde{\chi}_1}\tilde{\sigma}_{12}; \quad \tilde{\sigma}_{22} = -\frac{\tilde{\chi}_{12}}{\tilde{\chi}_2}\tilde{\sigma}_{12}, \quad (21)$$

$$\text{Det}[\tilde{\sigma}] = \frac{(\tilde{\chi}_{12}^2 - \tilde{\chi}_1\tilde{\chi}_2)}{\tilde{\chi}_1\tilde{\chi}_2}\tilde{\sigma}_{12}^2. \quad (22)$$

$\text{Det}[\tilde{\sigma}]$ has a factor $(\tilde{\chi}_{12}^2 - \tilde{\chi}_1\tilde{\chi}_2)$ in common with $\text{Det}[\Xi]$ which will cause $\text{Det}[\tilde{\sigma}]$ to also vanish if we choose this factor to be zero to make $\text{Det}[\Xi]$ vanish. In MSGUTs (also in YUMGUTs), Majorana mass of the right-handed neutrinos is determined by $(\tilde{\Sigma}) = \tilde{\sigma}$, which requires invertible VEV for Type-I see-saw contribution. Therefore, we analyse only the branch $(\tilde{\chi}_1 + \tilde{\chi}_2) = 0$ for vanishing $\text{Det}[\Xi]$. Equation (18) then implies

$$\tilde{\sigma}_{11}^2 = \frac{\tilde{F}_{11}\tilde{\chi}_{12}^2}{2(\tilde{\chi}_{12}^2 + \tilde{\chi}_1^2)}; \quad \tilde{F}_{11} = \tilde{F}_{22}; \quad \tilde{F}_{12} = 0. \quad (23)$$

We solve eq. (15) (linear in P) for all the components of P . Using the calculated P values, solve $\tilde{\chi}_1 = -\tilde{\chi}_2$, $\tilde{F}_{12} = 0$ and $\tilde{F}_{11} - \tilde{F}_{22} = 0$ for A_{11} , A_{12} and A_{22} . The remaining equations (eq. (14)) can be completely expressed in terms of W and ξ . We used a minimization method for a numerical solution of W for a convenient ξ . Using these numerical values of P , A , W , $\tilde{\sigma}$ and randomly chosen YUMGUT parameters $(\lambda, \eta, \gamma, \bar{\gamma}, h, f)$, we find M_H values from $\text{Det}[\mathcal{H}] = 0$. Then, Yukawas corresponding to all allowed values of M_H are determined. Yukawa eigenvalues, mixing angles and neutrino masses are presented in table 1 for $f \sim h$. Here we have acceptable fermion hierarchy and mixing but too small neutrino masses which is the main failure of MSGUT. One can boost Type-I see-saw contribution by suppressing f which implies $Y_u = Y_v$ and $Y_d = Y_l$. Type-II see-saw contribution is generated by VEV of $O[1, 3, -2]$ multiplet [9,17–19] which is now

Table 1. Yukawa eigenvalues and mixing angles for $N_g = 2$, $f = -0.13$. $\tilde{M}_{\nu^c} \equiv \lambda M_{\nu^c}/m = \{0.6969, 0.0636\}$. m/λ is taken to be 10^{16} GeV to estimate Δm_ν^2 . $\lambda = -0.038 + 0.005i$, $\eta = 0.4$, $\gamma = 0.32$, $\bar{\gamma} = -1.6$, $h = 0.34$, $\xi = 0.8719 + 0.5474i$.

Parameter	Value		
	I	II	III
M_H	$0.049 + 0.190i$	$0.599 + 0.791i$	$1.39 + 0.80i$
Y_u	0.1537, 0.0080	0.1293, 0.0118	0.0685, 0.0214
Y_d	0.0537, 0.0043	0.0562, 0.0051	0.0359, 0.0052
Y_l	0.0424, 0.0027	0.0712, 0.0065	0.0147, 0.0063
Y_ν	0.2515, 0.0233	0.0576, 0.0053	0.0911, 0.0028
$\theta_{\text{CKM}}(\text{deg.})$	5.15	2.27×10^{-6}	7.41
$\theta_{\text{PMNS}}(\text{deg.})$	14.5	2.32×10^{-5}	33.7
m_ν (meV)	0.0255, 0.2791	0.0013, 0.0144	0.0011, 0.0121
Δm_ν^2 (eV ²)	7.73×10^{-8}	2.06×10^{-10}	1.45×10^{-10}

a family group triplet. In the MSGUT (NMSGUT), Type I dominates over Type II. Type-II contribution needs to be re-examined in the YUMGUT. Complete superheavy spectra (in units of m/λ) for the solution found are presented in table 2. Only the SM singlet

Table 2. Mass spectrum of superheavy fields in units of $m/\lambda \sim 10^{16}$ GeV for $N_g = 2$. Only the spectra of $h[1, 2, \pm 1]$, $t[3, 1, \mp 2/3]$ (given in colours S. No. 1 (blue), S. No. 2 (green), S. No. 3 (red)) depend on the value of M_H chosen (see table 1 for the values of M_H).

Field [$SU(3), SU(2), Y$]	Masses
A[1, 1, 4]	4.093, 3.321, 0.137
B[6, 2, 5/3]	0.106, 0.099, 0.091
C[8, 2, 1]	1.727, 1.727, 1.224, 1.224, 0.614, 0.614
D[3, 2, 7/3]	1.919, 1.433, 1.191, 0.810, 0.205, 0.134
E[3, 2, 1/3]	1.475, 1.043, 0.716, 0.716, 0.677, 0.594, 0.506 0.404, 0.277, 0.087, 0.073, 0.050, 0.004
F[1, 1, 2]	1.794, 1.794, 1.681, 1.317, 0.289, 0.228, 0.018
G[1, 1, 0]	1.672, 1.665, 1.248, 1.248, 0.766, 0.766, 0.504, 0.469, 0.208 0.201, 0.079, 0.068, 0.055, 0.011, 0.009, 0
$h^{(1)}$ [1, 2, 1]	3.799, 2.812, 1.398, 1.182, 0.983, 0.74, 0.588, 0.511, 0.159, 0.024, 0.013
$h^{(2)}$ [1, 2, 1]	3.947, 2.961, 1.623, 1.247, 1.009, 0.726, 0.556, 0.51, 0.14, 0.044, 0.005
$h^{(3)}$ [1, 2, 1]	4.161, 3.196, 2.049, 1.289, 0.979, 0.710, 0.540, 0.520, 0.152, 0.029, 0.010
I[3, 1, 10/3]	0.210, 0.192, 0.003
J[3, 1, 4/3]	1.889, 1.889, 0.946, 0.740, 0.453, 0.278, 0.119, 0.086, 0.021, 0.006
K[3, 1, -8/3]	1.591, 1.237, 0.116
L[6, 1, 2/3]	1.066, 0.916, 0.757
M[6, 1, 8/3]	1.340, 0.958, 0.493
N[6, 1, -4/3]	1.795, 1.178, 0.345
O[1, 3, -2]	1.127, 0.886, 0.084
P[3, 3, -2/3]	0.902, 0.754, 0.595
Q[8, 3, 0]	0.163, 0.126, 0.083
R[8, 1, 0]	0.170, 0.119, 0.107, 0.086, 0.066, 0.047
S[1, 3, 0]	0.090, 0.058, 0.011
$t^{(1)}$ [3, 1, -2/3]	3.264, 2.802, 1.824, 1.496, 1.175, 1.019, 0.89 0.824, 0.598, 0.495, 0.343, 0.202, 0.055, 0.026, 0.007
$t^{(2)}$ [3, 1, -2/3]	3.418, 2.936, 1.873, 1.636, 1.2, 1.053, 0.909, 0.824 0.692, 0.532, 0.454, 0.211, 0.077, 0.018, 0.001
$t^{(3)}$ [3, 1, -2/3]	3.650, 3.156, 2.097, 1.747, 1.273, 1.116, 0.926, 0.824 0.779, 0.541, 0.466, 0.223, 0.116, 0.023, 0.002
U[3, 3, 4/3]	0.084, 0.070, 0.054
V[1, 2, -3]	0.227, 0.208, 0.003
W[6, 3, 2/3]	1.693, 1.324, 0.902
X[3, 2, -5/3]	1.666, 1.666, 0.149, 0.102, 0.072, 0.070, 0.066
Y[6, 2, -1/3]	0.167, 0.118, 0.058
Z[8, 1, 2]	0.100, 0.086, 0.070

sector $G[1, 1, 0]$ has pseudo-Goldstones which can act as DM candidates. The following symmetry breaking VEVs (in units of the MSGUT scale parameter m/λ) are used to calculate flavour structure given in table 1:

$$W = \begin{pmatrix} 0.141 - 0.203i & 0.3168 + 0.189i \\ 0.3168 + 0.189i & -0.2667 + 0.3075i \end{pmatrix} \quad (24)$$

$$P = \begin{pmatrix} -0.236 - 0.2001i & 0.1787 - 0.028i \\ 0.1787 - 0.028i & -0.2297 + 0.1202i \end{pmatrix} \quad (25)$$

$$A = \begin{pmatrix} -0.23 - 0.3521i & 0.3382 + 0.0777i \\ 0.3382 + 0.0777i & -0.4475 + 0.2227i \end{pmatrix} \quad (26)$$

$$\tilde{\sigma} = \tilde{\bar{\sigma}} = \begin{pmatrix} 0.0863 - 0.2366i & 0.1973 + 0.1041i \\ 0.1973 + 0.1041i & -0.0863 + 0.2366i \end{pmatrix} \quad (27)$$

$$\begin{aligned} \bar{D}_X &= 2(|p_+|^2 - |p_-|^2) + 3(|a_+|^2 - |a_-|^2) + 6(|w_+|^2 - |w_-|^2) \\ &\quad + \frac{1}{2}|\sigma_+|^2 - |\sigma_-|^2 + \frac{1}{2}|\bar{\sigma}_+|^2 - |\bar{\sigma}_-|^2 = -8.94. \end{aligned} \quad (28)$$

4.2 Realistic case ($N_g = 3$)

Symmetry breaking equations in this case are more complex and offer a number of phenomenologically interesting possibilities like light sterile neutrino and novel DM candidate from MSSM singlet sector $G[1, 1, 0]$. Now matter Yukawas can be written by the same procedure as in the $N_g = 2$ case (see [10] for the actual form of Ξ , $\Omega[V]$ and Yukawas). To avoid pseudo-Goldstones, we start with the non-degenerate case $\text{Rank}[\Xi] = 5$. In this case, we notice that

$$\text{Det}[\tilde{\sigma}] \sim \text{Det}[\Xi] \Rightarrow \text{Det}[\tilde{\sigma}] = 0 = \text{Det}[M_{\nu e}]. \quad (29)$$

It implies the existence of one or more light sterile neutrino depending upon the zero eigenvalues of $\tilde{\sigma}$ VEV. After integrating out the heavy right-handed neutrino, one can obtain the light neutrino mass matrix where sterile neutrino will get only Dirac mass. Rank reduction ($\text{Rank}[\Xi] = 4$) of homogeneous system provides a possible route to find invertible $\tilde{\sigma}$ VEV. We have also investigated the scenario if we consider only the five-dimensional traceless irrep of $O(3)$ and solved the least degenerate ($\text{Rank}[\Xi] = 4$) case. This option has relatively fewer parameters and so it is easier to perform numerical searches. We solved some equations analytically and the remaining numerically to find the solution. Example solutions for all the mentioned possibilities can be found in [10]. For illustration, in table 3, we provide Yukawa eigenvalues and mixing angles for $\text{Rank}[\Xi] = 4$ (6-dim). We have calculated the complete superheavy spectrum for all the cases considered to check the existence of pseudo-Goldstones which may be present when there is Higgs duplication. Spectra do not contain pseudo-Goldstone except the SM singlet $G[1, 1, 0]$ sector which does not affect unification. Main point to emphasize is that without any optimization solutions shown provide large lepton mixing, small quark mixing and acceptable Yukawa hierarchy. With optimization, one can aim to find the flavour-blind parameters of YUMGUT which can produce actual MSSM Yukawas.

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Table 3. Yukawa eigenvalues and mixing angles for $N_g = 3$ (Rank[Ξ] = 4), $f = -0.11 + 0.02i$. $\tilde{M}_{\nu^c} \equiv \lambda M_{\nu^c}/m$. m/λ is taken to be 10^{16} GeV to estimate Δm_{ν}^2 . $\lambda = 0.48 - 0.05i$, $\eta = -0.18$, $h = 0.26$, $\gamma = 0.12$, $\bar{\gamma} = -1.44$, $\xi = 1.7278 - 0.1734i$.

S. No.	M_H	Y_u	Y_d	$\{\theta_{13}, \theta_{12}, \theta_{23}\}^Q$ (deg.)
1	$-4.323+1.47i$	0.0007, 0.0021, 0.0215	0.001, 0.0019, 0.0219	9.0,15.9,15.6
2	$0.465+3.382i$	0.0018, 0.0148, 0.0182	0.0020, 0.0197, 0.0222	11.3,1.5,4.9
3	$0.76-2.193i$	0.0029, 0.0113, 0.0137	0.0054, 0.0233, 0.0385	1.5,6.2,7.4
4	$-0.002+0.968i$	0.0105, 0.040, 0.077	0.0035, 0.0174, 0.0408	5.2,3.7,2.8
5	$-0.508-0.209i$	0.0077, 0.053, 0.1126	0.0019, 0.0159, 0.0381	1.1,12.1,1.4
6	$-0.092-0.032i$	0.0041, 0.0467, 0.0558	0.0035, 0.0413, 0.0522	8.7,5.5,2.6
S. No.	Y_l	Y_ν	$\{\theta_{13}, \theta_{12}, \theta_{23}\}^L$ (deg.)	\tilde{M}_{ν^c}
1	0.0013, 0.0041, 0.0517	0.0023, 0.0064, 0.0468	3.6, 20.3, 23.3	5.9,5.3,1.5
2	0.0034, 0.0148, 0.0205	0.0032, 0.0126, 0.0162	27.5, 14.5, 47.0	5.9,5.3,1.5
3	0.0053, 0.0121, 0.0458	0.0033, 0.0102, 0.020	13.6, 11.1, 41.1	5.9,5.3,1.5
4	0.0048, 0.0174, 0.0473	0.0092, 0.0181, 0.0915	23.9, 14.5, 17.7	5.9,5.3,1.5
5	0.0042, 0.0224, 0.0382	0.0061, 0.0584, 0.0835	23.7, 26.1, 49.4	5.9,5.3,1.5
6	0.0043, 0.0497, 0.0621	0.0049, 0.0355, 0.0518	14.1, 37.6, 46.3	5.9,5.3,1.5
S. No.	$m_\nu/10^{-4}$ (meV)	$\Delta m_\nu^2/10^{-13}$ (eV ²)		
1	0.039, 0.187, 40.543	0.0033, 164.372		
2	0.079, 0.807, 4.09	0.0645, 1.6074		
3	0.12, 0.32, 6.946	0.0088, 4.8143		
4	1.33, 4.503, 23.439	1.851, 52.9128		
5	0.926,17.046, 34.448	28.9722, 89.6078		
6	1.03, 4.96, 9.806	2.3544, 7.1562		

5. Conclusion and discussions

We have proposed [10] dynamical generation of flavour based upon SUSY $SO(10)$ and family gauge group. In literature, $O(3)$ family symmetry with traceful representation is considered for non-renormalizable and non-GUT Yukawa-on models [5]. However, our model is renormalizable and GUT-based. $SO(10)$ Yukawa couplings are just single complex number. Thus parameter reduction is one of the main virtue of YUMGUTs. Consistent SSB is achieved with the introduction of ($O(N_g)$ symmetric two field S , ϕ) BM (hidden sector) superpotential.

We considered a number of generic possibilities, with random GUT scale parameters, which produce acceptable Yukawa eigenvalues and lepton and quark mixing angles, but small neutrino masses. Type-I contribution can be raised by suppressing f which provides unacceptable Yukawa structure. We have not considered the contribution of Type-II see-saw generated by the VEV of symmetric multiplet O^- . Although we expect it to be small as compared to Type I as in MSGUTs, some special points may yield significant contribution. Addition of **120**-plet, which along with **10**-plet is mainly responsible for generating charged fermion masses, is another way to get neutrino masses in experimentally measured range. To completely demonstrate this idea we need to actually produce

the SM mass mixing data respecting NMSGUT fitting features which will require a huge computational effort which will be justified if the flavour hierarchy is understood in a GUT context. A number of experimental signals such as light moduli fields and singlet pseudo-Goldstones [12], which can also be DM candidates, are associated with our proposal. Thus, our work has laid the basis for an extensive programme of future studies in unification and cosmology.

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