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New minimal $SO(10)$ GUT: A theory for all epochs

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Abstract. The supersymmetric $SO(10)$ theory (NMSO(10)GUT) based on the $\mathbf{210} + \mathbf{126} + \overline{\mathbf{126}}$ Higgs system proposed in 1982 has evolved into a realistic theory capable of fitting the known low energy particle physics data besides providing a dark matter candidate and embedding inflationary cosmology. It dynamically resolves longstanding issues such as fast dimension five-operator mediated proton decay in SUSY GUTs by allowing explicit and complete calculation of crucial threshold effects at M_{SUSY} and M_{GUT} in terms of fundamental parameters. This shows that $SO(10)$ Yukawas responsible for observed fermion masses as well as operator dimension-five-mediated proton decay can be highly suppressed on a 'Higgs dissolution edge' in the parameter space of GUTs with rich superheavy spectra. This novel and generically relevant result highlights the need for every realistic UV completion model with a large/infinite number of heavy fields coupled to the light Higgs doublets to explicitly account for the large wave function renormalization effects on emergent light Higgs fields. The NMSGUT predicts large-soft SUSY breaking trilinear couplings and distinctive sparticle spectra. Measurable or near measurable level of tensor perturbations – and thus large inflaton mass scale – may be accommodated within the NMSGUT by supersymmetric see-saw inflation based on an LHN flat direction inflaton if the Higgs component contains contributions from heavy Higgs components. Successful NMSGUT fits suggest a renormalizable Yukawon ultraminimal gauged theory of flavour based upon the NMSGUT Higgs structure.

Keywords. Supersymmetry; grand unification; $SO(10)$; flavour unification; inflation.

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1. Introduction

Grand unification theories (GUTs) have seen around 40 summers since the basic idea was first proposed by Pati and Salam in 1974 [1], yet retain their attraction as the most obvious progression of the gauge logic embodied in the Standard Model. Indeed, in particle physics, the only tangible and clear hints of physics beyond the Standard Model are the remarkable convergence of gauge couplings in the unification regime: 10^{15}–10^{17} GeV and the existence of neutrino masses (initially thought to be superfluous in both the electroweak and GUT contexts). To this, one should perhaps append the convergence of the pivotal [2] third-generation Yukawa couplings in the MSSM at high $\tan\beta$. The same gauge logic that structures the SM (and justifies the zeroth-order decoupling of right-handed neutrinos) also makes the Type-I see-saw mechanism [3], the most natural rationale for the milli-eV range neutrino masses actually observed. Supersymmetry seems essential to the structural stability of the SM and experience [4,5] with supersymmetric LR models and renormalizable $SO(10)$ GUTs shows it ensures complete calculability for UV completions of the MSSM. Thus, non-discovery of sparticles in the searches so far has still not dimmed the ardor of its many adherents. Arguments for considering some variety of the MSSM as the effective theory of any GUT became all the more cogent when, with the observation of solar and atmospheric neutrino oscillations at Super-Kamiokande, the scale of B-L breaking associated with a renormalizable see-saw mechanism emerged ($V_{\rm EW}^2/(10^{-12}$ GeV) $\sim 10^{16}$ GeV) in the GUT ball-park. The convergence of these two compelling arguments thus hints at a deep interconnection between B and L violation in a supersymmetric grand unified framework [6,7]. It is $SO(10)$ not $SU(5)$ GUTs that provide the most natural GUT framework, free of gauge singlets (which are really opaque, and thus anathemic, to the gauge logic pursued so successfully in the SM), for Type-I (and also Type-II) see-saw. Thus, eventually $SO(10)$ GUTs displaced $SU(5)$ theories and achieved a long-delayed vindication and recognition as minimal UV complete frameworks for neutrino oscillations. It was only natural that our pursuit of consistent minimal left–right supersymmetric R-parity preserving models [4] from the midnineties led in short order [5], after the epochal measurements of Super-Kamiokande to a realization that the very first complete supersymmetric $SO(10)$ GUT [8,9] proposed way back in 1982, just after the Georgi–Dimopoulos minimal SUSY $SU(5)$ model [10], was in fact the minimal supersymmetric GUT (MSGUT) [11]. This model is now called [12] – due to a transient glory of a truncated version with only $\mathbf{10} + \overline{\mathbf{126}}$ but not $\mathbf{120}$ coupling to matter fermion bilinears as the minimal theory [13] – the New/Next MSGUT(NMSGUT). In the best Popperian mode it has survived another decade of detailed investigation of its ability to fit all available SM, gauge, fermion and neutrino data, as well as the consistency of one-loop threshold corrections at both low- and high-energy scales made possible by its extreme parameter economy (a.k.a. minimality) and full calculability: simple virtues that are, alas, all too rare among the plethora of its (non-minimal) competitors. The model is based on a $\mathbf{210} \oplus \mathbf{126} \oplus \overline{\mathbf{126}}$ GUT Higgs system and already in 1982 [8] showed clearly that prevention of a RG flow catastrophe due to large pseudo-Goldstone supermultiplets requires a single step breaking of SUSY $SO(10)$ to the MSSM: later rediscovered in the context of another R-parity preserving but non-minimal model [5]. Being a complete theory of gauge physics, the model is capable of fitting and/or predicting most, if not all, of the commonly considered varieties of BSM physics, including neutrino masses, the g-2 muon anomaly, LSP dark matter, B violation (at acceptable rates: see below), leptogenesis driven by the parameters controlling neutrino mass etc. Remarkably, it also comes close to providing a workable embedding of inflation based on the D-flat directions involving lepton and Higgs doublet fields naturally present and contributing to the effective MSSM below the GUT scale. The model also predicts a distinctive and characteristic normal hierarchy of sfermion generations discoverable by the LHC or its successors. As striking was its early indication, on the basis of the parameters required for a successful fermion fit, that the trilinear soft SUSY breaking parameter A_0 must be large (as far back as 2008 [14], i.e. well before Higgs mass discovery made such a large A_0 respectable rather than ludicrously fine tuned: as then argued by naturalists).

The Standard Model parameters are fitted numerically in terms of definite sets of grand unified model coupling. For each set of couplings adequate for a fit within the experimental and theoretical accuracy, the model makes predictions for a vast range of phenomena ranging from exotic processes such as baryon number and lepton flavour violation to cosmological leptogenesis, dark matter relic density and inflation. The demonstration of the feasibility of such comprehensive fits of the SM data directly in terms of SUSY GUT parameters marks the NMSGUT as being cast in the mould of the SM rather than its piece-meal generalizations. The most striking themes and results that have emerged recently from our detailed investigation of the structural freedoms-in-necessity granted by the full calculability NMSGUT are:

(1) A simple generic mechanism for suppression of the long problematic [15] $d = 5$ fast proton decay generic to SUSY GUTs.
(2) An embedding of high-scale supersymmetric renormalizable inflexion [16] based on the left lepton (L)–Higgs doublet (H)–conjugate neutrino ($N \equiv \nu_L^c$) flat direction (labelled by the LHN chiral invariant) and utilizing the involvement of heavy partners of the MSSM Higgs. This may prove very attractive in the context of the BICEP2-driven focus on high scale inflation.
(3) Completely novel 'grand Yukawonification' models based on gauging of an $O(3)$ subgroup of the $U(3)$ flavour symmetry of the $SO(10)$ GUT fermion kinetic terms. This flavour symmetry is naturally broken at the GUT scale by the NMSGUT Higgs fields which promote themselves rather naturally to also carry the function of being Yukawons i.e., fields whose VEVs determine the Yukawa couplings in the effective MSSM besides simultaneously breaking $SO(10)$ while maintaining renormalizability: which is not possible in most Yukawon models.
(4) The models in point (3) above are made possible by a novel conflation of supergravity-mediated supersymmetry breaking with the breaking of gauged flavour symmetry in the hidden sector of the model using the so-called Bajc–Melfo calculable metastable SUSY breaking vacua [17,18].

Thus, the NMSGUT potentially provides a completely realistic and predictive theory of particle physics in all energy ranges and cosmological epochs. We have even speculated [19] that the Landau pole in the NMSGUT gauge coupling which occurs quite near the Planck scale should be interpreted as a physical cut-off on the perturbative dynamics associated with the scale where the NMSGUT condenses (via a strong coupled supersymmetric dynamics). This scale could then function as the Planck scale of an effective induced supergravity emergent at large length scales from the NMSGUT when the metric and gravitino fields introduced to define a coordinate independent microscopic GUT acquire kinetic terms due to quantum effects. Since points (2) and (3) are published as [16,17] and are also reported in these proceedings in the contributions of Ila Garg and Charanjit Kaur respectively, I shall touch upon them only briefly but focus on the results regarding points (1) and (4) and discuss some of their implications while referring the reader to the published papers [11,12,20–23] for details.

2. NMSGUT basics

The NMSGUT superpotential is built from the quadratic $SO(10)$ invariants and associated mass parameters

$$m : \mathbf{210^2}; \quad M : \mathbf{126} \cdot \overline{\mathbf{126}}; \quad M_H : \mathbf{10^2}; \quad m_\Theta : \mathbf{120^2} \tag{1}$$

and trilinear couplings

$$\begin{aligned} \lambda &: \mathbf{210^3}; \quad \eta : \mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}}; \quad \rho : \mathbf{120} \cdot \mathbf{120} \cdot \mathbf{210} \\ k &: \mathbf{10} \cdot \mathbf{120} \cdot \mathbf{210}; \quad \gamma \oplus \bar{\gamma} : \mathbf{10} \cdot \mathbf{210} \cdot (\mathbf{126} \oplus \overline{\mathbf{126}}) \\ \zeta \oplus \bar{\zeta} &: \mathbf{120} \cdot \mathbf{210} \cdot (\mathbf{126} \oplus \overline{\mathbf{126}}) \\ & \mathbf{16_A}.\mathbf{16_B}.(h_{AB}\mathbf{10} + f_{AB}\overline{\mathbf{126}} + g_{AB}\mathbf{120}). \end{aligned} \tag{2}$$

The couplings $h, f(g)$ are complex (anti)-symmetric in the flavour indices due to properties of the $SO(10)$ Clifford algebra. Either h or f is chosen as real and diagonal using the $U(3)$ flavour symmetry of the matter kinetic terms. The matter Yukawas thus contain 21 real parameters. Five phases say of $m, M, \lambda, \gamma, \bar{\gamma}$ are set to zero by phase conventions. One complex parameter (say M_H), is fine-tuned to keep two Higgs doublets $(H, \bar{H})$ of the effective MSSM light, leaving 23 magnitudes and 15 phases as parameters. Fine tuning fixes $H, \bar{H}$ composition as a mixture of (six pairs of the MSSM type) doublet fields in the GUT as a function of the superpotential parameters entering the null vectors of the $G_{3,2,1}$ irreps $[1, 2, \pm 1]$ mass terms. The mixture is described by the so-called 'Higgs fractions' [11,21].

The GUT scale VEVs (units m/λ) are known functions of x which is a solution of the cubic $(\xi = \lambda M/\eta m)$

$$8x^3 - 15x^2 + 14x - 3 + \xi(1 - x)^2 = 0. \tag{3}$$

The complete set of GUT scale mass matrices for the 26 different MSSM irrep types and tree level low-energy effective superpotential was calculated [12] extending the result for the MSGUT [20,21,23,24]. Using these and two-loop RG flows the gauge, superoptential and soft SUSY breaking parameters at GUT scales can be matched, using a downhill simplex search procedure, to the known values of the MSSM data described above [12]. This also yields a mini-split (10–100 TeV) supersymmetry sparticle spectrum with large A_0, μ parameters, light gauginos and Bino LSP, superheavy Higgsinos, sfermions in tens of TeV, normal inter-generational hierarchy and sometimes an smuon (or other sfermion) light enough (i.e., within 10% of the light Bino LSP) to co-annihilate with it and provide acceptable dark matter relic density [25]. The light smuon case is obviously attractive if supersymmetry is called upon to explain the muon magnetic moment anomaly and emerges in the right ball park for these solutions. Flavour violation in the quark and lepton sectors is also well controlled because of the multi-TeV masses of most of the sfermions. Moreover, A_0 is required to be large to allow the b quark mass to be fitted and this was found [12,14] serendipitously before Higgs discovery made it a requirement for supersymmetry.

3. GUT scale threshold corrections and baryon decay rate

The $SO(10)$ parameters determined by the realistic fit are substituted into the effective (dimension four) superpotential describing baryon violation which, after RG flow to low energies, is used to calculate the proton decay rate. As is well known, the generic result gives a lifetime of some 10^{27} years i.e., some seven orders of magnitude shorter than the current limits from the Super-Kamiokande experiment. The beautifully complete and predictive fits are thus of little use if this problem remains unsurmountable. However, as in previous tight spots, the NMSGUT points out a convincing and illuminating, generically applicable and dynamical, pathway out of the difficulty: precisely because of the available explicit solution described above.

Superpotential parameters renormalize only by wave function corrections. In the computable basis sets where heavy supermultiplet masses are diagonal, a generic heavy field type Φ (conjugate irrep $\bar{\Phi}$) mass matrix diagonalizes as

$$\bar{\Phi} = U^{\Phi}\bar{\Phi}'; \quad \Phi = V^{\Phi}\Phi' \quad \Rightarrow \quad \bar{\Phi}^T M \Phi = \bar{\Phi}'^T M_{\text{Diag}} \Phi'. \tag{4}$$

Circulation of heavy fields within the quantum loops which normalize the propagators of the light fields (f_A, f_A^c, H_f) which enter Higgs matter Yukawa couplings in the Lagrangian: $\mathcal{L} = [f_c^T Y_f f H_f]_F$ + h.c.+ implies [26] a finite wave function renormalization in the fermion and Higgs kinetic terms

$$\mathcal{L} = \left[\sum_{A,B} (\bar{f}_A^\dagger (Z_{\bar{f}})_A^B \bar{f}_B + f_A^\dagger (Z_f)_A^B f_B) + H^\dagger Z_H H + \bar{H}^\dagger Z_{\bar{H}} \bar{H}\right]_D + \cdots . \tag{5}$$

Unitary matrices $U_{Z_f}, \bar{U}_{Z_{\bar{f}}}$ diagonalize $(U^\dagger Z U = \Lambda_Z)$ $Z_{f,\bar{f}}$ to the positive definite form $\Lambda_{Z_f, Z_{\bar{f}}}$. We define a new basis to put the kinetic terms of the light matter and Higgs fields in canonical form:

$$f = U_{Z_f} \Lambda_{Z_f}^{-1/2} \tilde{f} = \tilde{U}_{Z_f} \tilde{f}; \quad \bar{f} = U_{Z_{\bar{f}}} \Lambda_{Z_{\bar{f}}}^{-1/2} \tilde{\bar{f}} = \tilde{U}_{Z_{\bar{f}}} \tilde{\bar{f}}$$

$$H = \frac{\tilde{H}}{\sqrt{Z_H}}; \quad \bar{H} = \frac{\tilde{\bar{H}}}{\sqrt{Z_{\bar{H}}}}. \tag{6}$$

Thus when matching the MSSM to the effective theory derived from the NMSGUT it is the loop corrected Yukawa couplings $\tilde{Y}_f$ of the new (canonically normalized) basis fields $\tilde{f}$, $\tilde{\bar{f}}$, $\tilde{H}$, $\tilde{\bar{H}}$, and not the original ones (Y_f) derived from the GUT at tree level, that must be equated to the MSSM couplings at the matching scale

$$\tilde{Y}_f = \Lambda_{Z_{\bar{f}}}^{-1/2} U_{Z_{\bar{f}}}^T \frac{Y_f}{\sqrt{Z_{H_f}}} U_{Z_f} \Lambda_{Z_f}^{-1/2} = \tilde{U}_{Z_{\bar{f}}}^T \frac{Y_f}{\sqrt{Z_{H_f}}} \tilde{U}_{Z_f} \tag{7}$$

and not the original tree-level ones that match the MSSM at the matching scale.

For a light field Φ_i the corrections have the form $(Z = 1 - \mathcal{K})$:

$$\mathcal{K}_i^j = -\frac{g_{10}^2}{8\pi^2} \sum_\alpha Q_{ik}^{\alpha *} Q_{kj}^\alpha F(m_\alpha, m_k) + \frac{1}{32\pi^2} \sum_{kl} Y_{ikl} Y_{jkl}^* F(m_k, m_l), \tag{8}$$

where $\mathcal{L} = g_{10}\, Q_{ik}^\alpha \psi_i^\dagger \gamma^\mu A_\mu^\alpha \psi_k$ describes the generic gauge coupling and $W = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k$, the generic Yukawa couplings, while $F(m_1, m_2)$ are symmetric 1-loop Passarino–Veltman functions.

Crucially, the $SO(10)$ Yukawa couplings $(h, f, g)_{AB}$ also enter into the coefficients L_{ABCD}, R_{ABCD} of the $d = 5$ baryon decay operators in the effective superpotential obtained by integrating out the heavy chiral supermultiplets that mediate baryon decay [12,20,21].

$\tilde{Y}_f$ must be diagonalized to mass basis (denoted by primes), so that $d = 5$, $\Delta B = \pm 1$ decay operator coefficients become

$$\begin{aligned} L'_{ABCD} &= \sum_{a,b,c,d} L_{abcd} (\tilde{U}'_Q)_{aA} (\tilde{U}'_Q)_{bB} (\tilde{U}'_Q)_{cC} (\tilde{U}'_L)_{dD}, \\ R'_{ABCD} &= \sum_{a,b,c,d} R_{abcd} (\tilde{U}'_{\bar{e}})_{aA} (\tilde{U}'_{\bar{u}})_{bB} (\tilde{U}'_{\bar{u}})_{cC} (\tilde{U}'_{\bar{d}})_{dD}. \end{aligned} \tag{9}$$

We find that when we search for a fit to the MSSM data with the constraint that L'_{ABCD}, R'_{ABCD} be sufficiently suppressed (i.e., yielding proton lifetime $\tau_p > 10^{34}$ yr) the search (programme) flows invariably towards parameter regions where $Z_{H,\bar{H}} \ll 1$. So $SO(10)$ Yukawa couplings required to match the MSSM are greatly reduced. The same couplings enter L'_{ABCD}, R'_{ABCD} quadratically thus suppressing them drastically. This mechanism is generically available to realistic multi-Higgs theories. Indeed we go so far as to say that any UV completion incapable of performing this computation will remain less than quantitative and thus is not a viable completion proposal at all. A tedious calculation determines the threshold corrections (see [27,28] for details). $Z \simeq 0$ also leads to smaller GUT couplings and compressed heavy spectra compared to previous fits. Higher loop corrections seem computationally prohibitive. However, we have calculated [25,29–31] the complete $SO(10)$ two-loop beta functions and two-loop threshold corrections also rely upon the same anomalous dimensions. So convoluting GUT scale mass spectra with our $SO(10)$ loop should determine two-loop threshold corrections as well. Recovering our one-loop results by this method is the necessary first step to proceed in this direction.

Searches for fits using the threshold-corrected baryon decay operators yield s-spectra similar to those found earlier, but with smaller values of all couplings (because of the constraints $Z_{H,\bar{H}} > 0$ imposed on the searches: which are badly violated if almost any of the superpotential parameters grow large), and acceptable $d = 5$ B decay rates: as shown in table 1. Imposing $\mathrm{Max}|O^{(4)}| < 10^{-22}\ \mathrm{GeV}^{-1}$ gives proton lifetimes above 10^{34} yr. $Z_{H,\bar{H}}$ approach zero (from above) while $Z_{f,\bar{f}}$ are close to 1: since **16**-plet Yukawas are all suppressed (see [25,27–29] for further details of these and and many related issues).

Table 1. $d = 5$ operator-mediated proton lifetimes τ_p (yr), decay rates $\Gamma(\mathrm{yr}^{-1})$ and branching ratios in the dominant meson$^+ + \nu$ channels.

Solution	$\tau_p(M^+\nu)$	$\Gamma(p \to \pi^+\nu)$	$\mathrm{BR}(p \to \pi^+\nu_{e,\mu,\tau})$	$\Gamma(p \to K^+\nu)$	$\mathrm{BR}(p \to K^+\nu_{e,\mu,\tau})$
1	9.63×10^{34}	4.32×10^{-37}	$\{1.3 \times 10^{-3}, 0.34, 0.66\}$	9.95×10^{-36}	$\{4.6 \times 10^{-4}, 0.15, 0.85\}$
2	3.52×10^{34}	2.14×10^{-36}	$\{1.7 \times 10^{-3}, 0.18, 0.81\}$	2.62×10^{-35}	$\{1.8 \times 10^{-3}, 0.19, 0.81\}$

Table 2. Soft SUSY breaking spectrum of the decoupled and mini-split-SUSY type that arises in a typical realistic fit. Note the large values of μ, B, A_0. Gauginos are the lightest sparticles, and the light Higgs is near the discovered value, but Higgsinos are superheavy. Note specially the distinctive *normal* super-hierarchy and the very light smuon.

Field	Mass (GeV)
$M_{\tilde{G}}$	1000.14
$M_{\chi^\pm}$	569.81, 125591.22
M_{χ^0}	$210.10_{\rm LSP}$, 569.81, 125591.20 , 125591.20
$M_{\tilde{\nu}}$	15308.069, 15258.322, 21320.059
$M_{\tilde{e}}$	1761.89, 15308.29, $211.57_{\rm smuon}$, 15258.60, 20674.72, 21419.56
$M_{\tilde{u}}$	11271.80, 14446.76, 11270.63 , 14445.80, 24607.51, 40275.87
$M_{\tilde{d}}$	8402.99, 11272.10, 8401.48 , 11270.95, 40269.19, 51845.93
M_A	377025.29
$M_{H^\pm}$	377025.30
M_{H^0}	377025.28
M_{h^0}	124.00

Our fits are associated with very distinctive sparticle spectra. An example is shown in table 2 which is of the general type used to evaluate the B-violation rates seen in table 1. Note the peculiar and remarkable smallness of the smuon mass which is a consequence of an RG flow driven by special features of the two-loop RG flow of soft SUSY breaking parameters in models with $M_H^2 > M_{\bar{H}}^2 < 0$. As discussed in [27], the RG flow in such theories can be such as to drive $M^2_{\tilde{\tau}_R}$ first negative and then to large positive values (due to the large value of the third-generation Yukawa coupling) while in the relatively flat evolution of the first two s-generations, the R-smuon lags the R-stau and the R-selectron the R-smuon due to the significant difference in their Yukawa couplings (figure 1). This can

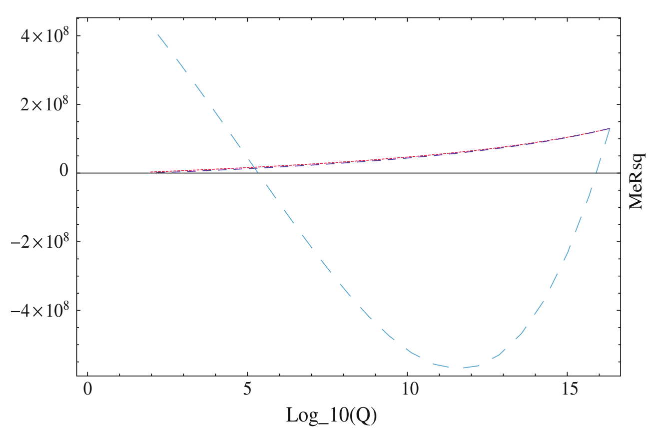

Figure 1. Running right-slepton masses squared showing development of small R-smuon (medium-dashed blue line almost obscured by the small-dashed red line of the R-selectron it leads) which far lags the development of R-stau (large-dashed blue line) which first turns negative and then becomes large driven by the large third-generation Yukawa coupling.

result in the peculiar feature that the running mass squared of the smuon is the smallest of all the sfermion masses and close to the LSP mass. Then the smuon (or other light sfermion as in other cases, up or down squarks of the light generations may emerge lightest) provides the necessary coannihilation channel for achieving an acceptable bino LSP dark matter [25]. The smuon case is doubly attractive as it will also contribute to the muon g-2 anomaly. We note however that the sfermion spectra shown here are not yet loop corrected and that the loop-corrected spectra in solutions which we have found so far do not present this feature.

Besides the immediate satisfaction of finding our model to be realistic and viable, our results imply that the criteria that must be satisfied by candidate UV completions of the (MS)SM should be significantly modified. Candidate models with a large or infinite number of fields must not only show that their parameters can consistently be constrained to yield some variant of the (MS)SM, but also that modification of light wave functions by coupling to heavy fields is both calculable and sensible in the sense of the possibilities considered and demonstrated above. In the absence of such a calculability and consistency, a UV completion model will remain at the level of a phantasmal possibility rather than a scientific model capable of braving this falsification gauntlet to progress to the status of a scientific theory. We have thus shown that for the NMSGUT, careful attention to the quantum communication between the low-energy effective theory and the UV completion through the light Higgs portal yields natural and generic suppression of fast proton decay in SUSY GUTs [28] due to the cumulative effects of the large number of GUT fields that allow the theory to approach and live on the 'Higgs dissolution edge' by renormalizing the light Higgs wave function and thus its Yukawa couplings, even when all individual GUT couplings are well within the perturbative regime.

4. Supersymmetric see-saw inflation

The necessity of cosmological inflation and even its description in terms of a single 'inflaton' slowly rolling down a potential plateau before oscillating around its true minimum to produce quanta of the low-energy theory in the post-inflationary reheating regime is by now well accepted [32]. However, the provenance of the inflaton field and potential is faced by an embarrassing multiplicity of candidates: many of which are compatible [32] with even the latest data [33]. From a particle physics – rather than gravitation physics – viewpoint, identification of an inflaton candidate from among the known particle degrees of freedom is most appealing. The suggestion [34] to utilize one (or more) of the D-flat directions (labelled by the chiral gauge invariants) of the MSSM Lagrangian as an inflaton candidate is thus in vogue. Such a suggestion does however face various stringent constraints such as ensuring a suitable scale of inflation together with the tiny Yukawa couplings for matter fermions required to ensure flatness of the potential during inflation. As neutrino-lepton Yukawa couplings may well be small, flat directions with sneutrino and lepton components are well suited for this role. In [35], an MSSM extended by conjugate neutrinos ($N\equiv \nu_L^c$) superfields to allow (tiny) Dirac neutrino masses and an extra gauge generator (B–L) was used as a framework for an inflaton, made up of the lepton and Higgs doublets (the flat direction is labelled as $\phi \equiv LHN$). Provided a stringent fine tuning is made between the trilinear soft SUSY breaking A_ϕ and the inflaton mass,

ϕ enjoys a renormalizable potential characterized by an inflection point at $\phi_0 \sim m_\phi/h_\nu$. With $m_\phi \sim 100$ GeV–10 TeV, $h_\nu \sim 10^{-12}$. Then $\phi_0 \sim 10^{11}$–10^{15} GeV and the energy density is characterized by a scale $V_0^{1/4} \equiv \Lambda \sim m_\phi/\sqrt{h_\nu} \sim$: 10^8–10^{10} GeV. This model is extremely fine tuned [36] as the inflaton mass M_ϕ is set by the SUSY breaking scale to lie in the TeV range. Moreover, as SUSY breaking parameters are subject to radiative corrections, it seems strange to impose extreme fine tuning on the trilinear coupling of the inflaton which arises only from the soft supersymmetry breaking parameters. It was natural [16] for us to ask the consequences, in this context, of using see-saw [3] masses for the light neutrinos i.e., heavy Majorana masses for the right-handed neutrinos and consequently tiny Majorana masses for the left-handed neutrinos. We showed that the soft parameters were then irrelevant so that tuning was necessary only in the superpotential. The inflaton mass is then set by the large right-handed neutrino masses ($m_\phi \sim M_{\nu^c} \sim 10^6$–$10^{12}$ GeV). Then superpotential Yukawas $h_\nu \sim 10^{-13}(M/\text{GeV})^{1/2}$ could thus be as large as 10^{-6} and have mild fine tuning: rather than the radiatively unstable and highly fine tuned [36] soft symmetry breaking parameters of the Dirac case. The embedding of the supersymmetric see-saw inflation (SSI) model in the NMSGUT was also attempted but could not achieve a large enough number of e-folds. However, the advent of BICEP2 has reopened the whole question of inflaton mass which suddenly made it plausible that it might be anywhere in the range between M_{ν^c} and M_{GUT}! The ratio of tensor to scalar modes r emerges as $r \sim 2(m_\phi/(10^{14}\text{GeV}))^3$ but r will be measurable in the near term only if $r > 10^{-1.5}$. If the value of r is in this ball park, then the inflaton mass would need to be much larger than even the commonly accepted upper limit of around 10^{12} GeV on the right-handed neutrino masses. Interestingly, the NMSGUT embedding studied by us earlier allows a generalization where mass of superheavy Higgs doublets mixed into the inflaton controls its mass allowing inflaton masses right upto the BICEP2-indicated range. This scenario allows raising the achievable number of e-folds by around five orders of magnitude! The contribution of Ila Garg in this volume provides further details.

5. Grand Yukawonification and the flavour-SUSY breaking link

The NMSO(10)GUT has achieved gauge and third-generation Yukawa unification consistent with B violation limits, along with excellent fits of other known fermion data. It makes an intriguing and distinctive prediction regarding sparticle spectra, and may well be compatible with the dominant cold dark matter plus inflation plus leptogenesis cosmological scenario, especially if the scale of inflation turns out to be near to that suggested by BICEP2 [37]. It is only natural to ask whether such a complete model can pretend to shed any light on the most outstanding mystery of particle physics: the flavour puzzle a.k.a. the fermion hierarchy i.e., the origin of the peculiar large intergenerational ratios of charged fermion masses combined with small quark but two large lepton mixing angles. At first sight, as we impose no discrete symmetries and as Yukawa couplings are parameters dialed to fit the data, it seems that this is asking too much. However, reflection on the way in which the SM Yukawa couplings emerge and the fact that the only convincing hint of flavour unification seems to be the convergence of third-generation Yukawa couplings at the GUT scale suggests that perhaps the Yukawa couplings may emerge via spontaneous breaking of the $U(3)$ flavour symmetry of the $SO(10)$ invariant matter kinetic terms. In [17] we suggested that our work [8,11,12,21,22,28] on MSGUTs naturally and minimally identifies MSGUT Higgs multiplets as candidate Yukawon multiplets. It thus yields a novel mechanism whereby the fermion hierarchy could emerge from a flavour symmetric and renormalizable GUT. In our work 'Yukawons' also carry representations of the gauge (SM/GUT) dynamics. In previous work, typically the dimension-1 Yukawa-on $\mathcal{Y}$ in the Higgs vertex makes it non-renormalizable ($\mathcal{L} = f^c \mathcal{Y} f H/\Lambda_{\mathcal{Y}} + \cdots$: where the unknown high-scale $\Lambda_{\mathcal{Y}}$ controls Yukawa-on dynamics). Minimal $SO(10)$ GUTs [8,9,11,12] provide a gauged $O(N_g)$ family symmetry route to 'Yukawonifcation'– with the GUT and family symmetry breaking at the same scale – obviating the need for non-renormalizable interactions and any extraneous scale $\Lambda_{\mathcal{Y}}$. As we shall see, the consistency conditions for the maintainability of the Higgs portal to the UV completion of the MSSM play a central role in determining just how the peculiarly lopsided and 'senseless' fermion hierarchy is produced from the flavour symmetric and grand unified UV completion. In minimal SUSY GUTs, one eschews invocation of discrete symmetries and insists only upon following the logic of $SO(10)$ gauge symmetry. This insistence, combined with careful attention to the implications of the emergence of a single light MSSM Higgs pair from the $2N_g(N_g+1)$ pairs in the $O(N_g)$ extended MSGUT, leads to an effectively unique extension of the $SO(10)$ gauge group by a $O(N_g)$ family gauge symmetry for the N_g generation case and the dynamical emergence of fermion hierarchy and mixing. The $\overline{\mathbf{126}}(\bar{\Sigma})$ also contributes to both neutrino and charged fermion masses. Gauging just an $O(N_g)$ subgroup of the $U(N_g)$ symmetry of the fermion kinetic terms seems the workable option, in contrast to a unitary family group, because the use of complex representations introduces anomalies and requires doubling of the Higgs structure to cancel anomalies and to permit holomorphic invariants to be formed for the superpotential. Worse, unitary symmetry enforces vanishing of half the emergent matter Yukawa couplings. $O(N_g)$ family symmetry suffers from none of these defects and gauging it ensures that no Goldstone bosons arise when it is spontaneously broken. We emphasize that in contrast with previous 'spurion/Yukawa-on' models (see e.g. [38]), our model is renormalizable and GUT-based.

The GUT superpotential has exactly the same form as the MSGUT (see [12,20,21,23] for comprehensive details):

$$\begin{aligned} W_{\text{GUT}} &= \text{Tr}(m\Phi^2 + \lambda\Phi^3 + M\bar{\Sigma}\cdot\Sigma + \eta\Phi\cdot\bar{\Sigma}\cdot\Sigma) \\ &\quad + \Phi\cdot H\cdot(\gamma\Sigma + \bar{\gamma}\cdot\bar{\Sigma}) + M_H H\cdot H), \\ W_F &= \Psi_A\cdot((hH) + (f\Sigma) + (g\Theta))_{AB}\Psi_B. \end{aligned} \quad (10)$$

We have shown how the **120**-plet is included in W_F but have studied only MSGUTs (i.e., with **10**, $\overline{\mathbf{126}}$). Inclusion of the **120**-plet does not affect GUT SSB. The only innovation in Higgs structure is that all the MSGUT Higgs fields now carry symmetric representations of the $O(N_g)$ family symmetry: $\{\Phi, \bar{\Sigma}, \Sigma, H\}_{AB}$; $A, B = 1, 2..N_g$ (under which the matter **16**-plets ψ_A are vector N_g-plets). Couplings h, f, g are single complex numbers, while the Yukawons carry symmetric ($\mathbf{H}$, $\bar{\mathbf{\Sigma}}$) and antisymmetric (Θ) representations of $O(3)$: as required by the transposition property of relevant $SO(10)$ invariants. For $N_g = 3$, real fermion mass parameters come down from 15 (Re[h_{AA}], f_{AB}) to just 3 (Re[h], f) without the **120**-plet (6 additional to just 2 with the **120**-plet). Thus, this type of renormalizable flavour-unified GUTs can legitimately be called Yukawon ultraminimal

GUTs (YUMGUTs). Further details on the gauge symmetry breaking and determination of Yukawas can be found in the contribution of Charanjit Kaur in this volume. We shall focus on our resolution [18] of a severe technical difficulty that crops up when we spontaneously break a gauged $O(3)$ subgroup of the $U(3)$ supersymmetric flavour symmetry. If this difficulty is resolved, the determination of the viability of the 'grand Yukawonification' model then becomes a matter of searching the relatively small remaining parameter space for viable parameter sets that fit the fermion data at M_X, while taking account of threshold corrections at low and high scales and while respecting constraints on crucial quantities like the proton lifetime [12,21,28]. Note that in this approach, not only are the hard parameters of the visible sector superpotential reduced by replacement of the flavoured parameters by bland family symmetric ones but also the soft supersymmetry breaking parameters are determined by the two parameters of the hidden sector superpotential and the Planck scale.

With just the family index carrying MSGUT Higgs present, the flavour D-terms cannot vanish. So additional fields with VEVs free to cancel the contribution $\bar{D}_X^A$ of the GUT sector to $O(N_g)$ D terms are needed. The extra F terms must be sequestered from the GUT sector to preserve the MSGUT SSB. The special role of the Bajc–Melfo supersymmetry breaking is that it provides flat directions in both the singlet and the gauge-variant parts of a symmetric chiral supermultiplet $S_{AB} = S_{BA}$. As it is very difficult to make the contribution of the visible sector GUT fields to the $O(N_g)$ D-terms vanish, the $\hat{S}$ flat direction performs the invaluable function of cancelling this contribution without disturbing the symmetry breaking in the visible sector. We proposed [18] Bajc–Melfo-type two-field superpotentials [39,40] (of structure $W = S(\mu_B\phi + \lambda_B\phi^2)$). Their potentials have local minima breaking supersymmetry ($\langle F_S\rangle \neq 0$) which leaves the VEV $\langle S\rangle$ undetermined (ϕ gets a VEV). If ϕ, S transform in symmetric representations of $O(N_g)$, then the undetermined VEV of the trace-free part of S is determined at the global supersymmetry level by the minimization of the $O(N_g)$ D-terms thus preserving supersymmetry from high-scale breaking. On the other hand, BM superpotentials and their metastable (local) supersymmetry breaking vacua function efficiently [18] as hidden sectors for supergravity GUT models: coupling to supergravity determines the $O(N_g)$ singlet part of S to have a VEV of order of the Planck scale. This felicitous and unexpected marriage of the NMSGUT to gauged flavour symmetry and supergravity, reduces the number of $SO(10)$ Yukawas drastically and makes a quite novel linkage between spontaneous violation of flavour and supersymmetry breaking. As the traceless parts of the fermionic components of S are massless at tree level (the trace part is eaten by the massive gravitino) they get masses only via radiative corrections and tend to remain very light. They could thus provide very light dark matter candidates as well as cause cosmological problems of the sort typically associated with moduli fields [41]. This requires further detailed study but it is interesting that such a simple extension of the NMSGUT can both reduce free parameters and provide moduli dynamics by taking on the meta-problem of the origin of flavour hierarchy. The structure used entails yet further stringent constraints as the masslessness of the moduli multiplets $\hat{S}_{AB}$ before supersymmetry breaking implies the existence of $N_g(N_g+1)/2 - 1$ SM singlet fermions generically lighter than the gravitino mass scale and possibly as light as a few GeV. In addition, the Polonyi mode S_s may also lead to difficulties in the cosmological scenario. Thus, such modes can be both a boon and a curse for familion GUT models: a boon because generic SUSY GUT models are hard put if asked to provide SUSY WIMPs of mass below 100 GeV as CDM candidates as suggested by the DAMA/LIBRA experiment [42] and a curse because there are strong constraints on the existence of such light moduli which normally demand that their mass be rather large (>10 TeV) due to the robust cosmological (Polonyi) problems arising from decoupled modes with Planck scale VEVs [43]. In contrast to the simple Polonyi model and string moduli, the BM moduli have explicit couplings to light fields through family D-term mixing and loops. Moreover, the MSGUT scenario favours [12,28] large gravitino masses >5–50 TeV. Thus, the Polonyi and moduli problems may be evaded. In any case, the cosmology needs to be considered seriously only after we have shown that the MSSM fermion spectrum is indeed generated by the 'Yukawonified' NMSGUT [44].

6. Discussion

The vast range of applicability of the NMSGUT, justifies its claim that it is a theory for all epochs. Not only does it present a well-controlled framework for realizing the longstanding dream of grand unification in a completely realistic fashion, it also makes distinctive predictions concerning the observation of supersymmetry and lepton flavour violation and dynamically palliates the proton decay problem that has chronically afflicted SUSY GUTs without introducing extraneous structures but in a way intrinsic to the GUT itself. Furthermore, it provides a surprisingly simple context for embedding high-scale supersymmetric inflation. The right-handed neutrinos and adequate, CKM–PMNS-linked, CP violation that are present can enable leptogenesis. Reaching still further, it even serves as a basis for the generation of flavour hierarchy by the spontaneous violation of a gauged flavour symmetry at the GUT scale and in the process makes the startling suggestion that supersymmetry breaking is linked to flavour violation via a novel BM hidden sector. This implies a striking and novel (for SUSY GUTs) prediction of possibly very light non-neutralino dark matter candidates. Interestingly, the self-coupling of this light DM is not constrained. The working out of the various flavour breaking scenarios while maintaining realism in the fermion and GUT SSB sector and accounting for low- and high-scale threshold corrections is indeed a formidable but manageable project that has already cleared several hurdles [12,22,28,45] that might well have falsified this model in the three decades since its proposal. These successes motivate our belief that a focus on the implication of the consistency conditions that define our world as we know it, in the context of the flavour symmetric NMSGUT, may further extend the scope of the NMSGUT to include a dynamical understanding of the flavour hierarchy. Moreover, the strong reduction in the number of parameters makes falsification much more practicable. Although it continues to live dangerously, we hope that this will be seen as a scientific virtue of the NMSGUT and its generalizations rather than as an unnecessary concreteness and optimism, with which it is sometimes reproached.

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