PRAMANA
— journal of
physics

Vol. 86, No. 1
January 2016
pp. 185–190

Predicting superdeformed rotational band-head spin in $A \sim 190$ mass region using variable moment of inertia model

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MS received 16 July 2014; revised 26 October 2014; accepted 27 October 2014
DOI: 10.1007/s12043-015-1013-9; ***e*Publication:** 27 August 2015

Abstract. The band-head spin (I_0) of superdeformed (SD) rotational bands in $A \sim 190$ mass region is predicted using the variable moment of inertia (VMI) model for 66 SD rotational bands. The superdeformed rotational bands exhibited considerably good rotational property and rigid behaviour. The transition energies were dependent on the prescribed band-head spins. The ratio of transition energies over spin $E\gamma/2I$ (RTEOS) vs. angular momentum (I) have confirmed the rigid behaviour, provided the band-head spin value is assigned correctly. There is a good agreement between the calculated and the observed transition energies. This method gives a very comprehensive interpretation for spin assignment of SD rotational bands which could help in designing future experiments for SD bands.

Keywords. Band-head spin; band-head moment of inertia; variable moment of inertia model; restoring force constant; transition energy ratio.

PACS No. 21.60.−n

1. Introduction

Superdeformed (SD) nuclei are one of the most challenging and interesting area of nuclear research, since the first SD band in ^{152}Dy was observed [1]. Over the past two decades more than 320 SD bands have been observed in different mass regions like $A \sim 60$, 80, 130, 150 and 190 [2,3]. The SD mass regions, $A \approx 60$–90, are very interesting as there are limited number of particles in these bands and exhibit highest rotational frequencies. Large number of SD bands have been observed but still many problems exist like assigning spin, parity and identifying excitation energy relative to the ground states. At present, the γ-ray energies are the only spectroscopic information universally available because of the non-observable discrete linking transitions between

SD states and the low-lying states at normal deformation (ND). In 1996, the discrete γ-rays connecting two different states, the yrast SD band ^{194}Hg(1) to ND states, were discovered and spins were assigned [4]. This helped to establish the spins and excitation energies of all members of ^{194}Hg(1) experimentally. Immediately after this study, the spins and excitation energies of the yrast SD band of ^{194}Pb [5] and ^{194}Hg(3) [6] were also established.

Unfortunately, after this, very few experiments could correctly assign spins of SD bands. This was mainly due to the absence of experimental information of spin and energies. The only way was to calculate the spin values theoretically. Several efforts for assigning spins of SD states have been proposed in $A \sim 190$ mass region. Some of the successful theoretical methods used are: energy expansion $I(I+1)$ [7], energy power series in terms of angular momentum [8,9], ω^2 expansion [10], *ab* formula [11–13] and the supersymmetric algebraic one including many-body interactions (SAM) [14–20]. Some of these methods were proposed to assign band-head spins and transition energies as a function of spin or rotational frequency ω. However, the spin assignments in these studies were not in good agreement with the experimental values.

In this paper a simple variable moment of inertia (VMI) model equation is proposed to assign band-head spin directly from experimentally observed transition energy ratio which requires less computation time. Transition energy ratio (R), from VMI model, is dependent on two parameters: restoring force constant (C) and moment of inertia ($\mathcal{J}_0$). These two parameters were calculated by a best-fitting method involving all the experimentally known transition energies with spin values. This method could correctly assign band-head spin while other spins are assigned in terms of I_0, I_0+2, I_0+4, and so on, along with γ energies for SD rotational bands. Band-head spin and transition energies for 66 SD rotational bands in $A \sim 190$ mass region for 22 isotopes of Au, Hg, Tl, Pb, Bi and Po, were calculated. The values obtained were compared with the experimental spectra and the literature values. The results are in good agreement with the earlier results.

This study also confirms the rigid behaviour of these bands for correct band-head spin assignment. The minimum r.m.s. (root mean square) deviation value, $\mathcal{X}$, depends on the correct assignment of the band-head spin (I_0). In some cases $\mathcal{X}$ was found insensitive to the suggested spins. This may be due to the r.m.s. deviation being close to each other for two or more spins making it difficult for unique spin assignment. In §2 we present the formalism for assigning band-head spin and fitted transition energies of SD rotational bands using VMI model. In §3 the obtained results are compared and all 66 SD bands observed in 22 isotopes in $A \sim 190$ mass region are discussed. Finally, the conclusion follows in §4.

2. Formalism

VMI model can theoretically explain the rotational energy in relation with angular momentum (I) of each level, given as the sum of the potential energy term $(\mathcal{J}_\mathrm{I} - \mathcal{J}_0)^2$ and rotational energy term $(\hbar^2 I(I+1)/2\mathcal{J}_\mathrm{I})$. These terms depend on the difference in moment of inertia ($\mathcal{J}_\mathrm{I}$) from that of the ground-state moment of inertia ($\mathcal{J}_0$). However, VMI model is a two-parameter formula which characterizes each nucleus by band-head moment of inertia ($\mathcal{J}_0$) and restoring force constant (C).

In VMI model, the energy level of $I_0 = 0$, in ground-state bands in even–even nuclei is represented as [21]

$$E_I(\mathcal{J}) = \frac{1}{2}C(\mathcal{J}_I - \mathcal{J}_0)^2 + \frac{1}{2}\left[\frac{I(I+1)}{\mathcal{J}_I}\right] \tag{1}$$

while the band-head energy levels ($I_0 \neq 0$) of rotational band is represented as [22]

$$E_I = E_0 + \frac{1}{2\mathcal{J}_I}[I(I+1) - I_0(I_0+1)] + \frac{1}{2}C(\mathcal{J}_I - \mathcal{J}_0)^2, \tag{2}$$

where E_0 is the band-head energy of the rotational band, $\mathcal{J}_0$ is the ground-state moment of inertia, variable $\mathcal{J}_I$ is the moment of inertia of the nucleus for each spin and C is the restoring force constant. In eq. (1) and the following expressions we assume $\hbar = 1$.

The variable moment of inertia $\mathcal{J}_I$ is determined by the equilibrium condition,

$$\partial E(\mathcal{J}_I)/\partial \mathcal{J}_I = 0. \tag{3}$$

This leads us to the expression

$$\mathcal{J}_I = \mathcal{J}_0/\left\{1 - [I(I+1) - I_0(I_0+1)]/2C\mathcal{J}_I^3\right\}. \tag{4}$$

This is equivalent to the cubic equation,

$$\mathcal{J}_I^3 - \mathcal{J}_I^2\mathcal{J}_0 - \{[I(I+1) - I_0(I_0+1)]/2C\} = 0. \tag{5}$$

The cubic equation has one real root for any finite positive value of $\mathcal{J}_0$ and C.

Combining eq. (4) with eq. (2), we get

$$E_I = E_0 + \left[\frac{I(I+1) - I_0(I_0+1)}{2\mathcal{J}_0}\right]\left[1 + \frac{I(I+1) - I_0(I_0+1)}{4C\mathcal{J}_0^3}\right]. \tag{6}$$

For the state, $I-2$

$$E_{I-2} = E_0 + \left[\frac{(I-2)(I-1) - I_0(I_0+1)}{2\mathcal{J}_0}\right]\left[1 + \frac{(I-2)(I-1) - I_0(I_0+1)}{4C\mathcal{J}_0^3}\right]. \tag{7}$$

For SD bands, the transition energy is expressed as

$$E\gamma(I \to I-2) = E(I) \to E(I-2).$$

Therefore, we get

$$E\gamma(I \to I-2) = \frac{1}{2\mathcal{J}_0}[I(I+1) - (I-2)(I-1)] + \frac{1}{8C\mathcal{J}_0^4}\left\{[I(I+1)]^2 - [(I-2)(I-1)]^2\right\}. \tag{8}$$

For an SD cascade

$$I_0 + 2n \to I_0 + 2n - 2 \to \cdots\cdots \to I_0 + 2 \to I_0.$$

The observed transition energies are

$$E\gamma(I_0+2n),\ E\gamma(I_0+2n-2),\ E\gamma(I_0+2n-4)\cdots E\gamma(I_0+4) \quad \text{and} \quad E\gamma(I_0+2).$$

The band-head spin is determined in terms of the observed transition energy ratios. The transition energy ratio (R) can be expressed as

$$R = \frac{E\gamma(I+2 \rightarrow I)}{E\gamma(I \rightarrow I-2)}$$

or

$$R = \frac{\frac{(I+2)(I+3)-I_0(I_0+1)}{2\mathcal{J}_0}\left[1+\frac{(I+2)(I+3)-I_0(I_0+1)}{4C\mathcal{J}_0}\right]-\left\{\frac{I(I+1)-I_0(I_0+1)}{2\mathcal{J}_0}\left[1+\frac{I(I+1)-I_0(I_0+1)}{4C\mathcal{J}_0}\right]\right\}}{\frac{I(I+1)-I_0(I_0+1)}{2\mathcal{J}_0}\left[1+\frac{I(I+1)-I_0(I_0+1)}{4C\mathcal{J}_0}\right]-\left\{\frac{(I-2)(I-1)-I_0(I_0+1)}{2\mathcal{J}_0}\left[1+\frac{(I-2)(I-1)-I_0(I_0+1)}{4C\mathcal{J}_0}\right]\right\}}. \quad (9)$$

The transition energy ratio (R) depends on two parameters C and $\mathcal{J}_0$. The band-head spin (I_0) is determined from R.

The r.m.s. deviations of calculated transition energies for different I_0 values are calculated. The r.m.s. value is the least for band-head spin value of a band. If I_0 shifts away from the accurate value by ± 1, rapid shift in r.m.s. deviation ($\mathcal{X}$) can be observed. The r.m.s. deviation,

$$\mathcal{X} = \left[\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E\gamma^{\text{cal}}(I_i) - E\gamma^{\text{exp}}(I_i)}{E\gamma^{\text{exp}}(I_i)}\right|^2\right]^{1/2}, \quad (10)$$

where n is the total number of transitions involved in the fitting.

As the lowest spin I_0 is known, all the spin values of the SD band levels can be determined. It is clear that the fitting procedure is quite simple and straightforward. Therefore, measured spins for these SD bands from VMI equation are successfully verified from the r.m.s. value. In some cases, the r.m.s. deviation differs from the band-head spin values suggested by the VMI model. The r.m.s. deviation is similar for two or more spin values which made it difficult to assign one spin values for the band. In table 1b* we have reported 8 SD bands, where the VMI results differ from r.m.s. deviation values.

3. Results and discussion

VMI model is used to predict the band-head spin of 66 SD rotational bands for 22 isotopes of Au, Hg, Tl, Pb, Bi and Po in $A = 190$ mass region. The two parameters, band-head moment of inertia ($\mathcal{J}_0$) and restoring constant (C), are determined by fitting the experimentally known transition energies using the best-fit method (BFM). The calculated transition energies are in agreement with the observed transition energies of Au, Hg, Tl, Pb, Bi and Po isotopes. Band-head spin is compared with experimentally known spin values which provide a significant test of validity of this approach. The results are given in table 1a* for 58 SD bands for which spin values are calculated. The results were in agreement with other methods in the literature. It is found that the r.m.s. deviation is minimum for the same spin value except for 8 isotopes, ^{191}Hg(b1, b2), ^{195}Hg(b1, b2), ^{192}Tl(b1, b2), ^{193}Pb(b2) and ^{195}Pb(b2) reported in table 1b*. The b1, b2 represents band 1 and band 2. In these isotopes the r.m.s. deviation may be similar for two or more spin

values making it difficult to assign the correct spin. Significant amount of band mixing occurs during energy transition that could influence r.m.s. deviation while fitting with least mean squares.

Nuclear softness parameter (σ) for SD bands were observed in the range $10^{-4} \leq \sigma \leq 10^{-6}$ as compared to the value for normal deformed (ND) bands in the range of $10^{-2} \leq \sigma \leq 10^{-4}$. The SD bands are observed to be much more rigid than ND bands [23–26]. SD bands become more and more rigid as deformation increases, suggesting a correlation between pairing nuclei and deformation bands. The rigid behaviour was more prominent when the ratio of transition energies over spin ($E\gamma/2I$) were plotted. This was found constant for rigid rotors, if accurate spin I_0, was assigned correctly. However, this may shift to hyperbola if I_0 deviates from the accurate value ± 1. The transition energies plotted for ^{194}Tl(b1) and ^{195}Pb(b1) are shown in figure 1*.

We also compared $E\gamma/2I$ vs. spin of VMI with the experimental plots. Both values are in good agreement. The r.m.s. deviation gives minimum possible value for the band-head spin. This helps to give comprehensive interpretation of the spin assignment of SD bands. Most of the SD bands were showing rigid behaviour except in few cases where the spin values were found to be the same as the experimental values and r.m.s. values were not in agreement, demanding further investigations. SD rotational bands like ^{191}Hg(b1, b2), ^{195}Hg(b1, b2), ^{192}Tl(b1, b2), ^{193}Pb(b2) and ^{195}Pb(b2) were not showing rigid behaviours. The energy spectra plots are shown in figure 2*.

Liang-Zhu *et al* [27] used ratio of transition energy over spin (RTEOS) method for assigning spin value in 190 and 150 mass regions in which changes in rigid rotational band transition energy were explained. This study explains changes in band-head spin value from the exact spin value, I_0 to $I_0 \pm 1$, claiming spin as a simulated function. Liang also pointed out that RTEOS is not sufficient in high spin values ($I_0 > 25$) due to the changes in RTEOS which is not sensitive to the relatively high spin values, concluding that SD bands in $A \sim 190$ mass region ($I_0 > 25$) are good rotational bands [27]. If more transition energies connecting lower spin states are found in the future, RTEOS can work even better. It is mentioned that RTEOS works well for rotational bands, but not for vibrational bands. Our study also found that when the transition energy ratio $E\gamma/2I$ vs. spin are plotted, most of the plots have shown rigid behaviour of these bands in $A \sim 190$ region provided correct values of band-head spin are assigned. The calculated transition energies were found to coincide with the observed values if it deviates for $I_0 + 1$ or $I_0 - 1$ values. The obtained values are compared with other theoretically calculated values.

In this paper the results are characterized as:

(1) Nuclei in which VMI and observed energy spectra are showing rigid behaviour and have good fitting. The obtained spin value is justified well by r.m.s. calculation. This is shown for ^{194}Tl(b1) and ^{195}Pb(b1) in figure 1*.
(2) The spin values calculated by VMI are not in agreement with the r.m.s. value. For eight SD bands ^{191}Hg(b1,b2), ^{195}Hg(b1,b2), ^{192}Tl(b1,b2), ^{193}Pb(b2) and ^{195}Pb(b2) are shown in figure 2*.

*Figures 1 and 2 and tables 1a and 1b can be accessed from http://www.ias.ac.in/pramana/v86/supplement.pdf.

4. Conclusion

This study was an effort to assign band-head spin (I_0) of SD rotational bands by VMI equation. We have assigned band-head spin of 66 SD bands in $A \sim 190$ mass region. The obtained γ energies are in good agreement with the observed spectra, highlighting that this method can efficiently calculate band-head spin for SD bands. This would help researchers to study spectroscopy of SD bands. This is only a preliminary work and the research is progressing to give systematic features of rotational bands of SD nuclei.

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