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Oscillating two-stream instability of laser wakefield-driven plasma wave

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Abstract. The laser wakefield-driven plasma wave in a low-density plasma is seen to be susceptible to the oscillating two-stream instability (OTSI). The plasma wave couples to two short wavelength plasma wave sidebands. The pump plasma wave and sidebands exert a ponderomotive force on the electrons driving a low-frequency quasimode. The electron density perturbation associated with this mode couples with the pump-driven electron oscillatory velocity to produce nonlinear currents driving the sidebands. At large pump amplitude, the instability grows faster than the ion plasma frequency and ions do not play a significant role. The growth rate of the quasimode, at large pump amplitude scales faster than linear. The growth rate is maximum for an optimum wave number of the quasimode and also increases with pump amplitude. Nonlocal effects, however reduce the growth rate by about half.

Keywords. Oscillating two-stream instability; plasma wave; laser wakefield accelerator.

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1. Introduction

The excitation of large amplitude large phase velocity plasma wave is the key factor in plasma-based electron accelerators [1–5]. The phase velocity of the plasma wave is approximately equal to the group velocity of the laser pulse ($\sim c$, the speed of light in vacuum) and hence, these plasma waves are highly suitable for high-energy particle acceleration. The plasma wave can be driven by a short-duration electron beam or an intense short pulse laser. The former scheme is employed in plasma wakefield accelerator (PWFA) [6–9] and the latter in laser wakefield accelerator (LWFA) [1,10,11]. Laser-induced wakefields were first observed as coherent far-infrared radiations from laser-produced plasmas by Hamster *et al* [12]. Recent experiments have demonstrated the generation of monoenergetic electrons of hundreds of MeV energy by short pulse laser plasma interaction [13–15]. Gorbunov *et al* [16] used an ultrashort petawatt laser for electron acceleration by exciting a nonlinear plasma wakefield which can accelerate injected electrons up to GeV energies. Andreev *et al* [17] have predicted acceleration over an extended interaction length of $\sim$10 cm for a CO_2 laser. Simulations indicate a wake amplitude $>$1 GeV/m for a 2 ps, 2.5 TW laser pulse. Wei *et al* [18] have reported more energetic electrons and ion accelerations up to 15 MeV. They have indicated that the ponderomotive force at the pulse front may be providing initial energy to electrons to be followed by plasma wave acceleration and direct laser acceleration. Gahn *et al* [19] have reported experimental results on multi-MeV electron beam generation by direct laser acceleration in high-density plasma channels. Chen *et al* [20] have observed electron acceleration from a helium gas jet of density 1.8×10^{19} cm^{-3}, employing two successive 0.8 μm, 8×10^{17} W/cm^2 laser pulses of 55 and 250 fs duration, respectively. Fritzler *et al* [21] have observed 55 MeV electrons from gas jet targets irradiated by 1 μm, 3×10^{18} W/cm^2, 70 fs laser pulses at an electron density of $n \sim 2.5 \times 10^{19}$ cm^{-3}.

An interesting issue in these acceleration schemes is the stability of the plasma wave. Particle-in-cell (PIC) simulations of LWFA indicate that the plasma wave develops short wavelength distortions on a time-scale comparable or shorter than an ion plasma period. Early studies on moderate intensity laser interaction with high-density plasmas have revealed the excitation of short wavelength plasma waves near the critical layer that could be attributed to the excitation of oscillating two-stream instability (OTSI). In this process, a long wavelength pump wave excites two short wavelength Langmuir wave sidebands and a purely growing zero-frequency mode. In regions where the electric fields of the pump and Langmuir waves are parallel, the plasma is pushed away to the regions where the fields are antiparallel. The depressed density regions attract more electric field energy from the neighbourhood leading to deeper density depressions and enhancement of the short-wavelength Langmuir waves [5]. The electron and ion responses to the low-frequency mode are adiabatic as growth rate Γ is shorter than kv_{the} and kv_{thi}, where k is the wave number of the mode and v_{the}, v_{thi} are the electron and ion thermal speeds. One may have OTSI relevant to laser-driven plasma wave as well. In the intense short pulse laser–plasma interaction, the growth rate of OTSI may far exceed kv_{thi} and adiabatic ion approximation may not remain valid. Gupta *et al* [22] have indicated the relevance of OTSI in the nonlinear saturation of laser-driven plasma beat wave. The sideband plasma waves divert the energy of plasma beat wave and saturate it.

In this paper, we examine the OTSI in the context of LWFA. The wakefield plasma wave has large phase velocity and large amplitude and is localized radially. It imparts large oscillatory velocity to electrons at frequency $\omega_0 \approx \omega_{\mathrm{p}}$ and wave number $k_0 \sim \omega_{\mathrm{p}}/c$, where ω_{p} is the electron plasma frequency. It couples a short-wavelength low-frequency mode ω, k; $\vec{k} > k_0$, $\omega \ll \omega_0$, to two Langmuir wave sidebands $(\omega_{1,2}, \vec{k}_{1,2})$. The pump and Langmuir wave sidebands exert a low-frequency ponderomotive force $\vec{F}_{\mathrm{p}}$, on electrons, driving the low-frequency mode, i.e., causing local density depression $n\,(\omega, \vec{k})$. This density perturbation in conjunction with the oscillating electron velocity $\vec{v}_0$ at $(\omega_0, \vec{k}_0)$ produces nonlinear density perturbations $n_{1,2}^{\mathrm{NL}}$ at $(\omega_{1,2}, \vec{k}_{1,2})$ that drive the sidebands.

In §2, the local theory of OTSI of laser wakefield-driven plasma wave is discussed. In §3, the nonlocal effects are studied. Section 4 summarizes the conclusion.

2. Local theory

Consider a homogeneous unmagnetized plasma of electron density n_0^0 and electron temperature T_{e}. The ions are singly ionized and cold. A short laser pulse propagates through it with the electric field,

$$\vec{E}_{\mathrm{L}} = \hat{x} A_{\mathrm{L}}(t,z) \exp[-i(\omega_{\mathrm{L}} t - k_{\mathrm{L}} z)],$$
$$A_{\mathrm{L}}^2 = \mathrm{A}_{\mathrm{L0}}^2 \exp[-(t - z/v_g)^2/\tau], \tag{1}$$

where $v_g = c(1-\omega_{\mathrm{p}}^2/2\omega_{\mathrm{L}}^2)$, $\omega_{\mathrm{p}} = (4\pi n_0^0 e^2/m)^{1/2}$, $\omega_{\mathrm{p}} \ll \omega_{\mathrm{L}}$, $-e$ and m are the electron charge and mass, and pulse duration τ is comparable to plasma period, $\tau \sim 2\pi\omega_{\mathrm{p}}^{-1}$.

The laser imparts an oscillatory velocity to electrons,

$$\vec{\mathrm{v}}_{\mathrm{L}} = e\vec{E}_{\mathrm{L}}/mi\omega_{\mathrm{L}}, \tag{2}$$

and exerts a ponderomotive force $\vec{F}_{\mathrm{p0}} = e\nabla\phi_{\mathrm{p0}}$ on them. In the non-relativistic limit,

$$\phi_{\mathrm{p0}} = -\frac{m}{2e}|v_{\mathrm{L}}|^2 = -\frac{eA_{\mathrm{L0}}^2 \mathrm{e}^{-(t-z/v_g)^2/\tau^2}}{2m\omega_{\mathrm{L}}^2}, \tag{3}$$

when $|v_{\mathrm{L}}|$ approaches c, $\phi_{\mathrm{p0}} \approx -\left(mc^2/2e\right)\left[\left(1 + a_{\mathrm{L}}^2/2\right)^{1/2} - 1\right]$, where $a_{\mathrm{L}} \approx |v_{\mathrm{L}}|/c$.

The ponderomotive force drives a plasma wave of potential

$$\phi_0 = A_0(r,z,t)\mathrm{e}^{-i(\omega_0 t - k_0 z)}. \tag{4}$$

In the non-relativistic limit,

$$A_0 \approx \frac{eA_{\mathrm{L0}}^2\sqrt{\pi}}{2.3m\omega_{\mathrm{L}}^2}$$

which is of the same order as the peak value of the ponderomotive potential.

The plasma wave provides oscillatory velocity to electrons, $\vec{v}_0 = -ek_0\phi_0\hat{z}/m\omega_0$ and couples to a low-frequency electrostatic mode of potential

$$\phi = A\exp\left[-i(\omega t - \vec{k}\cdot\vec{r})\right],$$

and two shorter wavelength Langmuir wave sidebands

$$\phi_j = A_j \exp\left[-i(\omega_j t - \vec{k}_j\cdot\vec{r})\right], \tag{5}$$

where

$$\omega_{1,2} = \omega \mp \omega_0 \quad \text{and} \quad \vec{k}_{1,2} = \vec{k} \mp \vec{k}_0 (\vec{k} > \vec{k}_0), \vec{k}_j = k_{jz}\hat{z} \text{ and } j = 1,2.$$

The sidebands contribute oscillatory velocities to electrons $\vec{v}_j = -e\vec{k}_j\phi_j/m\omega_j$, and in conjunction with the pump exert a low-frequency ponderomotive force on electrons at $(\omega,\vec{k})$

$$\vec{F}_{\mathrm{p}} = e\nabla\phi_{\mathrm{p}} = -(m/2)\nabla(\vec{v}_0\cdot\vec{v}_1 + \vec{v}_0^*\cdot\vec{v}_2),$$

i.e.,

$$\phi_{\mathrm{p}} = \frac{\vec{k}_1\cdot\vec{v}_0}{2\omega_1}\phi_1 + \frac{\vec{k}_2\cdot\vec{v}_0^*}{2\omega_2}\phi_2. \tag{6}$$

The ponderomotive and self-consistent potentials ϕ_{p} and ϕ produce electron and ion density perturbations, $n,\ n_{\mathrm{i}}$. In the limit $\omega \ll kv_{\mathrm{th}}$, where $v_{\mathrm{th}} = (2T_{\mathrm{e}}/m)^{1/2}$ is the electron thermal speed, $n,\ n_{\mathrm{i}}$ can be denoted in terms of electron and ion susceptibilities $\chi_{\mathrm{e}},\ \chi_{\mathrm{i}}$ as

$$n = \frac{k^2}{4\pi e}\chi_{\mathrm{e}}\left(\phi + \phi_{\mathrm{p}}\right), \tag{7}$$

$$n_{\mathrm{i}} = -(k^2/4\pi e)\chi_{\mathrm{i}}\phi \tag{8}$$

and

$$\chi_{\mathrm{e}} \approx 2\omega_{\mathrm{p}}^2/k^2v_{\mathrm{th}}^2 = \omega_{\mathrm{pi}}^2/k^2c_{\mathrm{s}}^2,$$

$$\chi_{\mathrm{i}} = -\omega_{\mathrm{pi}}^2/\omega^2,$$

where $c_{\mathrm{s}} = (T_{\mathrm{e}}/m_{\mathrm{i}})^{1/2}$ is the speed of sound and $\omega_{\mathrm{pi}} = \left(4\pi n_0^0e^2/m_{\mathrm{i}}\right)^{1/2}$ is the ion plasma frequency. Using n and n_{i} in the Poisson's equation $\nabla^2\phi = 4\pi e(n - n_{\mathrm{i}})$, we obtain

$$\varepsilon\phi = -\chi_{\mathrm{e}}\phi_{\mathrm{p}}, \tag{9}$$

where

$$\varepsilon = 1 + \chi_{\mathrm{e}} + \chi_{\mathrm{i}}.$$

The nonlinear density perturbations n_1^{NL} at the lower sideband, on solving the equation of continuity, $\partial n_1^{\mathrm{NL}}/\partial t + (1/2)\nabla\cdot\left(n\vec{v}_0^*\right) = 0$, can be written as

$$n_1^{\mathrm{NL}} = n\frac{\vec{k}_1\cdot\vec{v}_0^*}{2\omega_1} = \frac{\vec{k}_1\cdot\vec{v}_0^*}{2\omega_1}\frac{k^2}{4\pi e}\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{\varepsilon}\phi_{\mathrm{p}}. \tag{10}$$

Similarly, for the upper sideband, the density perturbation is

$$n_2^{\mathrm{NL}} = n\frac{\vec{k}_2\cdot\vec{v}_0}{2\omega_2} = \frac{\vec{k}_2\cdot\vec{v}_0}{2\omega_2}\frac{k^2}{4\pi e}\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{\varepsilon}\phi_{\mathrm{p}}. \tag{11}$$

The self-consistent potentials ϕ_1, ϕ_2 produce the linear density perturbations at the sidebands

$$n_j^{\mathrm{L}} = (k_j^2/4\pi e)\chi_{\mathrm{ej}}\phi_j, \quad j = 1, 2, \tag{12}$$

where χ_{ej} are the electron susceptibilities at sidebands $(\omega_j, \vec{k}_j)$. One may write χ_{ej}, including thermal effects, as $\chi_{\mathrm{ej}} = -(\omega_{\mathrm{p}}^2 + k_j^2v_{\mathrm{th}}^2)/\omega_j^2$. Using eqs (10)–(12) in the Poisson's equation $\nabla^2\phi_j = 4\pi e(n_j^{\mathrm{L}} + n_j^{\mathrm{NL}})$, we obtain

$$\begin{aligned}\varepsilon_1\phi_1 &= -4\pi e n_1^{\mathrm{NL}}/k_1^2,\\ \varepsilon_2\phi_2 &= -4\pi e n_2^{\mathrm{NL}}/k_2^2,\end{aligned} \tag{13}$$

where

$$\varepsilon_j = 1 + \chi_{\mathrm{ej}}.$$

Using eqs (10) and (11) in eq. (13), we obtain

$$\phi_1 = \frac{k^2}{k_1^2\varepsilon_1}\frac{\vec{k}_1\cdot\vec{v}_0^*}{2\omega_1}(1+\chi_{\mathrm{i}})\,\phi, \tag{14}$$

$$\phi_2 = \frac{k^2}{k_2^2 \varepsilon_2} \frac{\vec{k}_2 \cdot \vec{v}_0}{2\omega_2} (1 + \chi_i) \phi. \tag{15}$$

Using eqs (6), (14) and (15) in eq. (9), we obtain

$$\varepsilon = -\chi_e (1 + \chi_i) \left[\frac{k^2}{k_1^2 \varepsilon_1} \frac{\left|\vec{k}_1 \cdot \vec{v}_0\right|^2}{4\omega_1^2} + \frac{k^2}{k_2^2 \varepsilon_2} \frac{\left|\vec{k}_2 \cdot \vec{v}_0\right|^2}{4\omega_2^2} \right]. \tag{16}$$

For $k \gg k_0$, eq. (16) simplifies to

$$\varepsilon = -\chi_e (1 + \chi_i) \frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{4} \left(\frac{1}{\omega_1^2 \varepsilon_1} + \frac{1}{\omega_2^2 \varepsilon_2} \right), \tag{17}$$

where

$$\omega_1 = \omega - \omega_0, \quad \omega_2 = \omega + \omega_0.$$

We define a frequency mismatch

$$\Delta = \omega_0 - \left(\omega_p^2 + k^2 v_{th}^2 \right)^{1/2}$$

and simplify

$$\left(1/\omega_1^2 \varepsilon_1 + 1/\omega_2^2 \varepsilon_2 \right) \approx \Delta / \omega_0 (\Delta^2 - \omega^2).$$

Then eq. (17) can be written as

$$\varepsilon = -\chi_e (1 + \chi_i) \frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{4} \frac{\Delta}{\omega_0 (\Delta^2 - \omega^2)}. \tag{18}$$

In the limit $\omega_{pi} \ll \omega \ll k v_{th}$, ion motion can be neglected ($\chi_i = 0$) and eq. (18) turns out to be

$$\omega^2 = \Delta^2 + \frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{4\left(1 + k^2 c_s^2 / \omega_{pi}^2\right)} \frac{\Delta}{\omega_0}. \tag{19}$$

Instability occurs when Δ is negative and

$$|\Delta| < \frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{4\left(1 + k^2 c_s^2 / \omega_{pi}^2\right)}.$$

The growth rate turns out to be maximum when

$$\Delta = -\frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{8\omega_0 \left(1 + k^2 c_s^2 / \omega_{pi}^2\right)} \tag{20}$$

and the maximum growth rate is

$$\gamma_{\max} = \frac{k^2 v_{\rm th}^2}{8\omega_0\left(1 + k^2 c_{\rm s}^2/\omega_{\rm pi}^2\right)} \frac{v_0^2}{v_{\rm th}^2} = \frac{k^2 c_{\rm s}^2/\omega_{\rm pi}^2}{1 + k^2 c_{\rm s}^2/\omega_{\rm pi}^2} \frac{\omega_{\rm p}}{8} \frac{v_0^2}{v_{\rm th}^2}. \tag{21}$$

The condition $\gamma > \omega_{\rm pi}$ is satisfied when $v_0 \geq 3(m/m_{\rm i})^{1/4} v_{\rm th}$.

When we include the ion motion and treat the ions as cold, then eq. (18) can be written as

$$\left(\omega^2 - \Delta^2\right)\left[\omega^2\left(1 + \frac{\omega_{\rm pi}^2}{k^2 c_{\rm s}^2}\right) - \omega_{\rm pi}^2\right] = \left(\omega^2 - \omega_{\rm pi}^2\right) R_1 \Delta$$

or

$$\omega^4 - \omega^2(\Delta^2 + \omega_{\rm ac}^2 + R_1' \Delta) + \Delta^2 \omega_{\rm ac}^2 + \omega_{\rm pi}^2 R_1' \Delta = 0, \tag{22}$$

where

$$R_1 = \frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{4\omega_0} \frac{\omega_{\rm pi}^2}{k^2 c_{\rm s}^2},$$

$$R_1' = \frac{R_1}{1 + \omega_{\rm pi}^2/k^2 c_{\rm s}^2},$$

$$\omega_{\rm ac}^2 = \frac{k^2 c_{\rm s}^2}{1 + k^2 c_{\rm s}^2/\omega_{\rm pi}^2}.$$

Equation (22) gives a root

$$\omega^2 = \frac{1}{2}\left[\left(\Delta^2 + \omega_{\rm ac}^2 + R_1'\Delta\right) - \sqrt{\left(\Delta^2 + \omega_{\rm ac}^2 + R_1'\Delta\right)^2 - 4(\Delta^2 \omega_{\rm ac}^2 + \omega_{\rm pi}^2 R_1' \Delta)}\right]. \tag{23}$$

Instability occurs when Δ is negative and

$$|\Delta| < \frac{\omega_{\rm pi}^2}{\omega_{\rm ac}^2} R_1'.$$

We have solved eq. (23) numerically for the following parameters: $m_{\rm i}/m = 4000$, $|v_0|/v_{\rm th} = 0.5$. In figure 1, we have plotted the variation of normalized growth rate, $\gamma/\omega_{\rm p}$, with the normalized wave number perturbation, $k v_{\rm th}/\omega_{\rm p}$. The maximum growth rate occurs for an optimum value of the wave number, $k v_{\rm th}/\omega_{\rm p} \approx 0.10$ and the rate increases with the amplitude of the pump.

3. Non-local effects

Now we consider a plasma density profile,

$$n_0 = n_0^0\left(1 + \frac{r^2}{r_0^2}\right). \tag{24}$$

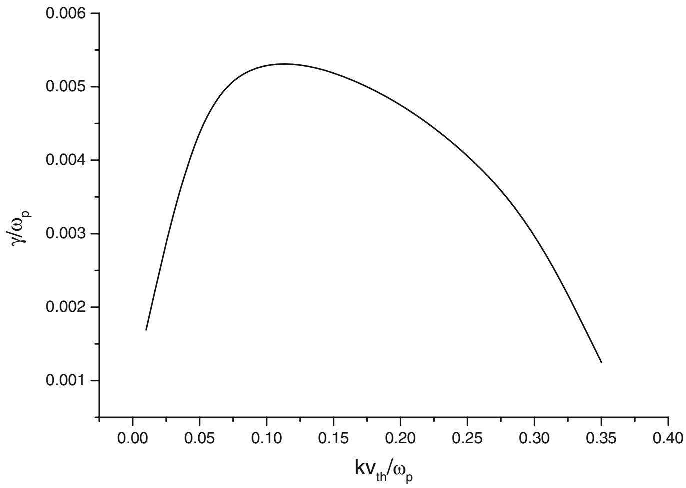

Figure 1. Variation in normalized growth rate, $\gamma/\omega_{\mathrm{p}}$, as a function of $kv_{\mathrm{th}}/\omega_{\mathrm{p}}$ for $m_{\mathrm{i}}/m = 4000$, $|v_0|/v_{\mathrm{th}} = 0.5$ in the case of local effects.

Such a profile is encountered when the spot size of the laser is small and the wakefield plasma wave creates a time average local density depression. In such a channel, the mode structure equations of the sidebands can be deduced as follows. We write eqs (13) as

$$\left(\omega_1^2 - k_1^2 v_{\mathrm{th}}^2 - \omega_{\mathrm{p}}^2\right)\phi_1 = -\frac{4\pi e}{k_1^2}\omega_1^2 n_1^{\mathrm{NL}}, \tag{25}$$

$$\left(\omega_2^2 - k_2^2 v_{\mathrm{th}}^2 - \omega_{\mathrm{p}}^2\right)\phi_2 = -\frac{4\pi e}{k_2^2}\omega_2^2 n_2^{\mathrm{NL}}. \tag{26}$$

From eqs (10) and (11) n_j^{NL} can be explicitly written as

$$n_1^{\mathrm{NL}} \approx \frac{k^2}{4\pi e}\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{\varepsilon}\frac{\left|\vec{k}\cdot\vec{v}_0\right|^2}{4\omega_0^2}\left(\phi_1 - \mathrm{e}^{-2i\omega_0 t}\phi_2\right),$$

$$n_2^{\mathrm{NL}} \approx \frac{k^2}{4\pi e}\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{\varepsilon}\frac{\left|\vec{k}\cdot\vec{v}_0\right|^2}{4\omega_0^2}\left(\phi_2 - \mathrm{e}^{-2i\omega_0 t}\phi_1\right).$$

Replacing $-k_1^2$ by $-k_{1z}^2 + \nabla_\perp^2$, $-k_2^2$ by $-k_{2z}^2 + \nabla_\perp^2$ in eqs (25) and (26) and writing $\phi_1 = A_1(x)\mathrm{e}^{-i(\omega t - k_{1z}z)}$, $\phi_2 = A_2(x)\mathrm{e}^{-i(\omega t - k_{2z}z)}$, we obtain

$$\frac{\partial^2 A_1}{\partial r^2} + \frac{1}{r}\frac{\partial A_1}{\partial r} + \left(\frac{\omega_1^2 - \omega_{\mathrm{p0}}^2\left(1 + r^2/r_0^2\right)}{v_{\mathrm{th}}^2} - k_z^2\right)A_1$$
$$= -\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{\varepsilon}\frac{\left|\vec{k}\cdot\vec{v}_0\right|^2}{4v_{\mathrm{th}}^2}\left(A_1 - A_2\right), \tag{27}$$

$$\frac{\partial^2 A_2}{\partial r^2} + \frac{1}{r}\frac{\partial A_2}{\partial r} + \left(\frac{\omega_2^2 - \omega_{\mathrm{p0}}^2\left(1 + r^2/r_0^2\right)}{v_{\mathrm{th}}^2} - k_z^2\right) A_2$$
$$= -\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{\varepsilon}\frac{\left|\vec{k}\cdot\vec{v}_0\right|^2}{4v_{\mathrm{th}}^2}(A_2 - A_1)\,. \tag{28}$$

Introducing

$$\xi = r/r_0',\quad r_0' = \left(r_0 v_{\mathrm{th}}/\omega_{\mathrm{p0}}\right),\quad \lambda_1 = \left[\left(\omega_1^2 - \omega_{\mathrm{p0}}^2\right)/v_{\mathrm{th}}^2 - k_z^2\right] r_0'^2,$$
$$\lambda_2 = \left[\left(\omega_2^2 - \omega_{\mathrm{p0}}^2\right)/v_{\mathrm{th}}^2 - k_z^2\right] r_0'^2 \quad \text{and} \quad C_1 = -\left[(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}\right]/4\varepsilon,$$

we may write eqs (27) and (28) as

$$\frac{\partial^2 A_1}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial A_1}{\partial \xi} + (\lambda_1 - \xi^2)A_1 = \frac{k^2|v_0|^2}{v_{\mathrm{th}}^2} C_1 r_0'^2 (A_1 - A_2)\,, \tag{29}$$

$$\frac{\partial^2 A_2}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial A_2}{\partial \xi} + (\lambda_2 - \xi^2)A_2 = -\frac{k^2|v_0|^2}{v_{\mathrm{th}}^2} C_1 r_0'^2 (A_1 - A_2)\,. \tag{30}$$

The pump plasma wave also satisfies a similar wave equation

$$\frac{\partial^2 \phi_0}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_0}{\partial r} + \left(\frac{\omega_0^2 - \omega_{\mathrm{p0}}^2\left(1 + r^2/r_0^2\right)}{v_{\mathrm{th}}^2} - k_{0z}^2\right)\phi_0 = 0. \tag{31}$$

This gives for the fundamental mode

$$\phi_0 = A_0 \mathrm{e}^{-i(\omega_0 t - k_{0z} z)},$$
$$A_0^2 = A_{00}^2 \mathrm{e}^{-r^2/r_0'^2},$$
$$\omega_0^2 = \omega_{\mathrm{p0}}^2 + \left(k_{0z}^2 + \frac{2}{r_0'^2}\right) v_{\mathrm{th}}^2.$$

We solve eqs (29)–(30) iteratively. First we ignore the right-hand sides, then these equations offer well-behaved solutions for the fundamental mode ($\lambda_1 = \lambda_2 = 2$), $\psi = \mathrm{e}^{-\xi^2/2}$,

$$A_1 = \psi(\xi)\,,$$
$$A_2 = \psi(\xi)\,.$$

On including the right-hand sides of eqs (29) and (30), the wave functions are presumed to remain unmodified, and only the eigenvalues are modified. Hence we write

$$A_1 = A_{10}\psi, \tag{32}$$

$$A_2 = A_{20}\psi. \tag{33}$$

Using these in eqs (29) and (30), multiplying the resulting equations by $\psi^* \xi\, \mathrm{d}\xi$ and integrating from 0 to ∞, we obtain

$$(\lambda_1 - 2)A_{10} = C_1(A_{10} - A_{20})\, r_0'^2 I_1/I_2, \tag{34}$$

$$(\lambda_2 - 2)A_{20} = -C_1(A_{10} - A_{20})\, r_0'^2 I_1/I_2, \tag{35}$$

where

$$I_1 = \frac{k^2}{v_{\mathrm{th}}^2}\int_0^\infty \left|v_0^2\right| \psi\psi^* \xi \mathrm{d}\xi,$$

$$I_2 = \int_0^\infty \psi\psi^* \xi\, \mathrm{d}\xi.$$

Equations (34) and (35) yield the nonlinear dispersion relation

$$(\lambda_1 - 2 - C_1 I_1 r_0'^2/I_2)(\lambda_2 - 2 - C_1 I_1 r_0'^2/I_2) = C_1^2 r_0'^4\, (I_1/I_2)^2$$

or

$$\varepsilon = -\frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{4} r_0'^2 \frac{I_1}{I_2}\left[\frac{1}{\lambda_1 - 2} + \frac{1}{\lambda_2 - 2}\right], \tag{36}$$

where

$$\lambda_1 - 2 = \frac{r_0'^2}{v_{\mathrm{th}}^2}\left[\omega_1^2 - \left(\omega_{\mathrm{p}}^2 + k^2 v_{\mathrm{th}}^2 + \frac{2}{r_0'^2} v_{\mathrm{th}}^2\right)\right],$$

$$\lambda_2 - 2 = \frac{r_0'^2}{v_{th}^2}\left[\omega_2^2 - \left(\omega_{\mathrm{p}}^2 + k^2 v_{\mathrm{th}}^2 + \frac{2}{r_0'^2} v_{\mathrm{th}}^2\right)\right].$$

We define a frequency mismatch

$$\Delta = \omega_0 - \left(\omega_{\mathrm{p}}^2 + k^2 v_{\mathrm{th}}^2 + \frac{2}{r_0'^2} v_{\mathrm{th}}^2\right)^{1/2} \approx -\frac{k^2 v_{\mathrm{th}}^2}{2\omega_{\mathrm{p}}}.$$

Then

$$\lambda_1 - 2 \approx -\frac{2r_0'^2}{v_{\mathrm{th}}^2}(\omega - \Delta)\omega_0,$$

$$\lambda_2 - 2 \approx \frac{2r_0'^2}{v_{\mathrm{th}}^2}(\omega + \Delta)\omega_0$$

and eq. (36) takes the form

$$\omega^2 = \Delta^2 + \frac{(1+\chi_{\mathrm{i}})\chi_{\mathrm{e}}}{4\varepsilon} k^2\, |v_0|_{\mathrm{av}}^2\, \Delta, \tag{37}$$

where

$$|v_0|_{\mathrm{av}}^2 = \int_0^\infty |v_0|^2\, \psi\psi^* \xi \mathrm{d}\xi \Big/ \int_0^\infty \psi\psi^* \xi \mathrm{d}\xi = v_{00}^2/2, \quad v_{00} = ek_0 A_0/m\omega_0.$$

Equation (37) gives

$$\omega^2 = \frac{1}{2}\left[\Delta^2 + \omega_{\mathrm{ac}}^2 + R_2\Delta - \sqrt{\left(\Delta^2 + \omega_{\mathrm{ac}}^2 + R_2\Delta\right)^2 - 4\left(\Delta^2\omega_{\mathrm{ac}}^2 + \omega_{\mathrm{pi}}^2 R_2\Delta\right)}\right], \tag{38}$$

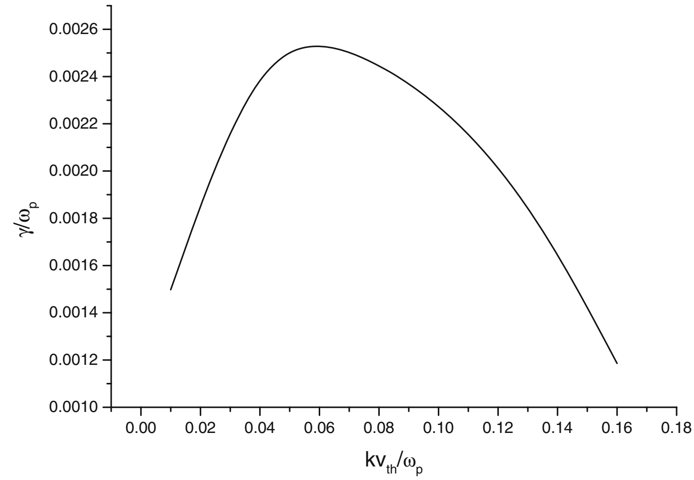

Figure 2. Variation in normalized growth rate, $\gamma/\omega_{\mathrm{p}}$, as a function of $kv_{\mathrm{th}}/\omega_{\mathrm{p}}$ for $m_{\mathrm{i}}/m = 4000$, $v_{00}/v_{\mathrm{th}} = 0.5$ in the case of nonlocal effects.

where

$$R_2 = \frac{k^2 v_{00}^2}{8\omega_0(1 + k^2 c_{\mathrm{s}}^2/\omega_{\mathrm{pi}}^2)},$$

$$\omega_{\mathrm{ac}}^2 = \frac{k^2 c_{\mathrm{s}}^2}{(1 + k^2 c_{\mathrm{s}}^2/\omega_{\mathrm{pi}}^2)}.$$

One may recall that $\Delta = -k^2 v_{\mathrm{th}}^2/2\omega_{\mathrm{p}}$ is negative and ω^2 is also negative, i.e., a purely growing instability occurs over a range of k values. In figure 2, we have plotted the normalized growth rate, $\gamma/\omega_{\mathrm{p}}$, as a function of $kv_{\mathrm{th}}/\omega_{\mathrm{p}}$ for $m_{\mathrm{i}}/m = 4000$, $v_{00}/v_{\mathrm{th}} = 0.5$. The growth rate is maximum $\left(\gamma = \gamma_{\max} \approx 0.256\omega_{\mathrm{p}}\right)$, when $kv_{\mathrm{th}}/\omega_{\mathrm{p}} = 0.05$. This growth rate is about half of the value one obtains, when local approximation is used.

4. Conclusion

A large-amplitude long-wavelength plasma wave undergoes two-stream instability producing short wavelength plasma waves in a plasma. The instability does not require the motion of ions. However, the growth rate is of the order of ion plasma frequency, and hence the ion motion is important. The growth rate attains a maximum value for an optimum wave number. For normalized pump amplitude $|v_0|/v_{\mathrm{th}} \approx 0.5$, the optimum wave number is $0.10v_{\mathrm{th}}/\omega_{\mathrm{p}}$ and the growth rate is $0.0054\omega_{\mathrm{p}}$. In the case of a plasma channel the pump and sideband Langmuir waves exhibit Gaussian amplitude profiles. The radial width of the eigenmode is the same for the pump and sideband waves. However, the OTSI dominantly occurs in the axial region of the laser. Hence, the growth rate of OTSI is reduced by a factor $\sim$2 by the nonlocal effects.

Thus, the OTSI tends to destroy the Langmuir wave in laser-driven electron accelerators in about 10 plasma periods.

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