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Next-to-leading order corrections to the valon model

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Abstract. A seminumerical solution to the valon model at next-to-leading order (NLO) in the Laguerre polynomials is presented. We used the valon model to generate the structure of proton with respect to the Laguerre polynomials method. The results are compared with H1 data and other parametrizations.

Keywords. Valon model; structure function; Laguerre polynomials.

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In the valon model [1–7], a valon is defined to be a dressed valance quark in quantum chromodynamics (QCD) with a cloud of gluons and sea quarks and antiquarks. In this model, the proton is considered as a bound state of three valons (UUD). They contribute independently in an inclusive hard collision with a Q^2-dependence that can be calculated in QCD at high Q^2. The proton structure function $F_2^{\mathrm{p}}(x, Q^2)$ is related to the valon structure function $f^{\mathrm{v}}(\frac{x}{y}, Q^2)$ for each valon v which can be shown as

$$F_2^{\mathrm{p}}(x, Q^2) = \sum_v \int_x^1 G_{v/N}(y) f^{\mathrm{v}}\left(\frac{x}{y}, Q^2\right) \mathrm{d}y, \tag{1}$$

where the summation in this equation is over the three valons and $G_{v/N}(y)$ indicates the probability for the v valon to have a momentum fraction y in the proton. The valon distributions are assumed to have a simple relation in a hadron as

$$G_{\mathrm{UUD}}(y_1, y_2, y_3) = g(y_1 y_2)^{\alpha} y_3^{\beta} \delta(y_1 + y_2 + y_3). \tag{2}$$

The U and D valon distributions can be defined by

$$\begin{aligned} G_{\mathrm{U}}(y) &= \int_0^{1-y} \mathrm{d}y_2 \int_0^{1-y-y_2} \mathrm{d}y_3 G_{\mathrm{UUD}}(y_1, y_2, y_3) \\ &= gB(\alpha+1, \beta+1) y^{\alpha}(1-y)^{\alpha+\beta+1} \end{aligned} \tag{3}$$

and

$$G_{\rm D}(y) = \int_0^{1-y} {\rm d}y_1 \int_0^{1-y-y_1} {\rm d}y_2 G_{\rm UUD}(y_1, y_2, y_3)$$
$$= gB(\alpha+1, \alpha+1) y^{\beta}(1-y)^{2\alpha+1}, \quad (4)$$

where $g(= (B(\alpha+1, \beta+1)B(\alpha+1, \beta+\alpha+2))^{-1})$ is given by the normalization factor and $B(i, j)$ is the Euler beta function with $\alpha = 0.65$ and $\beta = 0.35$ [1–7]. After doing the inverse Mellin transformation, the valon distributions have the forms

$$G_{\rm U}(y) = 7.98 y^{0.65}(1-y)^2 \quad (5)$$

and

$$G_{\rm D}(y) = 6.01 y^{0.35}(1-y)^{2.3}. \quad (6)$$

The valon structure functions can be given in terms of the flavour singlet (S) and nonsinglet (NS) terms as

$$F_2^{\rm U}(z, Q^2) = \frac{2}{9} z[G^{\rm S}(z, Q^2) + G^{\rm NS}(z, Q^2)] \quad (7)$$

and

$$F_2^{\rm D}(z, Q^2) = \frac{1}{9} z[2G^{\rm S}(z, Q^2) - G^{\rm NS}(z, Q^2)], \quad (8)$$

where

$$G^{\rm S}(z, Q^2) = \sum_{i=1}^{N_f} (G_{q_i/v} + G_{\bar{q}_i/v}) \quad (9)$$

and

$$G^{\rm NS}(z, Q^2) = \sum_{i=1}^{N_f} (G_{q_i/v} - G_{\bar{q}_i/v}). \quad (10)$$

In the moment representation we have

$$M_2(n, Q^2) = \int_0^1 x^{n-2} F_2(x, Q^2) {\rm d}x \quad (11)$$

and

$$M_\alpha(n, Q^2) = \int_0^1 x^{n-1} G_\alpha(x, Q^2) {\rm d}x \quad (12)$$

where $\alpha = v/N$, S, NS. Therefore, the proton moment is given as

$$M^{\rm p}(n, Q^2) = \sum_v M_{v/N}(n) M^{\rm v}(n, Q^2). \quad (13)$$

Here, the valon momentum distributions are defined as

$${\rm U}(n) \equiv M_{\rm U/p}(n) = \frac{B(\alpha+n, \alpha+\beta+2)}{B(\alpha+1, \alpha+\beta+2)},$$
$${\rm D}(n) \equiv M_{\rm D/p}(n) = \frac{B(\beta+n, 2\alpha+2)}{B(\alpha+1, 2\alpha+2)}, \quad (14)$$

and also the moments of parton distributions are

$$
\begin{aligned}
M_{\mathrm{u/v}}(n,s) &= 2\mathrm{U}(n)M^{\mathrm{NS}}(n,s),\\
M_{\mathrm{d/v}}(n,s) &= \mathrm{D}(n)M^{\mathrm{NS}}(n,s),\\
M_{\mathrm{g}}(n,s) &= [2\mathrm{U}(n)+\mathrm{D}(n)]M_{\mathrm{gq}}(n,s),\\
M_{\mathrm{sea}}(n,s) &= \frac{1}{2N_f}[2\mathrm{U}(n)+\mathrm{D}(n)][M^{\mathrm{S}}(n,s)-M^{\mathrm{NS}}(n,s)].
\end{aligned}
\tag{15}
$$

The evolution parameter s is defined as

$$
s = \ln\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}, \tag{16}
$$

where Λ and Q_0^2 are the QCD cut-off parameter and initial scale parameter, respectively. In the leading-order analysis (LO), the moments of quark (S and NS) and quark-to-gluon distribution inside the proton are defined as

$$
M^{\mathrm{NS}}(n,s) = \exp(-d_{\mathrm{NS}}s), \tag{17}
$$

$$
M^{\mathrm{S}}(n,s) = \frac{1}{2}(1+\rho)\exp(-d_+s) + \frac{1}{2}(1-\rho)\exp(-d_-s) \tag{18}
$$

and

$$
M_{\mathrm{gq}}(n,s) = \Delta^{-1}d_{\mathrm{gq}}[\exp(-d_+s)-\exp(-d_-s)]. \tag{19}
$$

Here, the anomalous dimensions are [1,2]

$$
d = \frac{\gamma_0^n}{2\beta_0}, \tag{20}
$$

where

$$
\gamma_{\mathrm{NS}}^n = \frac{\alpha_{\mathrm{s}}}{4\pi}\gamma_{\mathrm{qq}}^{(0),n} \tag{21}
$$

and

$$
\gamma_{ij}^n = \frac{\alpha_{\mathrm{s}}}{4\pi}\gamma_{ij}^{(0),n}, \quad i,j=\mathrm{q},\mathrm{g}, \tag{22}
$$

where γ_{ij} is the dependence to the splitting function. The running coupling constant at LO is

$$
\alpha_{\mathrm{s}}(Q^2) = \frac{4\pi}{\beta_0\ln\left(Q^2/\Lambda^2\right)}, \tag{23}
$$

where $\beta_0 = (33-2N_f)/3$, and the other parameters are given as

$$
\rho = (d_{\mathrm{NS}} - d_{\mathrm{gg}}), \tag{24}
$$

$$
\Delta = d_+ - d_- = [(d_{\mathrm{NS}}-d_{\mathrm{gg}})^2 + 4d_{\mathrm{gq}}d_{\mathrm{qg}}]^{1/2}, \tag{25}
$$

$$d_{\rm NS} = \frac{1}{3\pi b}\left[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}\right], \tag{26}$$

$$d_{\rm gq} = \frac{-2}{3\pi b}\frac{2+n+n^2}{n(n^2-1)}, \tag{27}$$

$$d_{\rm qg} = \frac{-N_f}{2\pi b}\frac{2+n+n^2}{n(n+1)(n+2)}, \tag{28}$$

$$d_{\rm gg} = \frac{-3}{\pi b}\left[\frac{-1}{12} + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} - \frac{N_f}{18} - \sum_{j=2}^{n}\frac{1}{j}\right], \tag{29}$$

$$d_{\pm} = \frac{1}{2}[d_{\rm NS} + d_{\rm gg} \pm \Delta], \tag{30}$$

where $b = \beta_0/4\pi$.

Using the next-to-leading order analysis (NLO), the running coupling constant is

$$\alpha_{\rm s}(Q^2) = \frac{4\pi}{\beta_0 \ln\left(Q^2/\Lambda^2\right)}\left(1 - \frac{\beta_1 \ln\ln(Q^2/\Lambda^2)}{\beta_0^2 \ln(Q^2/\Lambda^2)}\right), \tag{31}$$

where

$$\beta_1 = 102 - \frac{38N_f}{3},$$

and the moments of NS quark are proportional to [3,8–10]

$$M^{\rm NS} = \left[1 + \frac{\alpha_{\rm s}(Q^2) - \alpha_{\rm s}(Q_0^2)}{4\pi}\left(\frac{\gamma_{\rm NS}^{(1),n}}{2\beta_0} - \frac{\beta_1\gamma_{\rm qq}^{(0),n}}{2\beta_0^2}\right)\right]\left(\frac{\alpha_{\rm s}(Q^2)}{\alpha_{\rm s}(Q_0^2)}\right)^{\frac{\gamma_{\rm qq}^{(0),n}}{2\beta_0}}. \tag{32}$$

So the evolutions of the γ's up to NLO are given by

$$\gamma_{\rm NS}^{n} = \frac{\alpha_{\rm s}}{4\pi}\gamma_{\rm qq}^{(0),n} + \left(\frac{\alpha_{\rm s}}{4\pi}\right)^2\gamma_{\rm NS}^{(1),n} \tag{33}$$

$$\gamma_{ij}^{n} = \frac{\alpha_{\rm s}}{4\pi}\gamma_{ij}^{(0),n} + \left(\frac{\alpha_{\rm s}}{4\pi}\right)^2\gamma_{ij}^{(1),n}, \quad i,j = {\rm q},{\rm g}, \tag{34}$$

and also the moments of the S quark and quark-to-gluon distribution inside the proton are [3,8–10]

$$\begin{pmatrix} M^{\rm S}(n,Q^2 \\ M^{\rm g}(n,Q^2 \end{pmatrix} = \left(\left(\frac{\alpha_{\rm s}(Q^2)}{\alpha_{\rm s}(Q_0^2)}\right)^{\frac{\lambda_-^n}{2\beta_0}}\left[P_-^n - \frac{1}{2\beta_0}\frac{\alpha_{\rm s}(Q_0^2) - \alpha_{\rm s}(Q^2)}{4\pi}\right.\right.$$
$$\times P_-^n\gamma^n P_-^n - \left(\frac{\alpha_{\rm s}(Q_0^2)}{4\pi} - \frac{\alpha_{\rm s}(Q^2)}{4\pi}\left(\frac{\alpha_{\rm s}(Q^2)}{\alpha_{\rm s}(Q_0^2)}\right)^{\frac{\lambda_+^n - \lambda_-^n}{2\beta_0}}\right)$$
$$\left.\left.\times\frac{P_-^n\gamma^n P_+^n}{2\beta_0 + \lambda_+^n - \lambda_-^n}\right]\right)(\mathbf{1}) + \{\mathbf{Q}_{+\leftrightarrow -}\}. \tag{35}$$

Here $\gamma^n = \gamma^{(1),n} - (\beta_1/\beta_0)\gamma^{(0),n}$ and (**1**) is the unit matrix in eq. (35). The term $\{Q_{+\leftrightarrow -}\}$ implies that all subscripts are exchanged. Also, the other coefficients are given by [5,10]

$$\lambda_{\pm}^n = \frac{1}{2}[\gamma_{\rm qq}^{(0),n} + \gamma_{\rm gg}^{(0),n} \pm \sqrt{(\gamma_{\rm gg}^{(0),n} - \gamma_{\rm qq}^{(0),n})^2 + 4\gamma_{\rm qg}^{(0),n}\gamma_{\rm gq}^{(0),n}} \tag{36}$$

and

$$P_{\pm}^n = \pm\frac{\gamma^n - \lambda_{\mp}^n}{\lambda_{+}^n - \lambda_{-}^n}. \tag{37}$$

For our calculation at LO up to NLO, the initial scale is equal to $Q_0^2 = 0.2$ GeV^2 and the QCD cut-off parameters are $\Lambda^{\rm LO} = 0.255$ GeV and $\Lambda^{\rm NLO} = 0.217$ GeV, respectively. The parton distributions can be determined for a single value of s (or Q^2), by using a fit to moments with respect to a sum of beta functions. In turn, the moment of parton distributions are given by

$$zq_{\rm v}(z, Q^2) = N_{\rm v} z^{a_{\rm v}}(1-z)^{b_{\rm v}}, \quad {\rm v} = {\rm u, d}, \tag{38}$$

$$zq_{\rm sea}(z, Q^2) = a_{\rm s} z^{b_{\rm s}}(1-z)^{c_{\rm s}}(1 + e_{\rm s} z^{0.5} + d_{\rm s} z) \tag{39}$$

and

$$zg(z, Q^2) = a_{\rm g} z^{b_{\rm g}}(1-z)^{c_{\rm g}}(1 + d_{\rm g} z^{0.5}). \tag{40}$$

According to the H1 data [11], the parameters $a_i, b_i, \ldots$ are further considered to be functions of s, as can be seen in the Appendix. In figures 1–4, we show the shape of the distributions in eqs (38)–(40) for the valence, sea quarks and gluon distributions at $Q^2 = 45$ GeV^2. We have computed the predictions for distributions inside the proton in the kinematic range where it has been measured using the experimental data and have compared our predictions with the GJR parametrizations [12]. As we can see in these figures, the agreements at NLO are good.

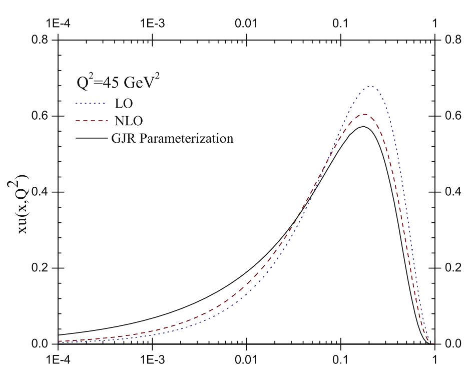

Figure 1. Distribution function for u valence quark in U valon at $Q^2 = 45$ GeV^2. Dot and dash curves are our results at LO and NLO, respectively as compared with the GJR parametrization [12].

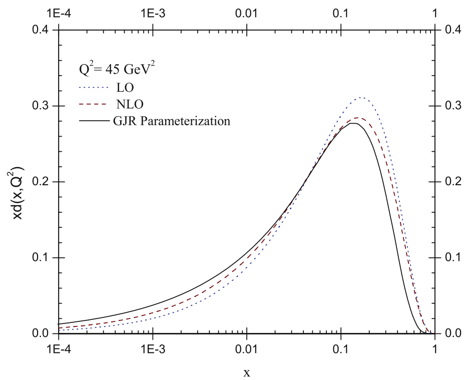

Figure 2. Distribution function for d valence quark in D valon at $Q^2 = 45$ GeV^2. Dot and dash curves are our results at LO and NLO, respectively as compared with the GJR parametrization [12].

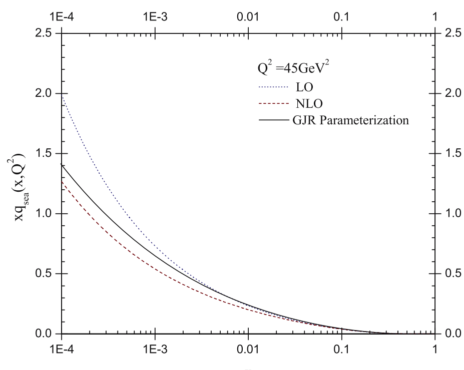

Figure 3. Distribution function for sea quarks per valon at $Q^2 = 45$ GeV^2. Dot and dash curves are our results at LO and NLO, respectively as compared with the GJR parametrization [12].

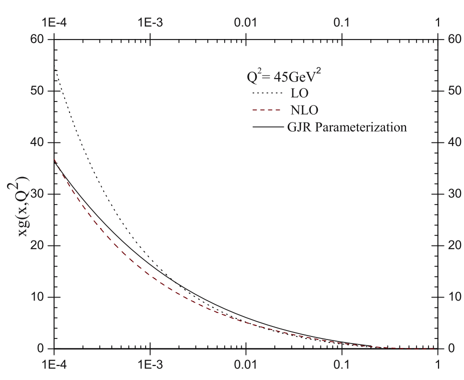

Figure 4. Distribution function for gluons per valon at $Q^2 = 45$ GeV^2. Dot and dash curves are our results at LO and NLO, respectively as compared with the GJR parametrization [12].

To obtain the proton structure function in valon model with respect to the Laguerre polynomials one needs to use an elegant and fast numerical method at LO up to NLO. Therefore, we concentrate on the Laguerre polynomials in our determinations. In the Laguerre method [13,14], the Laguerre polynomials are defined as

$$(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x), \tag{41}$$

where the orthogonality condition is defined as follows:

$$\int_0^\infty \mathrm{e}^{-x'} L_n(x')L_m(x')\mathrm{d}x' = \delta_{n,m}. \tag{42}$$

The general integrable function $f(\mathrm{e}^{-x'})$ is transformed into the sum as follows

$$f(\mathrm{e}^{-x'}) = \sum_0^N f(n)L_n(x'), \tag{43}$$

where

$$f(n) = \int_0^\infty \mathrm{e}^{-x'} L_n(x') f(\mathrm{e}^{-x'})\mathrm{d}x'. \tag{44}$$

In what follows we use the variable transformations, $x = \mathrm{e}^{-x'}$, $y = \mathrm{e}^{-y'}$ to obtain the valonic structure function form to the Laguerre polynomials form. Then we combine and expand each term of this equation on Laguerre polynomials according to eqs (43) and (44) and use the properties as

$$\int_0^{x'} \mathrm{d}y' L_n(x'-y')L_m(y') = L_{n+m}(x') - L_{n+m+1}(x'), \tag{45}$$

The equation obtained determines $F_2^{\mathrm{p}}(x, Q^2)$ in terms of the Laguerre polynomials at LO up to NLO according to the parton distributions as

$$\begin{aligned} F_2^{\mathrm{p}}(n, Q^2) = \sum_v \sum_{m=0}^{n} \tilde{G}_{v/p}(m)[F_2^{\mathrm{v}}(n-m, Q^2) \\ -F_2^{\mathrm{v}}(n-m-1, Q^2)], \end{aligned} \tag{46}$$

where

$$F_2^{\mathrm{v}}(n, Q^2) = \int_0^\infty \mathrm{d}x' \mathrm{e}^{-x'} F_2^{\mathrm{v}}(\mathrm{e}^{-x'}, Q^2) L_n(x') \tag{47}$$

and

$$\tilde{G}_{v/p}(m) = \int_0^\infty \mathrm{d}y' \mathrm{e}^{-y'} G_{v/p}(\mathrm{e}^{-y'}) L_m(y'). \tag{48}$$

Here $F_2^{\mathrm{v}}(x)$ is defined according to eqs (7) and (8) along with eqs (38)–(40) and their coefficients at LO and NLO (Appendix), and also $G_{v/p}(y)$ is defined according to eqs (5) and (6), respectively. Therefore, the proton structure function in valon model is defined by solving the recursion relation as

$$F_2^{\mathrm{p}}(x, Q^2) = \sum_{n=0}^{N} F_2^{\mathrm{p}}(n, Q^2) L_n \left(Ln \frac{1}{x} \right), \tag{49}$$

where $F_2^{\mathrm{p}}(n, Q^2)$ is the proton structure function with respect to the Laguerre model and is defined by eqs (46)–(48) as the coefficients in these equations are obtained with respect to the valon model at LO and NLO. This result is completely general and gives the expression for the proton structure function with respect to the Laguerre polynomials model. Here we can expand the integrable functions till a finite order $N = 30$, as we can converge these series in the numerical determinations. We computed the predictions for all detail of the proton structure function in the kinematic range where it has been measured by H1 Collaboration [11] and compared with GJR parametrization [12]. Our numerical predictions are presented as functions of x at $Q^2 = 45$ GeV^2. The result is presented in figure 5 along with a comparison with the H1 data and with the results obtained with the help of other standard parametrizations. We compared the results at LO and NLO with predictions of F_2^{p} in perturbative QCD where the input densities are given by GJR parametrizations [12]. The agreement between the Laguerre polynomials method for the proton structure function in valon model at NLO and data at low- and high-x are remarkably good. The good agreement indicates that the Laguerre polynomials method in valon model for the proton structure function at NLO has a good asymptotic behaviour and is compatible with both the data and the other standard models at x values. This model has this advantage that we get a very elegant solution for the proton structure function.

In summary, we have used the Laguerre polynomials method to describe the proton structure function in valon model at LO and NLO. The proton structure can be determined in terms of the valon distributions and valon structure functions with respect to the

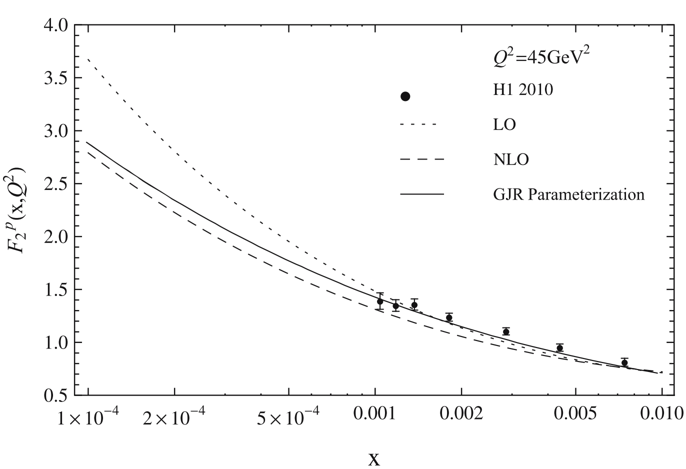

Figure 5. Proton structure function in valon model with respect to the Laguerre polynomials at $Q^2 = 45$ GeV^2. Dot and dash curves are our results at LO and NLO, respectively as compared with the GJR parametrization [12] and H1 data that are associated with total errors [11].

Laguerre polynomials. To confirm the method and results at NLO, the calculated values are compared with the H1 data on the proton structure function. It is shown that, there is a good agreement at NLO with the experimental H1 data for F_2^{p}, if one considers the total errors, and it is consistent with a higher order QCD calculations of F_2^{p} which essentially show an increase as x decreases. Also at NLO, this model gives a good description of the parton distributions at low- and high-x values.

Appendix

The functional form of the free parameters of eqs (38)–(40) are given by the following forms in terms of s at LO and NLO analyses.

Coefficients for u valence in U valon are:
At LO

$$\begin{aligned} a_{\mathrm{u}} &= 0.7278 + 0.306s - 0.166s^2, \\ b_{\mathrm{u}} &= -1.388 + 4.2863s - 1.0622s^2, \\ N_{\mathrm{u}} &= 0.2918 + 7.137s - 2.755s^2. \end{aligned} \tag{A.1}$$

At NLO

$$\begin{aligned} a_{\mathrm{u}} &= 1.1221 - 0.3809s + 0.0584s^2, \\ b_{\mathrm{u}} &= 2.2947 + 0.3889s + 0.0890s^2, \\ N_{\mathrm{u}} &= 6.9389 - 2.9848s + 0.5170s^2. \end{aligned} \tag{A.2}$$

Coefficients for d valence in D valon are:
At LO

$$\begin{aligned} a_{\mathrm{d}} &= 1.1071 - 0.3321s + 0.0418s^2, \\ b_{\mathrm{d}} &= 2.0465 + 0.4996s + 0.1124s^2, \\ N_{\mathrm{d}} &= 3.6678 - 1.3991s + 0.1982s^2. \end{aligned} \tag{A.3}$$

At NLO

$$\begin{aligned} a_{\mathrm{d}} &= 0.9575 - 0.296s + 0.0432s^2, \\ b_{\mathrm{d}} &= 2.0757 + 0.4545s + 0.0949s^2, \\ N_{\mathrm{d}} &= 2.7993 - 1.1193s + 0.1856s^2. \end{aligned} \tag{A.4}$$

Coefficients for sea quarks in each valon are:
At LO

$$\begin{aligned} a_{\mathrm{s}} &= -1.4436 + 2.8924s - 1.8425s^2 + 0.3848s^3, \\ b_{\mathrm{s}} &= -7.7792 + 15.1601s - 9.8912s^2 + 2.0796s^3, \\ c_{\mathrm{s}} &= 41.904 - 45.3909s + 12.6658s^2, \\ d_{\mathrm{s}} &= 1.1 + 0.4045s - 0.0887s^2, \\ e_{\mathrm{s}} &= 10.3428 - 22.0348s + 12.6556s^2 - 2.4302s^3. \end{aligned} \tag{A.5}$$

At NLO

$$\begin{aligned} a_{\rm s} &= 0.0284 - 0.0075s + 0.0151s^2, \\ b_{\rm s} &= -0.3223 + 0.0306s - 0.0297s^2, \\ c_{\rm s} &= 2.7174 - 2.1371s + 0.7825s^2, \\ d_{\rm s} &= 4.7717 - 5.0353s + 1.8486s^2, \\ e_{\rm s} &= -4.7272 + 3.5907s - 1.3427s^2. \end{aligned} \tag{A.6}$$

Coefficients for gluons in each valon are:
At LO

$$\begin{aligned} a_{\rm g} &= 13.8745 - 22.3304s + 12.7885s^2 - 2.4801s^3, \\ b_{\rm g} &= 4.6810 - 8.4594s + 4.7656s^2 - 0.9209s^3, \\ c_{\rm g} &= -24.5652 + 50.4661s - 30.147s^2 + 6.0738s^3, \\ d_{\rm g} &= -0.8839 + 0.0403s - 0.0174s^2. \end{aligned} \tag{A.7}$$

At NLO

$$\begin{aligned} a_{\rm g} &= 2.4924 - 1.3797s + 0.2349s^2, \\ b_{\rm g} &= 0.3278 - 0.6287s + 0.1026s^2, \\ c_{\rm g} &= 2.9282 + 0.4685s + 0.1035s^2, \\ d_{\rm g} &= -0.8539 + 0.0055s - 0.0075s^2. \end{aligned} \tag{A.8}$$

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