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Some aspects of optical spatial solitons in photorefractive media and their important applications

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Abstract. Some important properties of photorefractive spatial solitons and their applications have been reviewed in the present paper. Using band transport model, the governing principle of photorefractive nonlinearity has been addressed and nonlinear dynamical equations of spatial solitons owing to this nonlinearity have been discussed. Mechanisms of formation of screening and photovoltaic solitons of three different configurations, i.e., bright, dark and grey varieties have been examined. Incoherently coupled vector solitons due to single and two-photon photorefractive phenomena have been highlighted. Modulation instability of a broad quasicontinuous optical beam has also been discussed. Finally possible applications have been highlighted.

Keywords. Photorefractive nonlinearity; photovoltaic effect; optical spatial solitons; two-photon photorefractive phenomenon; modulation instability.

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1. Introduction

Nonlinear optics has led the way of several fundamental discoveries; elegant and fascinating among these is optical soliton. These solitons are extensively studied since last three decades not only because of their mathematical elegance but also due to their applications in photonics and optical communications [1–24]. Optical beams and pulses spread during propagation due to self-diffraction or dispersion. This broadening can be arrested in optical nonlinear media resulting in stable optical beams or pulses whose spreading is exactly balanced by optical nonlinearity and these self-trapped beams or pulses are known as optical solitons [2,3,5,7]. Optical spatial solitons are self-trapped optical beams whose spreading due to diffraction is exactly balanced by optical nonlinearity-induced self-lensing mechanisms. The optical beam induces a change in refractive index in the medium thereby creating an optical waveguide which subsequently guides the beam.

Thus, the beam first creates a waveguide and then is self-trapped and guided in the waveguide created by itself. On the other hand, an optical temporal soliton is a non-spreading optical pulse whose broadening due to group velocity dispersion is exactly balanced by the nonlinearity-induced self-phase modulation. More explicitly, the group velocity dispersion produces a chirp in the propagating pulse which is balanced by the opposite chirp created by nonlinearity. Exact balance of these opposite chirps results in the formation of optical temporal solitons.

1.1 *Optical solitons or optical solitary waves*

Way back in 1995, Zabusky and Kruskal [13] introduced the term solitons to reflect the particle-like properties of stable self-trapped waves in nonlinear media. The motivation behind this was the fact that the shape of these self-trapped wavepackets remain intact even after collision. Historically, this term was reserved for those wavepackets which obey integrable partial differential equations that can be solved by inverse scattering theory [10,11,13]. Solitons, which are solutions of these equations, remain invariant even after collision as they undergo elastic collision. Optical beam and pulse propagation in Kerr nonlinear media are governed by partial differential equations which are integrable via inverse scattering technique. Hence, optical solitons can be created in such media. However, most physical optical systems, for example, photorefractive media, possess non-Kerr types of nonlinearity in which optical beams or pulses are described by dynamical equations that are partial differential equations, but are not integrable by inverse scattering technique. Self-trapped solutions of these non-integrable equations are known as 'solitary waves'. These solitary waves are not solitons because collision between these solitary waves is not always elastic [8,9,14]. For example, under specific conditions, two photorefractive optical spatial solitary waves may coalesce to form a single solitary wave and a single solitary wave may undergo fission to give birth to new solitons. However, in many cases, these solitary waves play extremely important roles in several applications. In literature, despite their shortcomings, it is now quite common to loosely refer all self-trapped optical solitary waves as solitons regardless of whether they obey integrable partial differential equation or not.

1.2 *Emergence of photorefractive optical spatial solitons*

Guiding of optical beams in the self-created waveguide was first pointed out by Askaryan in 1962 [12]. Kelly [15] pointed out that in Kerr nonlinear media spatial solitons with two transverse dimensions (2D) are unstable and undergo catastrophic collapse. Soon it was realized that spatial solitons with only one transverse dimension (1D) are stable in Kerr nonlinear media [15]. Subsequently, it was also realized that catastrophic collapse of 2D solitons could be avoided in an optical medium possessing saturating nonlinearity [16]. Therefore, the idea of saturating nonlinearity was the key to the discovery of a plethora of optical spatial solitons. In spite of the prediction of formation of stable 2D solitons in saturating nonlinear media, no progress was made in creating these solitons due to lack of identification of appropriate optical materials that possess saturating nonlinearity. The breakthrough was made by Segev *et al* [17,18] in 1990 with the identification of

non-Kerr type of nonlinearity in photorefractive media and observation of optical spatial solitons due to such nonlinearities. Unlike Kerr solitons, the dynamics of photorefractive solitons, under a variety of photorefractive nonlinear effects, is governed by modified nonlinear Schrödinger equations. Photorefractive nonlinearity is non-instantaneous, typically non-local and inherently saturable and spatial solitons in these media can be created at microwatt power level using a very simple set-up in the laboratory, while their formation time ranges from microseconds to minutes.

Photorefractive optical spatial solitons possess several unique properties. For example, unlike Kerr solitons which undergo elastic collision, collisions of these solitons are inelastic, more diverse and interesting. The inelastic collision between two photorefractive spatial solitons may lead to soliton fusion or fission, with particle-like annihilation or birth of new solitons. Another unique feature is that one can create an induced waveguide using a weak soliton beam which subsequently can be used to guide another powerful beam at a different wavelength at which the material is less photosensitive [24,25]. It has been demonstrated that photorefractive spatial soliton-induced waveguides can be used for device applications such as directional couplers and high-efficiency frequency converters [26,27]. The unique features of photorefractive nonlinearity have opened up immense possibilities for new and novel devices, such as all-optical switching and routing, steering, interconnects, parallel computing, optical storage, etc. [28–31]. They are also promising in experimental verification of theoretical models, because they can be created at very low power. In this paper, we confine our discussion on the properties of photorefractive optical spatial solitons. This review gives only a partial description of the development of photorefractive optical spatial solitons, and interested readers are referred to several excellent recent reviews [14,32–35].

2. Photorefractive materials

Exposure of a photorefractive (PR) material with optical field of non-uniform intensity leads to the excitation of charge carriers of inhomogeneous density. These charge carriers then migrate due to drift or diffusion or both and create space-charge field which subsequently modifies the refractive index of the material via either linear electro-optic or quadratic electro-optic effect [30,31,35]. Change in refractive index of certain electro-optic materials due to optically-induced redistribution of charge carriers is known as photorefractive (PR) effect. Photorefractive effect has potential applications in holography [34], optical phase conjugation [30,31,36], optical signal processing and optical storage [31,34,35]. Generally, PR materials are classified in three different categories. Most commonly used PR materials are inorganic ferroelectrics, such as $LiNbO_3$, $KNbO_3$, $BaTiO_3$, etc. Recently, many experiments on spatial solitons have been performed [37] in centrosymmetric paraelectric potassium lithium tantalate niobate (KLTN). Some selected semiconductors such as InP, GaAs, CdTe, etc., also show PR property with large carrier mobility that produces fast dielectric response time, which is important for fast image processing. They have potential use in fast holographic processing of optical information. The third category of PR materials are polymers which show strong PR effect [34,35] at high applied voltage. PR pattern can be erased easily in polymers by decreasing the applied voltage.

3. Photorefractive optical nonlinearity

The band transport model of Kukhtarev–Vinetskii [38] has been widely used to describe the theoretical foundation of PR nonlinearity. In view of this model, we assume a PR medium with a completely full valance band and an empty conduction band which is illuminated by an optical field of non-uniform intensity. It has both donor and acceptor centres, uniformly distributed, whose energy states lie somewhere in the middle of the band gap. The donor electron states are at higher energy than that of the acceptor energy states. The non-uniform optical field excites unionized donors and creates charge carriers. These charge carriers move to the conduction band where they are free to move, to diffuse or to drift under the combined influence of self-generated and external electric field and are finally trapped by the acceptors. During this process, some of the electrons are captured by ionized donors and thus are neutralized, and finally a steady state is reached with the creation of internal space-charge field, that can be evaluated using donor ionization rate equation, electron continuity equation, current density (J) equation, Poisson's equation and the charge density (ρ) equation [14,21,39,40] as follows:

$$\frac{\partial N_{\mathrm{D}}^{+}}{\partial t} = (s_i I + \beta_T)(N_{\mathrm{D}} - N_{\mathrm{D}}^{+}) - \gamma_R N N_{\mathrm{D}}^{+}, \tag{1}$$

$$\frac{\partial N}{\partial t} - \frac{\partial N_{\mathrm{D}}^{+}}{\partial t} = \frac{1}{e}\vec{\nabla}\cdot\vec{J}, \tag{2}$$

$$\vec{J} = eN\mu\vec{E} + k_{\mathrm{B}}T\mu\vec{\nabla}N + k_{\mathrm{p}}s_i\left(N_{\mathrm{D}} - N_{\mathrm{D}}^{+}\right)I\vec{c}, \tag{3}$$

$$\vec{\nabla}\cdot\epsilon\vec{E} = \rho, \tag{4}$$

$$\rho\left(\vec{r}\right) = e\left(N_{\mathrm{D}}^{+} - N_{\mathrm{A}} - N\right), \tag{5}$$

where N, N_{D}, N_{A} and N_{D}^{+} are densities of electron, donor, acceptor and ionized donor, respectively; s_i is the photoexcitation cross-section, I is the intensity of light in terms of Poynting flux, β_T and γ_R are rate of thermal generation and electron trap recombination coefficient, respectively. T, e and μ are respectively the temperature, charge and mobility of the electron; k_{B} is the Boltzmann constant, k_{p} is the photovoltaic constant and $\vec{c}$ is the unit vector in the direction of the c-axis of the PR crystal. The current density arises due to the drift, diffusion and photovoltaic effect; E is the sum of externally applied field and the generated space-charge field E_{sc}. Most experimental investigations on PR solitons have been performed using one-dimensional waves. Therefore, it is appropriate to find material response in one transverse dimension (say x only). In the steady state, eqs (1)–(4) reduce to

$$s_i(I + I_{\mathrm{d}})(N_{\mathrm{D}} - N_{\mathrm{D}}^{+}) - \gamma_R N N_{\mathrm{D}}^{+} = 0, \tag{6}$$

$$\frac{\partial E_{\mathrm{sc}}}{\partial x} = \frac{e}{\epsilon_0\epsilon_r}(N_{\mathrm{D}}^{+} - N_{\mathrm{A}} - N), \tag{7}$$

$$J = eN\mu E_{\mathrm{sc}} + k_{\mathrm{B}}T\mu\frac{\partial N}{\partial x} + k_{\mathrm{p}}s_i\left(N_{\mathrm{D}} - N_{\mathrm{D}}^{+}\right)I, \tag{8}$$

$$\frac{\partial J}{\partial x} = 0, \tag{9}$$

where $I_{\mathrm{d}}(=\beta_T/s_i)$ is the dark irradiance which is also the homogeneous intensity that controls the conductivity of the crystal. Usually I is such that $N \ll N_{\mathrm{D}}$, $N \ll N_{\mathrm{A}}$

and $N_A \ll N_D^+$. The space-charge field E_{sc} under these approximations turns out to be

$$E_{sc} = E_0 \frac{I_\infty + I_D}{I + I_D} + E_p \frac{I_\infty - I}{I + I_D} - \frac{k_B T}{e} \frac{1}{I + I_D} \frac{\partial I}{\partial x}, \tag{10}$$

where E_0 and $E_p = k_p \gamma_R N_A / e\mu$ are the external bias field to the crystal and photovoltaic field, respectively. In the above derivation, we have assumed that the power densities of the optical field and space-charge field attain asymptotically constant values, i.e., $I(x \to \pm\infty, z) = I_\infty$ and $E_{sc}(x \to \pm\infty, z) = E_0$. The change in refractive index Δn in a non-centrosymmetric PR crystal [14,30,33–35], such as $BaTiO_3$, $LiNbO_3$, etc., is due to the linear electro-optic effect (Pockel's) and is given by

$$\Delta n = -\frac{1}{2} n^3 r_e E_{sc}, \tag{11}$$

$$= -\frac{1}{2} n^3 r_e \left[E_0 \frac{I_\infty + I_D}{I + I_D} + E_p \frac{I_\infty - I}{I + I_D} - \frac{k_B T}{e} \frac{1}{I + I_D} \frac{\partial I}{\partial x} \right], \tag{12}$$

where r_e and n are the effective linear electro-optic coefficient and average refractive index, respectively. In centrosymmetric PR materials, the index change is due to the quadratic electro-optic response to a photoinduced internal field and can be expressed [35,41] as

$$\Delta n = -\frac{1}{2} n_b^3 g_e \epsilon_0^2 (\epsilon_r - 1)^2 E_{sc}^2, \tag{13}$$

$$\Delta n = -\frac{1}{2} n_b^3 g_e \epsilon_0^2 (\epsilon_r - 1)^2 \left[E_0 \frac{I_\infty + I_D}{I + I_D} + E_p \frac{I_\infty - I}{I + I_D} - \frac{k_B T}{e(I + I_D)} \frac{\partial I}{\partial x} \right]^2, \tag{14}$$

where g_e, ϵ_r, ϵ_0 and n_b are the effective quadratic electro-optic coefficient, relative dielectric constant, free space permittivity and refractive index of the crystal, respectively; it is assumed that the DC polarization is in the linear regime. Depending on the sign of g_e, the nonlinearity is either self-focussing or defocussing. The nonlinear property of the PR media is evident from eqs (12) and (14) as the refractive index is intensity-dependent. Different terms in the above expressions are responsible for the existence of different types of solitons. Screening and photovoltaic solitons respectively owe their existence to the first and second terms in these expressions. When both first and second terms are dominant, one would expect screening photovoltaic solitons. The third term in each of these equations arises due to the diffusion process and is not in general responsible for the formation of solitons. However, it is responsible for the self-deflection of solitons [14,21,41].

4. Spatial optical solitons owing to single-photon PR phenomenon

4.1 *Historical development*

Way back in 1992, Segev *et al* [17,18] first detected PR solitons in quasisteady-state regime. Soon PR screening solitons were predicted and identified [19–21]. Two different varieties of steady-state screening solitons were subsequently investigated and identified in different biased non-centrosymmetric media under different experimental configurations [19,20,37–40,42–45]. Subsequently, screening solitons were also predicted and

observed in centrosymmetric materials, particularly in KLTN crystals [37,41]. Another family of optical solitons was also observed in unbiased PR crystals which exhibit photovoltaic effect. These solitons, popularly known as photovoltaic solitons, have been observed experimentally in 1D as well as in 2D configurations [46–48]. Besides these, PR solitons which are a combination of the screening and photovoltaic solitons were also predicted and successfully observed [42,49]. They owe their existence to both photovoltaic effect and spatially nonuniform screening of the applied field, and are also known as screening photovoltaic (SP) solitons. By controlling the magnitude of bias field they can be converted to screening solitons or photovoltaic solitons.

PR crystals like $LiNbO_3$ possess high second-order susceptibility, and hence can be used for parametric processes such as second-harmonic generation. Recently, it has become very popular, primarily due to its ultraslow PR relaxation time owing to which any written solitonic channels can act as optical waveguides for a long time even after turning off the soliton beam. These soliton-induced waveguides have been used both in switching devices [50] and for enhancing the second harmonic conversion efficiency [51]. Though non-centrosymmetric PR crystals possess large nonlinearity, their typical response time is in the range of milliseconds. Hence, these materials are not suitable for very fast reconfigurable waveguide channels and switches. On the other hand, using paraelectric centrosymmetric PR crystals, particularly KLTN, DelRe *et al* [52] demonstrated dynamic switching using electroholography [52–54]. Spatial solitons are created in paraelectrics that in turn are employed to guide the signal beams of non-photorefractive wavelength. The soliton-induced waveguide remains stable even at high power levels, and can be modified via the electro-optic effect by varying the external biasing field, ensuring fast change of propagation properties of the signal beam. Fast electro-optic response of KLTN makes them useful in important applications like optical communications and signal processing and this has paved the way to fast nanosecond applications of PR spatial solitons [52–54].

4.2 *Modified nonlinear Schrödinger equation (mNLSE)*

Most of the solitary wave experiments in PR media employ optical beams with one transverse dimension. Therefore, we assume the optical beam to be such that no dynamics is involved in y-direction and it is permitted to diffract only along x-direction. The external bias field E_0 and the optic axis of the crystal are directed along the x-axis. The extraordinary refractive index $\hat{n}_e$ in such cases [14,20,42,55] is given by $(\hat{n}_e)^2 = n_e^2 - n_e^4 r_e E_{sc}$, where n_e is the unperturbed extraordinary index of refraction. The electric field of the soliton forming beam is assumed to be $\vec{E} = \vec{x}\Phi(x,z)\exp[i(kz - \omega t)]$, where $k = k_0 n_e$, $k_0 = 2\pi/\lambda_0$ and λ_0 is the free-space wavelength of the optical field. Employing slowly-varying envelope approximation for Φ in the Maxwell's equations we obtain

$$i\frac{\partial\Phi}{\partial z} + \frac{1}{2k}\frac{\partial^2\Phi}{\partial x^2} - \frac{1}{2}k_0 n_e^3 r_e E_{sc}\Phi = 0. \tag{15}$$

By virtue of the space-charge field evaluated in §3, we obtain the following [14,21,56–60] modified nonlinear Schrödinger equation (mNLSE):

$$i\frac{\partial A}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A}{\partial s^2} - \frac{\beta(1+\rho)A}{1+|A|^2} - \frac{\alpha(\rho - |A|^2)A}{1+|A|^2} + \frac{\delta A}{1+|A|^2}\frac{\partial|A|^2}{\partial s} = 0, \tag{16}$$

where

$$\xi = \frac{z}{k_0 n_e x_0^2}, \quad s = \frac{x}{x_0}, \quad \alpha = (k_0 x_0)^2 (n_e^4 r_e/2) E_p,$$

$$\beta = (k_0 x_0)^2 (n_e^4 r_e/2) E_0, \quad \rho = \frac{I_\infty}{I_d},$$

$$\delta = (k_0^2 x_0 r_e n_e^4 k_B T)/(2e), \quad A = \sqrt{\frac{n_e}{2\eta_0 I_d}}\Phi$$

and η_0 is the intrinsic impedance of free space. The above mNLSE is the governing equation for varieties of bright, dark and gray solitons. Two parameters α and β play very important roles in the formation of these solitons, while δ, which is associated with the diffusion term, is not directly responsible for soliton formation. The diffusion processes are primarily responsible for bending of trajectories of propagating solitons, hence, large value of δ influences the trajectory of bending.

5. Screening optical spatial solitons

Screening solitons could be created in biased non-photovoltaic PR crystals (i.e., $\alpha = 0$). Hence, if we neglect diffusion then the equation for screening solitons reduces to

$$i\frac{\partial A}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A}{\partial s^2} - \beta(1+\rho)\frac{A}{1+|A|^2} = 0. \tag{17}$$

Bright screening solitons: The soliton forming beam for bright solitons vanishes at infinity, therefore $I_\infty = \rho = 0$. We express the stationary bright soliton as $A(s,\xi) = p^{1/2} y(s) \exp(i\nu\xi)$, where ν is the nonlinear shift in propagation constant and $y(s)$ is a normalized real function. In addition, $0 \le y(s) \le 1$, $y(0) = 1$, $\dot{y}(0) = 0$ and $y(s \to \pm\infty) = 0$. The parameter p represents the ratio of the peak intensity (I_{max}) to the dark irradiance I_d, where $I_{max} = I(s = 0)$. Substitution of $A(s,\xi)$ into eq. (17) yields

$$\frac{d^2 y}{ds^2} - 2\nu y - 2\beta\frac{y}{1+py^2} = 0. \tag{18}$$

Use of boundary condition and integration of the above equation leads to

$$s = \pm\frac{1}{(2\beta)^{1/2}}\int_y^1 \frac{p^{1/2}}{[\ln(1+p\hat{y}^2) - \hat{y}^2\ln(1+p)]^{1/2}} d\hat{y}. \tag{19}$$

The bright profile $y(s)$ can be obtained using numerical procedure and it can be easily shown that these solitons exist when $\beta > 0$, i.e., E_0 is positive.

Dark screening solitons: In appropriate media, eq. (17) also admits dark solitary wave solutions [14,21] which are embedded in a constant intensity background. Therefore, I_∞ is finite, and hence, ρ is also finite. In addition, they exhibit anisotropic field profiles with respect to s. We take the following ansatz for stationary solutions: $A(s,\xi) = \rho^{1/2} y(s) \exp(i\nu\xi)$, where ν is the nonlinear shift in propagation constant and $y(s)$ is a

normalized real odd function of s satisfying $y(0) = 0$, $y(s \to \pm\infty) = \pm 1$, $(\mathrm{d}y/\mathrm{d}s) = (\mathrm{d}^2y/\mathrm{d}s^2) = 0$ as $s \to \pm\infty$. Substituting A in eq. (17) we obtain

$$\frac{\mathrm{d}^2y}{\mathrm{d}s^2} - 2\nu y - 2\beta(1+\rho)\frac{y}{1+\rho y^2} = 0. \tag{20}$$

By virtue of integration and use of boundary condition, we immediately obtain

$$s = \pm\frac{1}{(-2\beta)^{1/2}}\int_y^0 \frac{\mathrm{d}\hat{y}}{[(\hat{y}^2-1) - \frac{(1+\rho)}{\rho}\ln\frac{1+\rho\hat{y}^2}{1+\rho}]^{1/2}}. \tag{21}$$

Obviously, these solitons exist only when $\beta < 0$, i.e., when E_0 is negative. Unlike their bright counterpart, these dark screening photovoltaic solitons do not possess bistable property.

Gray screening solitons: Besides bright and dark solitons, eq. (17) also admits another interesting class of solitary waves, which are known as gray solitons [14,21]. In this case too, wave power density attains a constant value at infinity, i.e., I_∞ is finite, and hence, ρ is finite. To obtain stationary solutions, we assume

$$A(s,\xi) = \rho^{1/2}y(s)\exp\left[i\left(\nu\xi + \int^s \frac{J\mathrm{d}\hat{s}}{y^2(\hat{s})}\right)\right], \tag{22}$$

where ν and J are nonlinear shift in propagation constant and a real constant, respectively; $y(s)$ is a normalized real even function of s with properties $y^2(0) = m$ $(0 < m < 1)$, $\dot{y}(0) = 0$, $y(s \to \pm\infty) = 1$ and all derivatives of $y(s)$ are zero at infinity. The parameter m describes grayness, i.e., the intensity $I(0)$ at the beam centre is $I(0) = mI_\infty$. Substitution of the above ansatz for A in eq. (17) yields

$$\frac{\mathrm{d}^2y}{\mathrm{d}s^2} - 2\nu y - 2\beta(1+\rho)\frac{y}{1+\rho y^2} - \frac{J^2}{y^3} = 0. \tag{23}$$

The values of J and ν can be obtained using boundary conditions mentioned earlier. Inserting these values and after integrating once, we get

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2 = 2\nu(y^2-1) + \frac{2\beta}{\rho}(1+\rho)\ln\left(\frac{1+\rho y^2}{1+\rho}\right) + 2(\nu+\beta)\left(\frac{1-y^2}{y^2}\right). \tag{24}$$

Profiles of dark solitons can be obtained easily by numerical integration of this equation. Unlike bright or dark solitons, the phase of gray solitons is not constant across s, instead it varies across s and they can exist only when $\beta < 1$ and $m < 1$.

6. Optical spatial vector solitons

In previous sections, our discussions were confined to optical spatial solitons which are solutions of a single dynamical equation. These solutions arise due to a single optical beam with specific polarization. However, there are instances where two or more optical beams may mutually get trapped and propagate without distortion. These beams could be of the same or different frequencies or polarization. They are mutually trapped and depend on each other in such a way that undistorted propagation of one is sustained by

other and vice versa. In order to describe self and mutually trapped propagation of more than one soliton-forming optical beams, we need to solve a set of more than one coupled solitary-waves. Solutions of this set of coupled NLS equations are called vector solitons if they preserve their shape.

6.1 *Incoherently coupled spatial vector solitons*

Steady-state incoherently coupled solitons are the most extensively studied vector solitons in PR media [14,56,57,61–71]. These solitons exist only when the two soliton-forming beams possess the same polarization and frequency and are mutually incoherent. Four different varieties of solitons i.e., bright–bright, dark–dark, bright–dark and gray–gray [56,57,66] have been observed. Since two beams are mutually incoherent, no phase matching is required and they experience equal effective electro-optic coefficients. The idea of two incoherently coupled solitons has been generalized and extended to soliton families where the number of constituent solitons are more than two.

6.2 *Coupled mNLSE owing to single-photon phenomenon*

We consider a pair of mutually incoherent optical beams of the same frequency and polarization which are propagating in a lossless PR crystal along z-direction. The optical c-axis of the crystal and polarization of both the beams are oriented along the x-direction. These beams are allowed to diffract only along the x-direction and y-dynamics has been implicitly omitted in the analysis. These beams are expressed as $\vec{E_j} = \vec{x}\Phi_j(x,z)\exp(ikz)$, $j = 1, 2$ and Φ_j is the slowly-varying envelope of optical field which satisfies the following equation:

$$i\frac{\partial \Phi_j}{\partial z} + \frac{1}{2k}\frac{\partial^2 \Phi_j}{\partial x^2} - \frac{k_0 n_e^3 r_{33} E_{sc}}{2}\Phi_j = 0. \tag{25}$$

Neglecting the diffusion effect, the space-charge field can be obtained from eq. (10) as

$$E_{sc} = E_0\frac{I_\infty + I_d}{I + I_d} + E_P\frac{I_\infty - I}{I + I_d}, \tag{26}$$

where $I(x,z) = n_e/(2\eta_0)(|\Phi_1|^2 + |\Phi_2|^2)$ is the total power density of the two beams, whose value at a distance far away from the centre of the crystal is $I_\infty = I(x \to \pm\infty)$. Substituting the expression of E_{sc} in eq. (25), we derive the following equation:

$$i\frac{\partial A_j}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A_j}{\partial s^2} - \beta(1+\rho)\frac{A_j}{(1+|A_j|^2+|A_{3-j}|^2)} - \alpha\frac{(\rho - |A_j|^2 - |A_{3-j}|^2)A_j}{(1+|A_j|^2+|A_{3-j}|^2)} = 0, \tag{27}$$

where $A_j = \sqrt{(n_e/2\eta_0 I_d)}\Phi_j$; α, β, ξ, s and ρ are defined earlier. The above set of two coupled Schrödinger equations can be examined for bright–bright, bright–dark, dark–dark, gray–gray screening, photovoltaic as well as screening photovoltaic

solitons [56,62–71]. In the theoretical front, numerical method to solve the above set of coupled equations was developed by Christodoulides *et al* [61]. Though this method has been used extensively, it fails to identify the existence of large family of solitons. Konar *et al* [57] have developed a method which captures those solitons which were missed out by Christodoulides *et al* [61]. Now we describe the method to identify bright–dark solitons only. For elaboration, readers are referred to [14,56,57,62–71].

Bright–dark soliton: We express $A_1 = p^{1/2} f(s) \exp(i\mu\xi)$ and $A_2 = \rho^{1/2} g(s) \exp(i\nu\xi)$, where $f(s)$ and $g(s)$ respectively represent envelope of bright and dark beams. Two positive quantities p and ρ represent the ratios of their maximum power density with respect to the dark irradiance I_d. Therefore, bright–dark soliton pair obeys the following coupled ordinary differential equations:

$$\frac{\mathrm{d}^2 f}{\mathrm{d}s^2} - 2\left[\mu + \frac{\beta(1+\rho)}{1+pf^2+\rho g^2}\right] f = 0 \tag{28}$$

and

$$\frac{\mathrm{d}^2 g}{\mathrm{d}s^2} - 2\left[\nu + \frac{\beta(1+\rho)}{1+pf^2+\rho g^2}\right] g = 0. \tag{29}$$

A particular solution of the above equations is obtained by assuming $f^2 + g^2 = 1$. Use of boundary conditions gives $\mu = -(\beta/\Lambda)\ln(1+\Lambda)$ and $\nu = -\beta$, where $\Lambda = (p-\rho)/(1+\rho)$. When peak intensities of two solitons are approximately equal ($\Lambda \ll 1$), the soliton solution [58,61] leads to $A_1 = p^{1/2}\,\mathrm{sech}[(\beta\Lambda)^{1/2}s]\exp[-i\beta(1-\Lambda/2)\xi]$ and $A_2 = \rho^{1/2}\tanh[(\beta\Lambda)^{1/2}s]\exp[-i\beta\xi]$, which exist only when $\beta\Lambda > 0$.

7. Two-photon photorefractive phenomenon

In previous sections, we have discussed properties of optical spatial solitons which owe their existence to the single-photon PR phenomenon. Recently, Ramadan *et al* [59] created bright spatial solitons in a biased BSO crystal using two-step excitation process. Electrons were first excited to the conduction band by a background beam, and then they were excited to higher levels in the conduction band by a second optical beam of larger wavelength. Using this two-step process in a biased BSO crystal, Ramadan *et al* [59] demonstrated the self-confinement of a red beam at 633 nm supported by another optical beam at 514.5 nm. Recently, Castro-Camus and Magana [60] also presented an identical model of two-photon PR phenomenon which includes a valance band (VB), a conduction band (CB) and an intermediate allowed level (IL). A gating beam of photon energy $\hbar\omega_1$ is used to maintain a quantity of excited electrons from the valance band (VB) to an intermediate allowed level (IL) which are subsequently excited to the conduction band by another signal beam with photon energy $\hbar\omega_2$. The signal beam can induce a spatial-dependent charge distribution leading to a nonlinear change of refractive index in the medium. Based on Castro-Camus and Magana's model, several authors have investigated two-photon screening and photovoltaic solitons [10,14,71–73] which owe their existence to the two-photon PR phenomenon.

7.1 *Optical nonlinearity and evolution equation of solitons*

In order to estimate optical nonlinearity owing to two-photon PR phenomenon, we need to evaluate the space-charge field which can be obtained from the set of rate, current and Poisson's equations proposed by Castro-Camus *et al* [60]. Instead of doing that here we refer interested readers to refs [14,73] and straightaway use the expression of space-charge field $E_{\rm sc}$ due to two-photon PR phenomenon [73]. We assume that the soliton-forming optical beam of intensity I_2 is propagating along the z-direction of the crystal which is permitted to diffract only along the x-direction. The optical beam is polarized along the x-axis which is also the direction of crystal c-axis and the external bias field. The soliton-forming beam is taken as $\vec{E} = \vec{x}\Phi(x,z)\exp[i(kz-\omega t)]$, where the symbols have been defined earlier. The crystal is biased with external voltage V and connected with external resistance R and it is illuminated with a gating beam of constant intensity I_1. We assume that both the power density and space-charge field are uniform at a large distance from the centre of the soliton-forming beam. Thus, $I_2(x\to\pm\infty,z) = I_{2\infty} =$ constant and $E_{\rm sc}(x\to\pm\infty,z) = E_0$. Neglecting the effect of diffusion, the space-charge field turns out to be

$$E_{\rm sc} = gE_{\rm a}\frac{(I_{2\infty}+I_{2\rm d})(I_2+I_{2\rm d}+\frac{\gamma_1 N_{\rm A}}{s_2})}{(I_2+I_{2\rm d})(I_{2\infty}+I_{2\rm d}+\frac{\gamma_1 N_{\rm A}}{s_2})} + E_{\rm p}\frac{s_2(gI_{2\infty}-I_2)(I_2+I_{2\rm d}+\frac{\gamma_1 N_{\rm A}}{s_2})}{(I_2+I_{2\rm d})(s_1I_1+\beta_1)}, \tag{30}$$

where $N_{\rm A}$ and N are acceptor density and electron density in the conduction band, respectively; γ_R, γ_1 and γ_2 are the recombination factors of the conduction to valence band transition, intermediate allowed level to valence band transition and conduction band to intermediate level transition, respectively; β_1 and β_2 are respectively, the thermoionization probability constant for transitions from valence band to intermediate level and intermediate level to conduction band; s_1 and s_2 are photoexcitation crosses. $E_{\rm p} = \kappa_{\rm p}N_{\rm A}\gamma_R/e\mu$ is the photovoltaic field, $I_{2\rm d} = \beta_2/s_2$ is the dark irradiance, $g = 1/(1+q)$, $q = (e\mu N_\infty SR/d)$, $N_\infty = N(x\to\pm\infty)$. In general, g is bounded between $0 \le g \le 1$. Under short circuit condition $R = 0$ and $g = 1$, implying that the external electric field is totally applied to the crystal. For open circuit condition $R\to\infty$, and thus, $g = 0$, i.e., no bias field is applied to the crystal. Employing eq. (30) and following the procedure employed earlier, the nonlinear Schrödinger equation for the normalized envelope can be obtained as [73,74]

$$i\frac{\partial A}{\partial\xi} + \frac{1}{2}\frac{\partial^2 A}{\partial s^2} - \beta g\frac{(1+\rho)(1+\sigma+|A|^2)A}{(1+|A|^2)(1+\sigma+\rho)} - \alpha\eta\frac{(g\rho-|A|^2)(1+\sigma+|A|^2)A}{1+|A|^2} = 0, \tag{31}$$

where $\rho = I_{2\infty}/I_{2\rm d}$, $A = \sqrt{(n_{\rm e}/2\eta_0 I_{2\rm d})}\Phi$, $\beta = (k_0x_0)^2(n_{\rm e}^4 r_{33}/2)E_{\rm a}$, $\eta = \beta_2/(s_1I_1+\beta_1)$, $\sigma = (\gamma_1 N_{\rm A}/s_2 I_{2\rm d}) = (\gamma_1 N_{\rm A}/\beta_2)$. Equation (31) can be employed to investigate screening, photovoltaic and screening photovoltaic solitons under appropriate experimental configuration.

Bright screening solitons: These solitons, for which $\alpha = 0$ and $\rho = 0$, have been studied by several researchers [14,60,72–75]. In the low-amplitude limit, i.e., when $|A|^2 \ll 1$, eq. (31) reduces to

$$i\frac{\partial A}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A}{\partial s^2} - \frac{\beta}{1+\sigma}\left(1+\sigma-\sigma|A|^2\right)A = 0. \tag{32}$$

The one-soliton solution of the above equation is given by

$$A(s,\xi) = p^{1/2}\mathrm{sech}\left[\left(\frac{\beta p\sigma}{1+\sigma}\right)^{1/2} s\right]\exp\left[i\frac{\beta(p\sigma-2\sigma-2)}{2(1+\sigma)}\xi\right]. \tag{33}$$

Dark screening solitons: For this case $\rho \neq 0$, and thus, when $|A|^2 \ll 1$, eq. (31) reduces to

$$i\frac{\partial A}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A}{\partial s^2} - \frac{\beta(1+\rho)}{(1+\sigma+\rho)}(1+\sigma-\sigma|A|^2)A = 0. \tag{34}$$

The dark soliton solution of the above equation turns out to be

$$A(s,\xi) = \rho^{1/2}\tanh\left[-\left(\frac{\beta\rho\sigma}{1+\sigma+\rho}\right)^{1/2} s\right]\exp\left[\frac{i\beta(1+\rho)(\rho\sigma-\sigma-1)\xi}{1+\sigma+\rho}\right]. \tag{35}$$

These solitons could be observed in SBN because, they have an intermediate level required for two-step excitation. In addition to bright and dark solitons, eq. (31) also predicts steady-state gray solitons which were investigated by Zhang *et al* [75]. Characteristics of these solitons are similar to one-photon PR gray spatial solitons. For example, they require bias field opposite to the optical c-axis and their FWHM is inversely proportional to the square root of the absolute value of the bias field. Proceeding in a similar way, we can study bright and dark photovoltaic solitons.

8. Modulation instability (MI)

Modulation instability (MI) is an inherent characteristic of wave propagation in nonlinear media. It refers to unstable propagation of a continuous or quasicontinuous wave (CW) in such a way that the wave disintegrates into a large number of localized coherent structures after propagating some distance through nonlinear physical media. MI occurs as a result of interplay between nonlinearity and dispersion in temporal domain, and between nonlinearity and diffraction in the spatial domain [2,5,76–85]. A continuous wave (CW) or quasicontinuous wave radiation propagating in a nonlinear medium may suffer instability with respect to weak periodic modulation of the steady state and results in the breakup of CW into a train of ultrashort pulses. In spatial domain, for a narrow beam, self-phase modulation exactly balances the diffraction and a robust spatial soliton is obtained, while a broad optical beam disintegrates into many filaments during propagation in the same self-focussing nonlinear medium.

MI has been extensively studied in a wide range of physical systems like fluids [86], plasmas [76], Bose–Einstein condensates [77], discrete nonlinear systems [78], negative index materials [79], soft matter [80], PR media [81,82], optical fibres [83], etc. MI typically

occurs in the same parameter region where solitons are observed. In fact, the filaments that emerge from the MI process are actually trains of almost ideal solitons. Therefore, the phenomenon of MI can be considered as a precursor of soliton formation and has been found in both coherent and incoherent beams. MI has been extensively investigated in PR media [85,87,88]. Single- as well as two-photon PR media have been considered to analyse instability characteristics. Moreover, not only non-centrosymmetric but also centrosymmetric PR media have been examined [85]. Unlike non-centrosymmetric media, in centrosymmetric media the characteristics of this instability is independent of the external applied field. In the next section, we examine the MI of a broad optical beam in a biased two-photon non-centrosymmetric photovoltaic PR medium.

8.1 *MI gain under linear stability framework*

In order to find out the MI gain, we consider an optical beam with large transverse spatial dimension. As we are confining our present interest on the stability of a broad bright beam of finite transverse extension, $I_{2\infty} = 0$ and hence $\rho = 0$. Therefore, the evolution equation of the broad optical field reduces to

$$i\frac{\partial A}{\partial \xi}+\frac{1}{2}\frac{\partial^2 A}{\partial s^2}-\beta g\frac{(1+\sigma+|A|^2)}{(1+|A|^2)(1+\sigma)}A+\alpha\eta\frac{|A|^2(1+\sigma+|A|^2)}{(1+|A|^2)}A=0. \quad (36)$$

Equation (36) admits a steady-state CW solution $A(\xi, s) = \sqrt{P}\exp[i\Psi(\xi)]$, where P is the initial input power at $\xi = 0$ and $\Psi(\xi)$ is the nonlinear phase shift which increases with propagation distance ξ according to the equation

$$\Psi(\xi) = -g\beta\frac{(1+\sigma+P)}{(1+P)(1+\sigma)}\xi+\alpha\eta\frac{P(1+\sigma+P)}{1+P}. \quad (37)$$

The initial stage of MI can be investigated in the linear stability framework, under which the stability of the steady-state solution is examined by introducing a perturbation in the amplitude of the beam envelope so that the perturbed field now becomes

$$A(\xi, s) = \left[\sqrt{P}+a(\xi, s)\right]\exp[i\Psi(\xi)], \quad (38)$$

where $a(\xi, s)$ is an arbitrarily small complex perturbation field such that $a(\xi, s) \ll \sqrt{P}$. Substituting the perturbed field in eq. (36) and retaining terms linear only in the perturbed quantity, the evolution equation for the perturbation field is obtained as

$$i\frac{\partial a}{\partial \xi}+\frac{1}{2}\frac{\partial^2 a}{\partial s^2}+\alpha\eta P(a+a^*)+\frac{\alpha\eta\sigma P(a+a^*)}{(P+1)^2}+\frac{g\beta\sigma P(a+a^*)}{(1+\sigma)(1+P)^2}=0, \quad (39)$$

where $*$ denotes complex conjugate. The spatial perturbation $a(\xi, s)$ is assumed to be composed of two side-band plane waves

$$a(\xi, s) = u\cos(K\xi-\Omega s)+iv\sin(K\xi-\Omega s), \quad (40)$$

where u and v are the real amplitudes of the perturbing field, K and Ω are the wave number and spatial frequency of the perturbations, respectively. Substitution for the perturbation field into its evolution equation yields

$$\begin{pmatrix}\Pi^- & -K\\ K & \Pi^+\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}=0, \quad (41)$$

where

$$\Pi^- = -\Pi^+ + \frac{2g\beta\sigma P}{(1+\sigma)(1+P)^2} + \frac{2\alpha\eta\sigma P}{(1+P)^2} + 2\alpha\eta P$$

and

$$\Pi^+ = \Omega^2/2.$$

Equation (41) possesses a non-trivial solution only when the following dispersion relation holds good:

$$K^2 = -\frac{\Omega^2}{2}\left(\frac{2g\beta\sigma P}{(1+\sigma)(1+P)^2} + \frac{2\alpha\eta\sigma P}{(1+P)^2} + 2\alpha\eta P - \Omega^2/2\right). \tag{42}$$

If the wave number of the perturbation becomes complex, then the instability will set in with exponential growth of the perturbation field $a(\xi, s)$ resulting in the filamentation of the broad beam into a number of filaments. Thus, the propagating broad optical beam will be unstable. The growth rate $g(\Omega)(= 2\mathrm{Im}(K))$ of the MI is obtained as

$$g(\Omega) = \sqrt{2}\Omega\left(\frac{2g\beta\sigma P}{(1+\sigma)(1+P)^2} + \frac{2\alpha\eta\sigma P}{(1+P)^2} + 2\alpha\eta P - \Omega^2/2\right)^{1/2}. \tag{43}$$

8.2 *Gain spectrum of instability*

MI gain is achievable in a photovoltaic PR (PVPR) crystal for a given range of frequency and specified range of values of $g, \sigma, \eta, \alpha, \beta$ and P. Parameters η and σ are always positive, while α and β can be both positive or negative depending on the media and the polarity of the external bias field. In an unbiased PVPR media ($E_a = 0$, i.e., $\beta = 0$), thus K is always positive if $\alpha < 0$. Hence, usually in such media, modulation instability

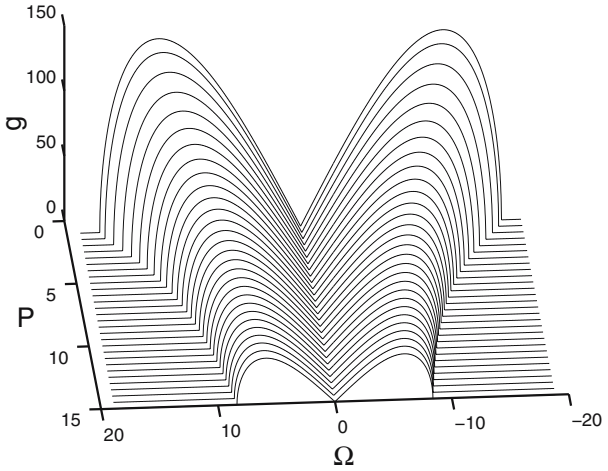

Figure 1. MI gain spectrum $g(\Omega)$ as a function of perturbation frequency Ω.

cannot set in. However, in such media with the application of external field of appropriate magnitude and polarity MI can set in and grow as long as

$$\frac{2g\beta\sigma P}{(1+\sigma)(1+P)^2} > 2|\alpha|\eta P\left(1+\frac{\sigma}{(1+P)^2}\right)+\frac{\Omega^2}{2}. \tag{44}$$

Therefore, with the application of external electric field, it is possible to initiate modulation instability and control the growth of instability in those media where MI was hitherto prohibited. On the other hand, if for a given value of beam power MI growth rate is finite in an unbiased PVPR medium with positive value of α, then the instability growth rate can be enhanced or decreased with the application of external electric field of appropriate

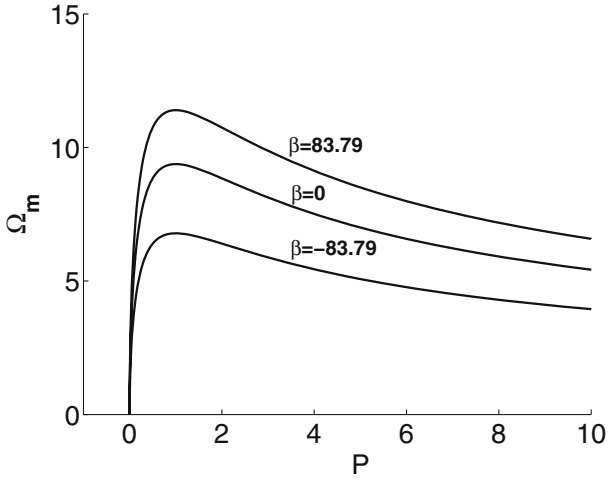

Figure 2. Variation of Ω_m with P for three values of β.

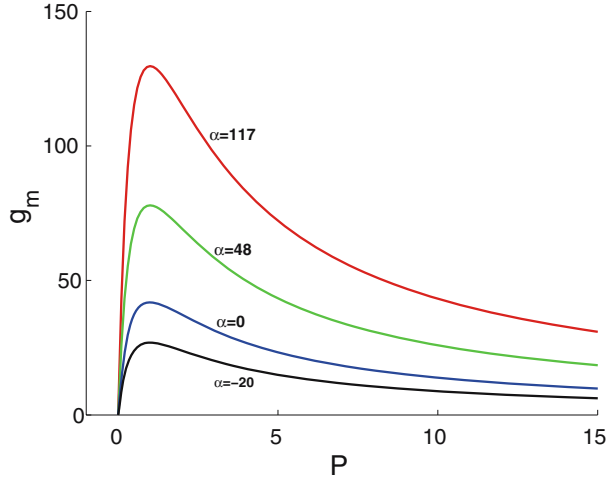

Figure 3. Variation of maximum growth g_m with the normalized beam power for different values of α.

magnitude and polarity. Or in other words, growth rate of the instability can be controlled with the application of external field of chosen polarity and magnitude. MI can take place only below the critical frequency Ω_c which is given by

$$\Omega_c = \pm \left[\frac{g\beta\sigma P}{(1+\sigma)(1+P)^2} + \frac{\alpha\eta\sigma P}{(1+P)^2} + \alpha\eta P \right]^{1/2}. \tag{45}$$

The instability is most efficient and reaches its maximum at $g = g_m$ when $\Omega = \Omega_m$, where $g_m = (\Omega_c^2/2)$ and $\Omega_m = (\Omega_c^2/\sqrt{2})$. To examine the MI growth, we take a typical Cu:KNSBN crystal. Intermediate energy level is included in Cu:KNSBN crystal and photovoltaic field is in the direction of optic axis. At $\lambda_0 = 0.5\,\mu$m, crystal parameters are $n_e = 2.27$, $E_p = 2.8 \times 10^6\,\mathrm{Vm}^{-1}$, $\eta = 1.5 \times 10^{-4}$, $\sigma = 10^4$, $r_{33} = 200 \times 10^{-12}\,\mathrm{m/V}$. The scaling parameter $x_0 = 10\,\mu$m, $\alpha = 117.3$ and $g = 1$. We take three different values of E_a, in particular, $E_a = -2 \times 10^6\,\mathrm{Vm}^{-1}$, 0 and $2 \times 10^6\,\mathrm{Vm}^{-1}$ corresponding to $\beta = -83.79$, 0 and 83.79 respectively. Figure 1 depicts the MI gain spectrum $g(\Omega)$ as a function of perturbation frequency Ω. The variation of Ω_m with P for three values of β has been depicted in figure 2. Initially, Ω_m increases with the increase in the value of P, then decreases with the increase in P. In order to examine the role played by α on the growth of instability, we have demonstrated in figure 3, the variation of maximum growth g_m with the normalized beam power for three different values of α. As expected, a higher value of α enhances the growth of instability. In conclusion, with the application of external electric field, it is possible to initiate modulation instability and control the growth of instability in those media where MI was hitherto prohibited.

9. Conclusion

A brief review of some selected developments in the field of optical spatial solitons in PR media has been presented. Underlying mechanism responsible for the formation of solitons has been discussed for both single- and two-photon PR media. Vector solitons, particularly, incoherently coupled solitons due to single photon and two-photon PR phenomena have been highlighted. Existence of some missing solitons has been pointed out. Modulation instability which is a precursor to soliton formation has also been considered. Important applications of PR solitons have been highlighted.

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