Novel energy sharing collisions of multicomponent solitons

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Abstract. In this paper, we discuss the fascinating energy sharing collisions of multicomponent solitons in certain incoherently coupled and coherently coupled nonlinear Schrödinger-type equations arising in the context of nonlinear optics.

Keywords. Coupled nonlinear Schrödinger equations; Hirota's bilinearization method; bright soliton solution; soliton collision; energy sharing collision.

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1. Introduction

Solitons are fascinating nonlinear entities with huge potential for technological applications due to their remarkable collision properties. Following the pioneering numerical work of Zabusky and Kruskal [1] on soliton collisions, there has been a number of research papers on soliton interaction and it still remains a frontier topic of research. It is well known that solitons are solitary waves that asymptotically preserve their amplitude and speed during their collisions with other solitary waves except for a phase-shift [2].

Multicomponent solitons (MSs) are intriguing nonlinear objects in which a given soliton is split among several components. These solitons are also known as vector solitons or multicolour solitons. From a mathematical perspective, these MSs arise as solutions of certain multicomponent integrable nonlinear partial differential equations. In integrable systems, such MSs have the same central position and travel with the same velocity. These MSs appear in a wide range of physical systems that include nonlinear optics [3,4], plasma physics [4], water waves [5], biophysics [6] and Bose–Einstein condensates [7]. Here, our focus will be on particular MSs arising in the context of nonlinear optics.

In the context of nonlinear optics, the MSs arise as solutions of integrable multiple coupled nonlinear Schrödinger-type equations which describe the dynamics of simultaneous propagation of multiple waveguide modes in Kerr-like media [3]. Such multicomponent

systems show interesting propagation and collision dynamics as a result of various non-linear effects. When two or more optical modes copropagate inside a fibre, they can interact with each other through the fibre nonlinearity. In general, such interactions are governed by coupled nonlinear Schrödinger (CNLS) family of equations. Based on the presence and absence of coherent (phase-dependent) nonlinearities, these CNLS equations can be classified into two classes, namely coherently coupled nonlinear Schrödinger (CCNLS) equations and incoherently coupled nonlinear Schrödinger (ICNLS) equations. A physically interesting set of ICNLS equations arising in nonlinear optics is

$$iq_{j,z} + q_{j,tt} + 2\left(|q_1|^2 + \sigma|q_2|^2\right)q_j = 0, \quad j = 1, 2, \tag{1}$$

in which the nonlinear couplings are due to self-phase modulation (SPM) and cross-phase modulation (XPM) and depend only on the local intensities of the copropagating fields, but insensitive to their phases [3]. For $\sigma = 1$, eq. (1) reduces to the integrable Manakov system with q_j, $j = 1, 2$, being the envelope of the jth mode, z and t represent the normalized distance along the fibre and the retarded time, respectively and describes an intense electromagnetic pulse propagation in birefringent fibre [8]. For $\sigma = -1$, system (1) becomes the mixed-ICNLS system. Lazarides and Tsironis [9] have obtained this mixed ICNLS system as governing equations for electromagnetic pulse propagation in isotropic and homogeneous nonlinear left-handed materials, by taking the effective permittivity and effective permeability to be intensity dependent and following a reductive perturbational approach. Here q_1 and q_2 are the electric and magnetic field components of the electromagnetic pulse, respectively, the subscripts z and t denote partial derivatives with respect to normalized distance and retarded time respectively. Mixed-ICNLS system (1) can also be obtained as the modified Hubbard model (Lindner–Fedyanin system) in the long-wavelength approximation by taking the electron–phonon interaction into account [10]. These Manakov and mixed-ICNLS systems find important applications in optical communication and in artificial metamaterials. They have been intensively studied in literature [2,8–19]. Also, the integrable multicomponent generalization of ICNLS system (1) can be written as

$$iq_{j,z} + q_{j,tt} + 2\left(\sum_{j=1}^{m} \sigma_j|q_j|^2\right)q_j = 0, \quad j = 1, 2, 3, ..., m, \tag{2}$$

where $\sigma_j = \pm 1$ represent the nature of nonlinear coupling, which is of either focussing (defocussing) type for $\sigma_j = 1$ ($\sigma_j = -1$) or mixed type ($\sigma_j = 1$ for $j = 1, 2, ..., p$ and $\sigma_j = -1$ for $j = p + 1, p + 2, ..., m$). The above system admits bright soliton solutions for the Manakov (focussing) case and it supports both bright and bright-dark soliton solutions for the mixed case [11–18]. Particularly, the bright multisoliton solutions of the multicomponent generalization of Manakov system have been obtained by Kanna *et al* using the Hirota bilinearization method and a detailed investigation on the soliton collisions, such as energy sharing collision and elastic-type interactions, have been explored [14–17]. In this paper, we shall review the results of the two-component systems only.

In general cases, like picosecond pulse propagation in non-ideal low birefringent multimode fibres or beam propagation in weakly anisotropic Kerr-type nonlinear media, the coherent effects due to the interaction of copropagating fields should also be considered [3,20]. The propagation of coherently coupled orthogonally polarized waveguide modes

in Kerr-type nonlinear medium is governed by the following two-component coherently coupled nonlinear Schrödinger (CCNLS)-type equations [3,20,21]:

$$iq_{1,z} + \delta q_{1,tt} - \mu q_1 + (|q_1|^2 + \sigma|q_2|^2)q_1 + \lambda q_2^2 q_1^* = 0, \tag{3a}$$

$$iq_{2,z} + \delta q_{2,tt} + \mu q_2 + (\sigma|q_1|^2 + |q_2|^2)q_2 + \lambda q_1^2 q_2^* = 0, \tag{3b}$$

where q_1 and q_2 are slowly varying complex amplitudes in each polarization mode, z and t are the propagation direction and transverse direction, respectively, μ is the degree of birefringence and δ is the group velocity dispersion.

In the above equations, the nonlinearities arise from the SPM ($|q_k|^2 q_j$, $j = k = 1, 2$), XPM ($|q_k|^2 q_j$, $j, k = 1, 2$, $k \neq j$) and four-wave mixing process (FWM: $q_k^2 q_j^*$, $j, k = 1, 2$, $k \neq j$), among which the first two are phase-independent while the third one is phase-dependent (coherent) nonlinearity. Also, eq. (3) is non-integrable. However, for specific choices of system parameters (δ, μ, σ and λ) it becomes integrable and exists in different physical situations [21]. The corresponding integrable CCNLS system is

$$iq_{1,z} + q_{1,tt} + \gamma(|q_1|^2 + 2|q_2|^2)q_1 - \gamma q_2^2 q_1^* = 0, \tag{4a}$$

$$iq_{2,z} + q_{2,tt} + \gamma(2|q_1|^2 + |q_2|^2)q_2 - \gamma q_1^2 q_2^* = 0. \tag{4b}$$

Hereonwards we refer to the above system as 2-CCNLS system. The above system governs the dynamics of pulse propagation in nonlinear gyrotropic media [22] as well as in an isotropic nonlinear Kerr medium for particular choices of third-order susceptibilities. System (4) is shown to be integrable by Painlevé analysis and soliton solutions were obtained as the linear superposition of two nonlinear Schrödinger (NLS) solitons [21].

An integrable multicomponent generalization of the above 2-CCNLS system (4) is,

$$iq_{j,z} + q_{j,tt} + \gamma\left(|q_j|^2 + 2\sum_{l=1,l\neq j}^{2} |q_l|^2\right)q_j$$
$$-\gamma \sum_{l=1,l\neq j}^{2} q_l^2 q_j^* = 0, \quad j = 1, 2, 3, ..., m. \tag{5}$$

In ref. [24], Kanna *et al* have also investigated bright soliton dynamics in another type of 2-CCNLS system similar to the two-component version of (4). This 2-CCNLS system shows novel energy switching collision of bright solitons which will be discussed later. Additionally, there exists another integrable 2-CCNLS system with nonlinearities having opposite signs in the two components, for which the soliton solutions and bound states are constructed in ref. [25]. As the solitons in this system undergo standard elastic collision, we do not discuss this system in this review.

The soliton solutions of multicomponent Manakov and mixed-ICNLS systems (2) were obtained using the Hirota's bilinearization method [26] by transforming the nonlinear equations (2) into the bilinear form and by recursively solving the resulting set of equations in a standard way. On the other hand, for m-CCNLS system (5), we have to apply non-standard bilinearization procedure. A standard bilinearization procedure will result in a greater number of bilinear equations than the number of bilinearizing variables, which results in soliton solutions with less number of arbitrary parameters. In order to get more

general soliton solutions, we introduce an auxiliary function during the bilinearization of the m-CCNLS system which gives equal number of bilinear equations and variables [23–27]. To be more clear with the presentation, we now give the bilinear equations for the m-CCNLS system (5)

$$(i D_z + D_t^2)(g^{(j)} \cdot f) = \gamma s g^{(j)*}, \quad j = 1, 2, ..., m, \tag{6a}$$

$$D_t^2(f \cdot f) = 2\gamma \sum_{j=1}^{m} |g^{(j)}|^2, \tag{6b}$$

$$s \cdot f = \sum_{j=1}^{m} (g^{(j)})^2, \tag{6c}$$

obtained by using the rational transformation $q_j = (g^{(j)}/f)$, $j = 1, 2, ..., m$, with the introduction of an auxiliary function s. Here $g^{(j)}$ (f) is complex (real) function of z, t, D_z and D_t represent the Hirota's differential operators [26] and $*$ indicates the complex conjugation. The exact forms of $g^{(j)}$, f and s, which result in the soliton solutions of (5), are obtained by suitably expressing them as power series and recursively solving the resulting set of equations at various powers of expansion parameter from the bilinear equations (6). One can refer to [23] for more details regarding the soliton solutions of (5).

The m-CCNLS system exhibits a variety of interesting solitons due to the existence of additional coherent nonlinearities resulting from four-wave mixing process. Based on the presence and absence of coherent nonlinearities (respectively for $s \neq 0$ and $s = 0$ in (6)), the obtained one-soliton solution (given in Appendix A) can be classified into two types, namely (i) coherently coupled solitons and (ii) incoherently coupled solitons, using the soliton parameters ($\alpha_u^{(j)}$, $u = 1, 2$, $j = 1, 2, 3, ..., m$). Particularly, in ref. [23], it has been shown that for the choice $\sum_{j=1}^{m} (\alpha_u^{(j)})^2 = 0$, $u = 1, 2$, the auxiliary function s becomes zero and the corresponding one-soliton solution is said to be incoherently coupled soliton (ICS). But for the choice $\sum_{j=1}^{m} (\alpha_u^{(j)})^2 \neq 0$, $u = 1, 2$, s becomes non-zero and the resulting one-soliton solution is said to be coherently coupled soliton (CCS). In general, ICS exhibits standard sech-type (single-hump) soliton profile whereas the CCSs can have novel double-hump and flat-top profiles in addition to the single-hump (non-sech type) structures. A detailed analysis of these ICS and CCS is given in ref. [23].

The main objective of the present paper is to give a clear picture of various energy sharing collisions of bright solitons in the three integrable nonlinear systems, namely the Manakov, the mixed-ICNLS and the m-CCNLS systems. For this purpose, we use the soliton solutions obtained earlier and demonstrate the collisions graphically. We present the collision scenario of solitons in the Manakov and the mixed-CCNLS systems in §2 and §3, respectively. Bright soliton collision in the m-CCNLS system is given in §4 and the final section is allotted for conclusion.

2. Soliton collisions in the ICNLS (Manakov) system: Type-I energy sharing collision

To begin, we consider the collision of solitons in the celebrated Manakov system (eq. (1) with $\sigma = 1$). Manakov himself has explicitly obtained one- and two-soliton solutions using the inverse scattering transform method. He has shown that in a two-soliton collision

process, soliton polarization does not change only when their initial polarizations are parallel or orthogonal. Later, Radhakrishnan *et al* have shown that the solitons in the Manakov system (1) exhibit certain novel inelastic (energy sharing) collisions [11] in contrast to single-component NLS system. Kanna *et al* have obtained multisoliton solutions for the multicomponent Manakov system (2) using the Hirota's method [15]. Thus, the system has been well studied in [8,11,13–16] and the existence of N-soliton (for arbitrary N) solution and also its proof has been obtained. In this section, we restrict our review to the interaction of two solitons in the Manakov system.

2.1 *Two-soliton solution and its collision dynamics*

The two-soliton solution of Manakov system obtained by Radhakrishnan *et al* [11] can be compactly written in terms of Gram determinant [17] as

$$q_j = \frac{g^{(j)}}{f}, \quad j = 1, 2, \tag{7a}$$

where

$$g^{(j)} = \begin{vmatrix} A_{11} & A_{12} & 1 & 0 & e^{\eta_1} \\ A_{21} & A_{22} & 0 & 1 & e^{\eta_2} \\ -1 & 0 & B_{11} & B_{12} & 0 \\ 0 & -1 & B_{21} & B_{22} & 0 \\ 0 & 0 & -\alpha_1^{(j)} & -\alpha_2^{(j)} & 0 \end{vmatrix}, \qquad f = \begin{vmatrix} A_{11} & A_{12} & 1 & 0 \\ A_{21} & A_{22} & 0 & 1 \\ -1 & 0 & B_{11} & B_{12} \\ 0 & -1 & B_{21} & B_{22} \end{vmatrix}, \tag{7b}$$

in which

$$A_{ij} = \frac{e^{\eta_i + \eta_j^*}}{k_i + k_j^*}$$

and

$$B_{ij} = \kappa_{ji} = \frac{\left(\alpha_j^{(1)} \alpha_i^{(1)*} + \alpha_j^{(2)} \alpha_i^{(2)*} \right)}{(k_j + k_i^*)}, \quad i, j = 1, 2.$$

The above most general bright two-soliton solution is characterized by six arbitrary complex parameters k_1, k_2, $\alpha_1^{(j)}$ and $\alpha_2^{(j)}$, $j = 1, 2$ and it corresponds to the collision of two bright solitons.

Now, we discuss the collision dynamics of two bright solitons in the Manakov system. One finds that enhancement and suppression of soliton intensities in different components occur as a consequence of energy exchange between the two colliding solitons as well as the two components. This exchange phenomenon also satisfies the energy conservation of both the solitons before and after collision and also the conservation of energy in individual components which has been discussed in detail in refs [11,13–15]. For illustrative purpose, we show the energy sharing collision characterized by intensity redistribution, amplitude-dependent phase-shift and change in relative separation distances in the Manakov system in figure 1. The parameters chosen are: $k_1 = 1 + i$, $k_2 = 1.5 - 0.8i$, $\alpha_1^{(1)} = 2.5$, $\alpha_1^{(2)} = 0.2$, $\alpha_2^{(1)} = 1.5$ and $\alpha_2^{(2)} = -0.6$. The two solitons S_1 and S_2 are well separated before and after collision in both the components q_1 and q_2. In the q_1 component the intensity of soliton S_1 gets suppressed while that of soliton S_2 is enhanced after interaction and the reverse scenario takes place in the q_2 component.
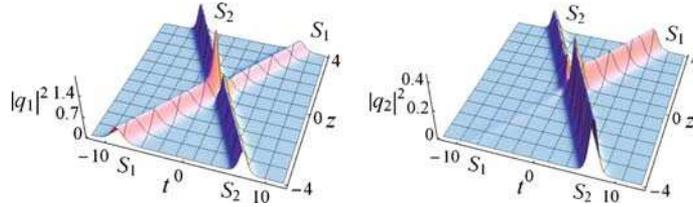
Figure 1. Type-I energy sharing collision of Manakov solitons in 2-ICNLS system.

2.2 *Asymptotic analysis of two-soliton solution of the Manakov system*

The understanding of this fascinating collision process can be facilitated by making an asymptotic analysis of the two-soliton solution of the Manakov case [15]. We perform the analysis for the choice $k_{1R}, k_{2R} > 0$ and $k_{1I} > k_{2I}$. For any other choice the analysis is similar. The study shows that due to collision, the amplitudes of the colliding solitons S_1 and S_2 change from $(A_1^{1-}k_{1R}, A_2^{1-}k_{1R})$ and $(A_1^{2-}k_{2R}, A_2^{2-}k_{2R})$ to $(A_1^{1+}k_{1R}, A_2^{1+}k_{1R})$ and $(A_1^{2+}k_{2R}, A_2^{2+}k_{2R})$, respectively. Here the superscripts in A_i^j denote the solitons (number (1, 2)), the subscripts represent the components (number (1, 2)) and $\pm$ signs stand for $z \to \pm\infty$. One can find that

$$A_j^{l+} = T_j^l A_j^{l-}, \quad j, l = 1, 2, \tag{8a}$$

where

$$\begin{pmatrix} A_1^{1-} \\ A_2^{1-} \end{pmatrix} = \begin{pmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \end{pmatrix} \frac{e^{-R_1/2}}{(k_1 + k_1^*)}, \tag{8b}$$

$$\begin{pmatrix} A_1^{2-} \\ A_2^{2-} \end{pmatrix} = \begin{pmatrix} e^{\delta_{11}} \\ e^{\delta_{12}} \end{pmatrix} \frac{e^{-(R_1+R_3)/2}}{(k_2 + k_2^*)}, \tag{8c}$$

and the transition amplitudes are given by

$$T_j^1 = \left(\frac{(k_2 + k_1^*)(k_1 - k_2)}{(k_1 + k_2^*)(k_1^* - k_2^*)} \right)^{1/2} \left[\frac{1 - \lambda_2}{\sqrt{1 - \lambda_1\lambda_2}} \right], \quad j = 1, 2, \tag{8d}$$

$$T_j^2 = -\left(\frac{(k_2 + k_1^*)(k_1^* - k_2^*)}{(k_1 - k_2)(k_1 + k_2^*)} \right)^{1/2} \left[\frac{\sqrt{1 - \lambda_1\lambda_2}}{1 - \lambda_1} \right], \quad j = 1, 2. \tag{8e}$$

In the above expressions

$$e^{R_j} = \frac{\kappa_{jj}}{k_j + k_j^*}, \qquad e^{R_3} = \frac{|k_1 - k_2|^2}{(k_1 + k_1^*)(k_2 + k_2^*)|k_1 + k_2^*|^2} \begin{vmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{vmatrix},$$

$$e^{\delta_{1j}} = \frac{(k_1 - k_2)}{(k_1 + k_j^*)(k_2 + k_j^*)} \begin{vmatrix} \alpha_1^{(j)} & \alpha_2^{(j)} \\ \kappa_{21} & \kappa_{22} \end{vmatrix}, \quad \lambda_1 = \frac{\kappa_{21}}{\kappa_{11}} \frac{\alpha_1^{(j)}}{\alpha_2^{(j)}} \text{ and } \lambda_2 = \frac{\kappa_{12}}{\kappa_{22}} \frac{\alpha_2^{(j)}}{\alpha_1^{(j)}},$$

in which

$$\kappa_{ji} = \frac{\left(\alpha_j^{(1)} \alpha_i^{(1)*} + \alpha_j^{(2)} \alpha_i^{(2)*} \right)}{(k_j + k_i^*)}, \quad i, j = 1, 2.$$

In general, $|T_j^l|^2 \neq 1$ and hence intensity (energy) redistribution occurs among the two colliding solitons as well as among the components. However, during the interaction process the total energy of each soliton is conserved, that is $|A_1^{l\pm}|^2 + |A_2^{l\pm}|^2 = 1$, $l = 1, 2$. Another noticeable observation in this interaction process is that the intensity of each mode is separately conserved, that is $\int_{-\infty}^{\infty} |q_j|^2 dt = $ constant, $j = 1, 2$. Also, the colliding solitons S_1 and S_2 undergo amplitude-dependent phase-shifts Φ_1 and Φ_2, respectively, given by

$$\Phi_1 = -\Phi_2 = \frac{1}{2} \ln \left[\frac{|k_1 - k_2|^2 (\kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21})}{|k_1 + k_2^*|^2 \kappa_{11}\kappa_{22}} \right]. \tag{9}$$

Ultimately, these phase-shifts make the relative separation distance between the solitons $t_{12}^{\pm}$ (position of S_2 (at $z \to \pm\infty$) minus position of S_1 (at $z \to \pm\infty$) also to vary during collision, depending upon the amplitudes. The change in the relative separation distance is found to be

$$\Delta t_{12} = t_{12}^- - t_{12}^+ = \frac{(k_{1R} + k_{2R})}{k_{1R} k_{2R}} \Phi_1.$$

We call such a collision scenario as type-I energy sharing collision (ESC). Such energy sharing collision occurs for $(\alpha_1^{(1)}/\alpha_2^{(1)}) \neq (\alpha_1^{(2)}/\alpha_2^{(2)})$, which is quite general. But when we choose $(\alpha_1^{(1)}/\alpha_2^{(1)}) = (\alpha_1^{(2)}/\alpha_2^{(2)})$, the two solitons exhibit elastic collision only. This interesting collision behaviour has also been experimentally verified in birefringent fibres [28] and in photorefractive media [29]. The most important application of the energy sharing collision property is a theoretical possibility for constructing logic gates for optical computers.

3. Soliton collisions in the mixed-ICNLS system: Type-II energy sharing collision

Next, we consider the mixed 2-ICNLS system (1) with $\sigma = -1$. This system (1) admits three types of soliton solutions, namely bright–bright, bright–dark and dark–dark. It was found that the bright solitons exhibit a special type of energy sharing collision whereas the dark solitons always undergo elastic collision [16,18]. The Gram determinant form of the bright N-soliton solution (for arbitrary N) has been obtained by Kanna *et al* [16]. In this section, we revisit the collision dynamics of two bright solitons in detail. For this purpose, we consider the bright two-soliton solution of the mixed 2-CNLS equations which is given by eq. (7) with the redefinition of B_{ij} as

$$B_{ij} = \kappa_{ji} = \frac{(\alpha_j^{(1)}\alpha_i^{(1)*} - \alpha_j^{(2)}\alpha_i^{(2)*})}{(k_j + k_i^*)}, \quad i, l = 1, 2. \tag{10}$$

Note that the form of the above two-soliton solution remains the same as that of the Manakov case except for the crucial difference in the expressions for κ_{ji}.

It was found that the mixed 2-CNLS equations admit energy sharing collision of bright solitons in a quite different manner from the collision scenario of the Manakov system [16]. It has been shown that in mixed CNLS equations, during a two-soliton collision process, there is a possibility of either enhancement or suppression of intensity in a given soliton in all the components [16]. Here also the collision process is characterized as in

the focussing case. The most important consequence of this energy sharing collision is the possibility of soliton amplification in all the components. Figure 2 shows that after collision the first soliton S_1 in the component q_1 gets enhanced in its amplitude while the soliton S_2 is suppressed. Interestingly, the same kind of changes are observed in the second component q_2 as well. As the two-soliton solution of mixed ICNLS equation is the same as that of the Manakov system except for κ_{ji}, the asymptotic expressions are also same as given by (8) with κ_{il} as given in eq. (10). The analysis reveals the fact that the colliding solitons change their amplitudes in each component according to the conservation equation

$$|A_1^{j-}|^2 - |A_2^{j-}|^2 = |A_1^{j+}|^2 - |A_2^{j+}|^2 = 1, \quad j = 1, 2, \tag{11}$$

where $A_j^{l\pm}$ are given by eq. (8) with modified κ_{ji}s. This condition allows the given soliton to experience the same effect in each component during collision, which may find potential application in the noiseless amplification of a pulse. It can be easily observed from the conservation relation (11) that each component of a given soliton experiences the same kind of energy switching during collision process. The other soliton (say S_2) experiences an opposite kind of energy switching due to the conservation law $\int_{-\infty}^{\infty} |q_j|^2 \mathrm{d}t = $ constant, $j = 1, 2$.

For the standard elastic collision property ascribed to the scalar solitons to occur here we need the magnitudes of the transition intensities to be unity which is possible for the specific choice $(\alpha_1^{(1)}/\alpha_2^{(1)}) = (\alpha_1^{(2)}/\alpha_2^{(2)})$. The other quantities characterizing this collision process, along with this energy redistribution, are the amplitude-dependent phase-shifts and change in relative separation distances. The corresponding expressions take the same form as that of the Manakov model with the redefinition of κ_{ji} as in (11). We refer to this collision process in which a given soliton experiences the same kind of energy switching in both components as type-II energy sharing collision.

Thus, the type-II energy sharing collision scenario is entirely different from the one observed in the Manakov system where one soliton gets suppressed in one component and is enhanced in the other component with commensurate changes in the other soliton. The collision scenario shown in figure 2 can also be viewed as an amplification process in which the soliton S_1 represents a signal or data carrier, while the soliton S_2 represents an energy reservoir or pump. The main advantage of this amplification process is that it does not require any external amplification medium and therefore the amplification of S_1 does not introduce any noise [16].
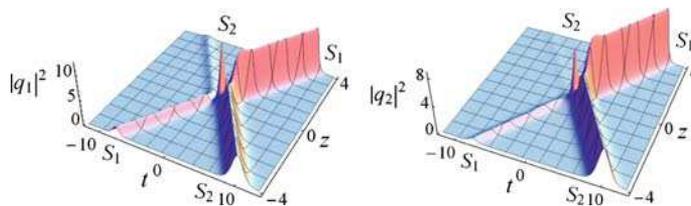
Figure 2. Type-II energy sharing collision of bright solitons in mixed 2-ICNLS system for $k_1 = 1.5 + i$, $k_2 = 1 - i$, $\alpha_1^{(1)} = 1 + i$, $\alpha_1^{(2)} = 0.8 + 0.2i$, $\alpha_2^{(1)} = 1 - i$ and $\alpha_2^{(2)} = 0.5$.

4. Soliton collisions in the m-CCNLS equations: Type-III and type-IV energy sharing collisions

The bright solitons of the m-CCNLS system (5) display interesting collision properties. Unlike in the other multicomponent nonlinear systems, namely Manakov system and mixed-ICNLS system, the m-CCNLS system shows novel energy switching mechanism for a given soliton while the other soliton reappears elastically after collision. The exact two-soliton solution describing different types of collision scenario in m-CCNLS system is given in Appendix B. Importantly, based on the one-soliton solution, we can classify the two-soliton collisions of the m-CCNLS system into three cases: (i) collision between a CCS and an ICS ($\sum_{j=1}^{m}(\alpha_1^{(j)})^2 \neq 0$ and $\sum_{j=1}^{m}(\alpha_2^{(j)})^2 = 0$), (ii) collision between two CCSs ($\sum_{j=1}^{m}(\alpha_u^{(j)})^2 \neq 0$, $u = 1, 2$) and (iii) collision between two ICSs ($\sum_{j=1}^{m}(\alpha_u^{(j)})^2 = 0$, $u = 1, 2$).

In order to understand these collision dynamics more clearly, we have performed an asymptotic analysis of the two-soliton solution given in the Appendix with k_{1R}, $k_{2R} > 0$ and $k_{1I} > k_{2I}$. In the following, we discuss various combinations of soliton collisions of the m-CCNS system (5) for $m \geq 2$. Here and in the following sections, the two colliding solitons are represented as S_1 and S_2.

(i) *Collision between a CCS and ICS: Type-III energy sharing collision*

Let us consider the collision of two bright solitons, in which S_1 is of CCS type while S_2 is an ICS. From the detailed asymptotic analysis, which we skipped here, the amplitude of the given CCS S_1 and ICS S_2 before collision (A_j^{u-}) can be related to that of after collision (A_j^{u+}), by the transition amplitudes ($T_j^{(u)}$) as

$$A_j^{u+} = T_j^{(u)} A_j^{u-}, \quad u = 1, 2, \quad j = 1, 2, 3, ..., m, \tag{12a}$$

where

$$T_j^{(1)} = \left(\frac{(k_1^* + k_2)(k_1 - k_2)\left|(\alpha_1^{(j)}\kappa_{22} - \alpha_2^{(j)}\kappa_{12}) + \alpha_2^{(j)*}\Omega\right|^2}{(k_1 + k_2^*)(k_1^* - k_2^*)\,\kappa_{22}^2\,|\alpha_1^{(j)}|^2} \right)^{1/2}, \tag{12b}$$

$$T_j^{(2)} = \frac{(k_1^* + k_2)(k_1^* - k_2^*)}{(k_1 - k_2)(k_1 + k_2^*)}, \quad j = 1, 2, 3, ..., m, \tag{12c}$$

in which

$$\Omega = \frac{\gamma \sum_{j=1}^{m}(\alpha_1^{(j)}\alpha_2^{(j)})}{(k_1 - k_2)}$$

and

$$\kappa_{uv} = \frac{\gamma}{(k_u + k_v^*)} \sum_{j=1}^{m}(\alpha_u^{(j)}\alpha_v^{(j)*}), \quad u, v = 1, 2.$$

The above-mentioned transition amplitudes ($T_j^{(u)}$) determine the collision nature of a given soliton S_u, $u = 1, 2$, in a particular component q_j, $j = 1, 2, 3, ..., m$.

From eq. (12), one can understand that the solitons undergo elastic collision when their transition intensities become unimodular, which results in the solitons with same intensities before and after collision. Especially, the ICS S_2 exhibits elastic collision always as $|T_j^{(2)}|^2 = 1$ without any restriction on the soliton parameters. However, the CCS S_1 undergoes energy switching collision for general choice of soliton parameters. Only for a specific choice when $\sum_{j=1}^{m}(\alpha_1^{(j)})^2 = 0$, one can expect an elastic collision, but this is not possible as this choice restricts S_1 to be an ICS. Here we observe that in a given component one soliton retains its intensity while the other undergoes change in its intensity after collision, which shows the non-conservation of energy in that component. In order to conserve the total energy of the system, the corresponding soliton undergoes opposite type of intensity switching in another component. Apart from the change or invariance in the intensity/amplitude, the colliding solitons CCS S_1 and ICS S_2 experience phase-shifts

$$\Phi_1 = \frac{1}{k_{1R}} \ln \left(\frac{(k_1 - k_2)(k_1^* - k_2^*)}{(k_1 + k_2^*)(k_1^* + k_2)} \right)$$

and

$$\Phi_2 = -\left(\frac{2k_{1R}}{k_{2R}} \right) \Phi_1,$$

respectively after collision. This will result in a change in the relative separation distance between the solitons before collision ($t_{12}^- = ((\theta_{11} - \epsilon_{11})/2k_{2R}) - (\epsilon_{11}/4k_{1R})$: position of soliton S_2 minus position of soliton S_1 before collision) and after collision ($t_{12}^+ = (R_2/2k_{2R}) - (\theta_{11} - R_2)/4k_{1R}$: position of soliton S_2 minus position of soliton S_1 after collision) and this can be written as

$$\Delta t_{12} = t_{12}^- - t_{12}^+ = \left(1 + \frac{2k_{1R}}{k_{2R}} \right) \Phi_1.$$

Except the transition amplitudes $T_j^{(u)}$, both the phase-shift and the relative separation distance are independent of $\alpha_u^{(j)}$ parameters. Note that the reverse type of energy switching scenario is also possible for CCS S_2 in the two components which can be obtained for a proper choice of $\alpha_u^{(j)}$. We refer to this energy sharing collision with energy switching occurring in CCS only with opposite nature in the two components q_1 and q_2 as type-III energy sharing collision.

To be more clear, we explicitly demonstrate the CCS–ICS collision in 2-CCNLS and 3-CCNLS systems, which can be generalized to m-CCNLS system, with $m > 3$. In figure 3, we have shown the energy switching collision of CCS S_1 with ICS S_2 for $k_1 = 2.3+i$, $k_2 = 2.5-i$, $\gamma = 2$, $\alpha_1^{(1)} = 0.75i$, $\alpha_1^{(2)} = 1.9$, $\alpha_2^{(1)} = 1+i$ and $\alpha_2^{(2)} = 1-i$. Here CCS S_1 changes its profile from a double-hump (single-hump) to a single-hump (double-hump) structure with enhancement (suppression) of intensity in the q_1 (q_2) component, but the ICS S_2 exhibits elastic collision in both components.

It should be noted that the present CCNLS system (5) conserves total energy but the energy in the individual component is not conserved. In order to understand this, we obtain the form of energy conservation from (4), simply for 2-CCNLS case as

$$i \frac{d}{dz} \int_{-\infty}^{\infty} |q_1|^2 \, dt = \gamma \int_{-\infty}^{\infty} (q_1^{*2} q_2^2 - q_1^2 q_2^{*2}) \, dt$$
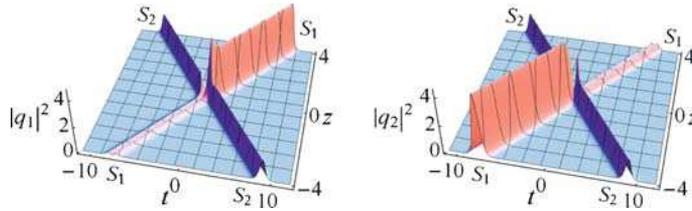
Figure 3. Type-III energy sharing collision of CCS with ICS in 2-CCNLS system.

and

$$i \frac{d}{dz} \int_{-\infty}^{\infty} |q_2|^2 \, dt = \gamma \int_{-\infty}^{\infty} (q_1^2 q_2^{*2} - q_1^{*2} q_2^2) \, dt.$$

This shows that the energy in individual component is not conserved ($\frac{d}{dz} \int_{-\infty}^{\infty} |q_j|^2 \, dt \neq 0$, $j = 1, 2$) but the total energy is conserved ($\frac{d}{dz} \int_{-\infty}^{\infty} (|q_1|^2 + |q_2|^2) dt = 0$). As a consequence, the ICS induces significant energy switching in the CCS with an amplitude-dependent phase-shift and reappears elastically after interaction.

The 3-CCNLS system admits more possible ways of energy switching for CCS. One possible way is depicted in figure 4 for $k_1 = 1.5+i$, $k_2 = 2-i$, $\gamma = 2$, $\alpha_1^{(1)} = 1$, $\alpha_1^{(2)} = 1.5$, $\alpha_1^{(3)} = 2$, $\alpha_2^{(1)} = 2+i$, $\alpha_2^{(2)} = 2-i$ and $\alpha_2^{(3)} = \sqrt{6}\,i$. It is evident from figure 4 that the CCS S_1 changes its profile from single-hump to double-hump with suppression in its intensity in the q_1 and q_3 components. However, in the q_2 component, CCS S_1 just increases its intensity without change in the nature of the profile. Similar to 2-CCNLS system, in 3-CCNLS system too, the ICS S_2 remains the same before and after collision in all the three components. In fact, one can have various combinations of energy switching collision of CCS with ICS for different choices of soliton parameters.
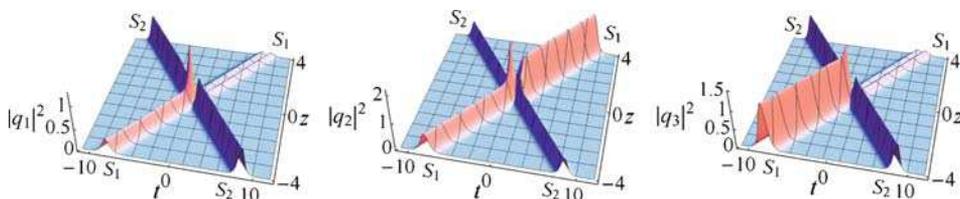
(ii) *Collision of two CCSs*

Here the two CCSs can be obtained for the choice $\sum_{j=1}^{m} (\alpha_u^{(j)})^2 \neq 0$, $u = 1, 2$. Then the relations between the amplitudes of solitons before and after collision are obtained by an asymptotic analysis of two-soliton solution (B.1) as
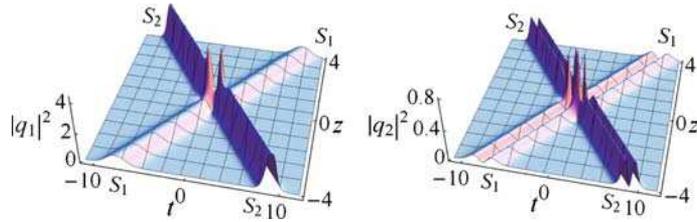
$$A_j^{1+} = \frac{(k_1 - k_2)(k_1^* + k_2)}{(k_1^* - k_2^*)(k_1 + k_2^*)} A_j^{1-}$$

and

$$A_j^{2+} = \frac{(k_1^* - k_2^*)(k_1^* + k_2)}{(k_1 - k_2)(k_1 + k_2^*)} A_j^{2-}, \quad j = 1, 2, 3, ..., m.$$

Figure 4. Type-III energy sharing collision of CCS with ICS in 3-CCNLS system.

Figure 5. Elastic collision of coherently coupled solitons in 2-CCNLS system.

From these expressions, we can easily find that the CCSs always undergo elastic collision with different profile structures as the corresponding relations for intensities become $|A_j^{u+}|^2 = |A_j^{u-}|^2$, $u = 1, 2$, $j = 1, 2, 3, ..., m$. But these solitons, CCS S_1 and CCS S_2, exhibit phase-shifts after collision

$$\Phi_1 = \frac{1}{k_{1R}} \ln \left(\frac{(k_1 - k_2)(k_1^* - k_2^*)}{(k_1 + k_2^*)(k_1^* + k_2)} \right)$$

and

$$\Phi_2 = - \left(\frac{k_{1R}}{k_{2R}} \right) \Phi_1,$$

respectively with a change in the relative separation distance ($\Delta t_{12} = (1 + (k_{1R}/k_{2R}))\Phi_1$) between the two CCSs.

For illustrative purpose, we have shown the collision between two CCSs in the 2-CCNLS and 3-CCNLS systems, respectively in figures 5 and 6 for $k_1 = 1.5 + i$, $k_2 = 2 - i$, $\gamma = 2$, $\alpha_1^{(1)} = 1.7i$, $\alpha_1^{(2)} = 1$, $\alpha_2^{(1)} = 2i$, $\alpha_2^{(2)} = 1.2$ and $k_1 = 1.5 + i$, $k_2 = 2 - i$, $\gamma = 2$, $\alpha_1^{(1)} = 0.25$, $\alpha_1^{(2)} = -0.71$, $\alpha_1^{(3)} = 1.2i$, $\alpha_2^{(1)} = 1$, $\alpha_2^{(2)} = 1.4i$, $\alpha_2^{(3)} = 0.75i$. In figure 5, two CCSs having single-hump (double-hump) profiles in q_1 (q_2) component undergo elastic collision. In figure 6, the collision takes place between two double-hump CCSs in q_1, a single-hump and double-hump CCSs in q_2 and two single-hump CCSs in q_3 components. Other combinations of soliton profiles for the elastic collision of CCSs can also be achieved by tuning the $\alpha_u^{(j)}$ parameters.

(iii) *Collision of two ICSs: Type-IV energy sharing collision*

Based on the choice for the ICS in the one-soliton solution, here we obtain two ICSs for the choice $\sum_{j=1}^m (\alpha_u^{(j)})^2 = 0$, $u = 1, 2$. When two such ICSs collide with each other we get exciting energy sharing collision in the m-CCNLS system (5). From the asymptotic
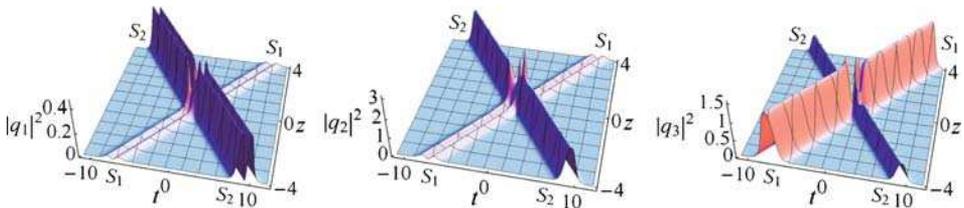
Figure 6. Elastic collision of two coherently coupled solitons in 3-CCNLS system.

analysis of (B.1), the relation between the amplitudes of ICSs before and after collision is obtained as

$$A_j^{u+} = T_j^{(u)} A_j^{u-}, \quad u = 1, 2, \quad j = 1, 2, 3, ..., m. \tag{13a}$$

Transition amplitude $T_j^{(u)}$ of soliton S_u, $u = 1, 2$, appearing in the above equation can be written as

$$
T_j^{(1)} = \frac{\left(1 - \hat{\lambda}_1 + \frac{\alpha_2^{(j)*}\Omega}{\alpha_1^{(j)} \kappa_{22}}\right)}{\sqrt{1 - \hat{\lambda}_1 \hat{\lambda}_2 + \frac{|\Omega|^2}{\kappa_{11}\kappa_{22}}}}
$$
$$
\times \left(\frac{(k_1 - k_2)(k_1^* + k_2)}{(k_1^* - k_2^*)(k_1 + k_2^*)}\right)^{1/2}, \quad j = 1, 2, 3, ..., m, \tag{13b}
$$

$$
T_j^{(2)} = -\frac{\sqrt{1 - \hat{\lambda}_1 \hat{\lambda}_2 + \frac{|\Omega|^2}{\kappa_{11}\kappa_{22}}}}{\left(1 - \hat{\lambda}_2 + \frac{\alpha_1^{(j)*}\Omega}{\alpha_2^{(j)} \kappa_{11}}\right)}
$$
$$
\times \left(\frac{(k_1^* - k_2^*)(k_1^* + k_2)}{(k_1 - k_2)(k_1 + k_2^*)}\right)^{1/2}, \quad j = 1, 2, 3, ..., m, \tag{13c}
$$

where

$$\Omega = \frac{\gamma}{(k_1 - k_2)} \sum_{l=1}^{m}(\alpha_1^{(l)}\alpha_2^{(l)}),$$

$$\kappa_{uv} = \frac{\gamma}{(k_u + k_v^*)} \sum_{j=1}^{m}(\alpha_u^{(j)}\alpha_v^{(j)*}), \quad u, v = 1, 2,$$

$$\hat{\lambda}_1 = \frac{\alpha_2^{(j)}\kappa_{12}}{\alpha_1^{(j)}\kappa_{22}} \quad \text{and} \quad \hat{\lambda}_2 = \frac{\alpha_1^{(j)}\kappa_{21}}{\alpha_2^{(j)}\kappa_{11}}.$$

In general, the ICSs undergo energy sharing collision which involves a change in their amplitudes after collision ($|T_j^{(u)}|^2 \neq 1$, $u = 1, 2$, $j = 1, 2, 3, ..., m$). However, for special choice of $\alpha_1^{(1)}$ parameters ($\alpha_1^{(1)}/\alpha_2^{(1)} = \alpha_1^{(2)}/\alpha_2^{(2)} = \alpha_1^{(3)}/\alpha_2^{(3)} = \cdots = \alpha_1^{(m)}/\alpha_2^{(m)}$), we can also get elastic collisions as this choice will lead to $|T_j^{(1)}|^2 = |T_j^{(2)}|^2 = 1$, $j = 1, 2, 3, ..., m$. Also, the colliding ICSs S_u, $u = 1, 2$, exhibit phase-shift

$$\Phi_1 = \frac{1}{2k_{1R}} \ln\left[\frac{|k_1 - k_2|^2}{|k_1 + k_2^*|^2}\left(1 - \hat{\lambda}_1 \hat{\lambda}_2 + \frac{|\Omega|^2}{\kappa_{11}\kappa_{22}}\right)\right]$$

and

$$\Phi_2 = -\frac{k_{1R}}{k_{2R}}\Phi_1,$$

respectively which lead to a change in the relative separation distance between the two ICSs, $\Delta t_{12} = (1 + (k_{1R}/k_{2R}))\Phi_1$.
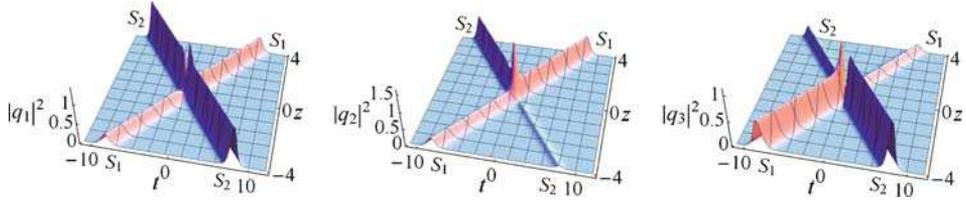
Figure 7. Novel type-IV energy sharing collision of two ICSs in 3-CCNLS system.

To facilitate the understanding of ICS–ICS collision, first we discuss their collision in the 2-CCNLS system. Here we find that the ICSs always undergo elastic collision. It can be verified that for the 2-CCNLS system the transition amplitudes (intensities) given by eq. (13) are always unimodular. However, in the 3-CCNLS system, the ICSs show energy sharing collision in which the energy is not conserved in each q_j component but the total energy among all the components is conserved. We have shown such energy sharing collision of two ICSs in figure 7 for $k_1 = 1.5 + i$, $k_2 = 2 - i$, $\gamma = 2$, $\alpha_1^{(1)} = \sqrt{2}$, $\alpha_1^{(2)} = \sqrt{2}$, $\alpha_1^{(3)} = 2i$, $\alpha_2^{(1)} = \sqrt{8}\,i$, $\alpha_2^{(2)} = \sqrt{6}$ and $\alpha_2^{(3)} = \sqrt{2}$. It is interesting to note that only CCS undergoes intensity change during its collision with ICS (i.e. CCS–ICS collision) and ICS remains unaltered. But in collision between two ICSs, the intensity gets altered in both solitons. In figure 7, the two colliding solitons always undergo an enhancement in their intensities in q_1 and q_2 components while they experience suppression in the q_3 component. This type of collision between two ICSs takes place only in the m-CCNLS system with $m \geq 3$ and one cannot expect such energy sharing collision in the 2-CCNLS system, which always exhibits elastic collision. We refer to this collision scenario in which the solitons undergo the same kind of switching in a given component with commensurate changes in the other components as type-IV energy sharing collision. In addition to this type-IV energy sharing collision, the elastic collision of two ICSs can also result for the choice

$$\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}} = \frac{\alpha_1^{(3)}}{\alpha_2^{(3)}}.$$

5. Conclusion

We have investigated various types of soliton collisions in multicomponent nonlinear Schrödinger-type systems with different nonlinearities. We have revealed type-I and type-II energy sharing collisions in the two-component Manakov and mixed-ICNLS systems, respectively, where the solitons undergo opposite and same type of energy sharing nature among the two components. Such energy sharing collisions can also be observed in their multicomponent counterparts with $m \geq 3$. Then we have discussed the interesting energy switching collision scenario in coherently coupled nonlinear Schrödinger system which admits various soliton profiles like single-hump, double-hump and flat-top structures. This energy switching collision does not conserve the energy in a given component, instead the total energy of all the components is conserved. Particularly, we have demonstrated that during its collision with the ICS, the CCS undergoes energy switching leaving

the ICS unaltered. We have referred this collision as type-III energy sharing collision. Also, another interesting energy sharing collision has been identified in the m-CCNLS system with $m \geq 3$. Additionally, elastic collision processes of different bright solitons are observed for special choices of soliton parameters. The reported four types of energy sharing collisions of bright solitons will find applications in the context of soliton collision-based optical computing, optical switching devices, etc.

Acknowledgements

Appendix A. Bright one-soliton solution of the m-CCNLS system (5)

The bright one-soliton solution of the m-CCNLS eq. (5) obtained by using the non-standard approach of Hirota's bilinearization method [23] can be written as

$$q_j = \frac{\alpha_1^{(j)} e^{\eta_1} + e^{2\eta_1 + \eta_1^* + \delta_{11}^{(j)}}}{1 + e^{\eta_1 + \eta_1^* + R_1} + e^{2\eta_1 + 2\eta_1^* + \epsilon_{11}}}, \quad j = 1, 2, 3, ..., m, \tag{A.1}$$

where

$$e^{\delta_{11}^{(j)}} = \frac{\gamma \alpha_1^{(j)*} \Gamma_1}{2(k_1 + k_1^*)^2}, \quad e^{R_1} = \frac{\kappa_{11}}{(k_1 + k_1^*)}, \quad e^{\epsilon_{11}} = \frac{\gamma^2 |\Gamma_1|^2}{4(k_1 + k_1^*)^4},$$

in which

$$\Gamma_1 = \sum_{j=1}^m (\alpha_1^{(j)})^2 \quad \text{and} \quad \kappa_{11} = \frac{\gamma \sum_{j=1}^m |\alpha_1^{(j)}|^2}{(k_1 + k_1^*)}.$$

Here the auxiliary function s is obtained as $s = \sum_{j=1}^m (\alpha_1^{(j)})^2 e^{2\eta_1}$.

Appendix B. Bright two-soliton solution of the m-CCNLS system (5)

The bright two-soliton solution of system (5) can be written as [23]

$$q_j = \frac{g^{(j)}}{f}, \quad j = 1, 2, 3, ..., m, \tag{B.1}$$

where

$$g^{(j)} = \sum(\alpha_u^{(j)} e^{\eta_u}) + \sum(e^{2\eta_u + \eta_v^* + \delta_{uv}^{(j)}}) + \sum(e^{\eta_1 + \eta_2 + \eta_u^* + \delta_u^{(j)}})$$
$$+ \sum(e^{2\eta_u + 2\eta_v^* + \eta_{3-u} + \mu_{uv}^{(j)}}) + e^{\eta_1 + \eta_1^* + \eta_2 + \eta_2^*}$$
$$\times \left(\sum e^{\eta_u + \mu_u^{(j)}} + \sum e^{\eta_1 + \eta_2 + \eta_u^* + \phi_u^{(j)}} \right), \quad j = 1, 2, 3, ..., m,$$

$$f = 1 + \sum(e^{\eta_u + \eta_u^* + R_u}) + e^{\eta_1 + \eta_2^* + \delta_0} + e^{\eta_2 + \eta_1^* + \delta_0^*}$$
$$+ \sum(e^{2\eta_u + 2\eta_v^* + \epsilon_{uv}}) + e^{\eta_1^* + \eta_2^*} \sum(e^{2\eta_u + \tau_u}) + e^{\eta_1 + \eta_2} \sum(e^{2\eta_u^* + \tau_u^*})$$
$$+ e^{\eta_1 + \eta_1^* + \eta_2 + \eta_2^*} \left(e^{R_3} + \sum e^{\eta_u + \eta_v^* + \theta_{uv}} + e^{\eta_1 + \eta_1^* + \eta_2 + \eta_2^* + R_4} \right),$$

and the auxiliary function s takes the form

$$s = \sum(\Gamma_u e^{2\eta_u}) + \Gamma_3 e^{\eta_1 + \eta_2} + \sum(e^{\eta_u + 2\eta_{3-u} + \eta_v^* + \lambda_{uv}})$$
$$+ e^{2\eta_1 + 2\eta_2} \left(\sum e^{2\eta_u^* + \lambda_u} + e^{\eta_1^* + \eta_2^* + \lambda_3} \right).$$

Here $\eta_u = k_u(t + ik_u z)$, $u = 1, 2$, the summation is taken over u and v for $u, v = 1, 2$, and the expressions for various other quantities are given below.

$$e^{R_u} = \frac{\kappa_{uu}}{(k_u + k_u^*)}, \quad e^{\delta_0} = \frac{\kappa_{12}}{(k_1 + k_2^*)}, \quad e^{\delta_0^*} = \frac{\kappa_{21}}{(k_2 + k_1^*)}, \quad e^{\delta_{uv}^{(j)}} = \frac{\gamma \alpha_v^{(j)*} \Gamma_u}{2(k_u + k_v^*)^2},$$

$$e^{\delta_u^{(j)}} = \frac{\gamma \alpha_u^{(j)*} \Gamma_3 + (k_1 - k_2)(\alpha_1^{(j)} \kappa_{2u} - \alpha_2^{(j)} \kappa_{1u})}{(k_1 + k_u^*)(k_2 + k_u^*)},$$

$$e^{\epsilon_{uv}} = \frac{\gamma^2 \Gamma_u \Gamma_v^*}{4(k_u + k_v^*)^4}, \quad e^{\tau_u} = \frac{\gamma^2 \Gamma_3^* \Gamma_u}{2(k_u + k_1^*)^2 (k_u + k_2^*)^2},$$

$$e^{\lambda_{uv}} = \frac{(k_1 - k_2)^2 \kappa_{uv} \Gamma_{3-u}}{(k_u + k_v^*)(k_{3-u} + k_v^*)^2}, \quad e^{\mu_{uv}^{(j)}} = \frac{\gamma^2 (k_1 - k_2)^2 \alpha_{3-u}^{(j)} \Gamma_u \Gamma_v^*}{4(k_u + k_v^*)^4 (k_{3-u} + k_v^*)^2},$$

$$e^{\theta_{uv}} = \frac{\gamma^2 |k_1 - k_2|^4}{4\tilde{D}(k_u + k_v^*)^2} \Gamma_u \Gamma_v^* \kappa_{3-u \ 3-v}, \quad e^{\lambda_u} = \frac{\gamma^2 (k_1 - k_2)^4 \Gamma_1 \Gamma_2 \Gamma_u^*}{4(k_1 + k_u^*)^4 (k_2 + k_u^*)^4},$$

$$e^{\lambda_3} = \frac{\gamma^2 (k_1 - k_2)^4}{2\tilde{D}} \Gamma_1 \Gamma_2 \Gamma_3^*, \quad e^{\phi_u^{(j)}} = \frac{\gamma^3 (k_1 - k_2)^4 (k_1^* - k_2^*)^2}{8\tilde{D}(k_1 + k_u^*)^2 (k_2 + k_u^*)^2} \alpha_{3-u}^{(j)*} \Gamma_1 \Gamma_2 \Gamma_u^*,$$

$$e^{R_3} = \frac{|k_1 - k_2|^2 (\kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21}) + \gamma^2 |\Gamma_3|^2}{(k_1 + k_1^*)|k_1 + k_2^*|^2 (k_2 + k_2^*)}, \quad e^{R_4} = \frac{\gamma^4 |k_1 - k_2|^8}{16\tilde{D}^2} \Gamma_1 \Gamma_2 \Gamma_1^* \Gamma_2^*,$$

$$e^{\mu_u^{(j)}} = \frac{(k_1 - k_2)^2 \gamma}{2\tilde{D}} \Gamma_u (k_{3-u} + k_1^*)(k_{3-u} + k_2^*)[\gamma \alpha_{3-u}^{(j)} \Gamma_3^*$$
$$+ (k_1^* - k_2^*)(\alpha_1^{(j)*} \kappa_{3-u2} - \alpha_2^{(j)*} \kappa_{3-u1})],$$

where

$$\tilde{D} = (k_1 + k_1^*)^2 (k_1^* + k_2)^2 (k_1 + k_2^*)^2 (k_2 + k_2^*)^2,$$

$$\kappa_{uv} = \frac{\gamma}{(k_u + k_v^*)} \sum_{j=1}^{m} (\alpha_u^{(j)} \alpha_v^{(j)*}),$$

$$\Gamma_1 = \sum_{j=1}^{m} (\alpha_1^{(j)})^2, \quad \Gamma_2 = \sum_{j=1}^{m} (\alpha_2^{(j)})^2, \quad \Gamma_3 = \sum_{j=1}^{m} (\alpha_1^{(j)} \alpha_2^{(j)})$$

and $u, v = 1, 2$, $j = 1, 2, 3, ..., m$.

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