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Regge-like initial input and evolution of non-singlet structure functions from DGLAP equation up to next-next-to-leading order at low x and low Q^2

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Abstract. This is an attempt to study how the features of Regge theory, along with QCD predictions, lead towards the understanding of unpolarized non-singlet structure functions $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ at low x and low Q^2. Combining the features of perturbative quantum chromodynamics (pQCD) and Regge theory, an ansatz for $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ structure functions at small x was obtained, which when used as the initial input to Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation, gives the Q^2 evolution of the non-singlet structure functions. The non-singlet structure functions, evolved in accordance with DGLAP evolution equations up to next-next-to-leading order are studied phenomenologically in comparison with the available experimental and parametrization results taken from NMC, CCFR, NuTeV, CORUS, CDHSW, NNPDF and MSTW Collaborations and a very good agreement is observed in this regard.

Keywords. Quantum chromodynamics; Regge theory; non-singlet structure function.

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1. Introduction

Deep inelastic scattering (DIS) structure functions are the objects of intensive investigation both theoretically and experimentally in order to understand the underlying theory of strong interaction. Significant progresses have been observed in the field of experimental investigation of structure functions (see the recent reviews [1–6]). Recent developments of dedicated experimental facilities allows one to measure the structure functions with far greater precision than before. On the other hand, there are several theoretical approaches such as Regge theory, quantum chromodynamics (QCD) etc., for describing the strong interaction processes observed at high-energy particle colliders. But, standard and the most widespread for the theoretical investigation of DIS structure functions is the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations [7–10], developed within perturbative QCD (PQCD) formalism. Although, in PQCD, the structure functions are predicted in accordance with DGLAP equations, predictive power of DGLAP evolution equations is limited. They cannot predict the initial values from which the evolution starts. There are several numerical as well as analytical methods [11–19] to solve DGLAP equations, but usually structure functions from DGLAP evolution equations are determined by considering an initial distribution of the structure function at a fixed Q^2, which is characterized by some free parameters and the parameters are determined by fitting the parametrization with the available experimental data. On the other hand, this requirement of initial distribution at fixed Q^2 can be obtained from Regge theory and there are many experiments [1,19–29] which use the Regge theory for predicting the initial input of DGLAP evolution equation to obtain the structure functions. Although many parametrizations are available in previous works to predict the initial distribution of structure functions, most of them are with several parameters. Again, a parametrization with a large number of parameters creates difficulties in obtaining best fitting, which in turn leads towards the inaccuracy in results. Thus, explorations of the possibility of obtaining accurate solutions of DGLAP evolution equations with less number of parameters are always interesting. Under these motivations, here we have investigated the usefulness of a simple Regge-inspired model as the initial input to DGLAP evolution equation to determine the structure functions. This investigation is performed by means of unpolarized non-singlet structure functions $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$, which are considered as the starting ground for theoretical description of DIS structure functions. Besides being interesting in themselves, another significant advantage is that QCD analysis by means of non-singlet structure functions is comparatively technically simpler. Again, our investigation is restricted within the kinematical region for $x \leq 0.045$ and $Q^2 \leq 20\,\mathrm{GeV}^2$.

The paper is organized as follows. Section 2 introduces models for $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ structure functions by combining features of Regge theory and pQCD. In §3, DGLAP evolution equations in leading order (LO), next-to-leading order (NLO) and next-next-to-leading order (NNLO) are solved for $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ structure functions and phenomenological analysis is performed in comparison with different results taken from NMC [29], CCFR [30], NuTeV [31], CHORUS [32], CDHSW [33], NNPDF [34] and MSTW [35] Collaborations and finally, in §4, the paper is concluded with a brief discussion.

2. Regge-inspired models for non-singlet structure functions

In Regge theory, the x dependency of the non-singlet structure functions $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ are described with the power laws [1,23,36,37],

$$F_2^{\mathrm{NS}}(x) = B_2^{\mathrm{NS}} x^{1-\alpha_{A_2}(0)} \tag{1}$$

and

$$xF_3(x) = B_3^{\mathrm{NS}} x^{1-\alpha_{A_2}(0)}, \tag{2}$$

respectively. Here $\alpha_{A_2}(0)$ is the intercept of the A_2 Regge trajectory. For $\alpha_{A_2}(0) \approx 0.5$ this behaviour is stable against the LO QCD evolution. Thus the stable small x behaviour of the unpolarized non-singlet structure functions can be expressed, with respect to Regge theory as

$$F_2^{\mathrm{NS}}(x) = B_2^{\mathrm{NS}} x^{0.5} \tag{3}$$

and

$$xF_3(x) = B_3^{\mathrm{NS}} x^{0.5}. \tag{4}$$

Besides being x-dependent, the structure functions, in accordance with QCD predictions, are also dependent on Q^2. The Bjorken scaling violation or the Q^2 dependence of the structure functions is one of the significant predictions of QCD and recent experiments also reveal the evidence of Q^2 dependency of the structure functions even at small x. Thus, in compliance with Regge theory, all the dependence on Q^2 is expected to be in $B^{\mathrm{NS}}_{(i=2,3)}(Q^2)$. Hence incorporating the Q^2 behaviour to the structure functions, eqs (3) and (4) in terms of the function $B^{\mathrm{NS}}_{(i=2,3)}(Q^2)$, gives the QCD-modified Regge-like model for both x as well as Q^2-dependent non-singlet structure functions at small x as

$$F_2^{\mathrm{NS}}(x, Q^2) = B_2^{\mathrm{NS}}(Q^2) x^{0.5} \tag{5}$$

and

$$xF_3(x, Q^2) = B_3^{\mathrm{NS}}(Q^2) x^{0.5}. \tag{6}$$

The above QCD-inspired Regge-like form of the structure functions can be used as the initial input for solving DGLAP evolution equations in order to obtain the Q^2 behaviour of $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ structure functions.

3. Solutions of DGLAP evolution equations up to NNLO

The DGLAP evolution equations which describe the Q^2 behaviour of unpolarized non-singlet structure functions $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ are given by

$$\frac{\partial F_i(x, Q^2)}{\partial \ln Q^2} = \int_0^1 \frac{\mathrm{d}\omega}{\omega} F_i\left(\frac{x}{\omega}, Q^2\right) P_i(\omega). \tag{7}$$

Here $F_i(x, Q^2)$ denotes both the structure functions and $P_i(\omega)$ are the respective splitting functions associated with the structure functions. However, the splitting functions for both the unpolarized non-singlet structure functions $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ are same which are defined up to NNLO by [38,39]

$$P_i(\omega) = \frac{\alpha(Q^2)}{2\pi} P_i^0(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^2 P_i^1(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P_i^2(\omega), \tag{8}$$

where $P_i^0(\omega)$, $P_i^1(\omega)$ and $P_i^2(\omega)$ are the corresponding LO, NLO and NNLO corrections to the splitting functions.

In LO, NLO and NNLO, the running coupling constant $\alpha(t)/2\pi$ has the forms [40],

$$\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} = \frac{2}{\beta_0 t}, \tag{9}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NLO}} = \frac{2}{\beta_0 t}\left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right] \tag{10}$$

and

$$\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NNLO}} = \frac{2}{\beta_0 t}\left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2}\left[\left(\frac{\beta_1}{\beta_0}\right)^2 \times (\ln^2 t - \ln t - 1) + \frac{\beta_2}{\beta_0}\right]\right], \tag{11}$$

where $\beta_0 = 11 - \frac{2}{3}N_{\mathrm{F}}$, $\beta_1 = 102 - \frac{38}{3}N_{\mathrm{F}}$ and $\beta_2 = \frac{2857}{18} - \frac{6673}{18}N_{\mathrm{F}} + \frac{325}{54}N_{\mathrm{F}}^2$ are the one-, two- and three-loop corrections to the QCD β-function.

Substituting the respective splitting functions along with the corresponding running coupling constant in (7), the DGLAP evolution equations in LO, NLO and NNLO become

$$\frac{\partial F_i(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}}\left[\frac{2}{3}\{3 + 4\ln(1-x)\}F_i(x,t) + I_1(x,t)\right], \tag{12}$$

$$\frac{\partial F_i(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NLO}}\left[\frac{2}{3}\{3 + 4\ln(1-x)\}F_i(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NLO}}^2 I_2(x,t) \tag{13}$$

and

$$\frac{\partial F_i(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NNLO}}\left[\frac{2}{3}\{3 + 4\ln(1-x)\}F_i(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NNLO}}^2 I_2(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NNLO}}^3 I_3(x,t), \tag{14}$$

respectively, in terms of the variable $t = \ln(Q^2/\Lambda^2)$ instead of Q^2, where Λ is the QCD cut-off parameter. The integral functions are given by

$$I_1(x,t) = \int_x^1 \frac{\mathrm{d}\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}F\left(\frac{x}{\omega}, t\right) - 2F_i(x,t)\right\}, \tag{15}$$

$$I_2(x,t) = \int_x^1 \frac{\mathrm{d}\omega}{\omega} P^2(\omega) F_i\left(\frac{x}{\omega}, t\right) \tag{16}$$

and

$$I_3(x,t) = \int_x^1 \frac{\mathrm{d}\omega}{\omega} P^2(\omega) F_i\left(\frac{x}{\omega}, t\right). \tag{17}$$

One can now solve the DGLAP equations by considering $F_i^{\mathrm{NS}}(x,t) = A_i^{\mathrm{NS}} x^{0.5}$ as the initial input. First, the solution of DGLAP equations in LO is discussed explicitly and then extended the same formalism to obtain the solutions in NLO and NNLO.

The DGLAP evolution equations (12) which describe the Q^2 behaviour of $F_2^{\mathrm{NS}}(x, Q^2)$ and $F_3^{\mathrm{NS}} = xF_3(x, Q^2)$ structure functions in LO are given by [24,25]

$$\frac{\partial F_i^{\mathrm{NS}}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \left[\frac{2}{3}\{3 + 4\ln(1-x)\} F_i^{\mathrm{NS}}(x,t) + \frac{4}{3}\int_x^1 \frac{\mathrm{d}\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega} F_i^{\mathrm{NS}}\left(\frac{x}{\omega}, t\right) - 2F_i^{\mathrm{NS}}(x,t)\right\}\right]. \tag{18}$$

Here the equations are expressed in terms of the variable $t = \ln(Q^2/\Lambda^2)$ instead of Q^2, where Λ is the QCD cut-off parameter.

Now, substituting

$$F_i^{\mathrm{NS}}(x,t) = B_i^{\mathrm{NS}}(t) x^{0.5} \tag{19}$$

and

$$F_i^{\mathrm{NS}}\left(\frac{x}{\omega}, Q^2\right) = B_i^{\mathrm{NS}}(t)\left(\frac{x}{\omega}\right)^{0.5} = F_i^{\mathrm{NS}}(x,t)\omega^{-0.5} \tag{20}$$

in eq. (18) we get

$$x^{0.5}\frac{\mathrm{d}B_i^{\mathrm{NS}}(t)}{\mathrm{d}t} = \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \left[\frac{2}{3}\{3 + 4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{\mathrm{d}\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}\right] B_i^{\mathrm{NS}}(t) x^{0.5} \tag{21}$$

or

$$\frac{\mathrm{d}B_i^{\mathrm{NS}}(t)}{B_i^{\mathrm{NS}}(t)} = \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \left[\frac{2}{3}\{3 + 4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{\mathrm{d}\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}\right] \mathrm{d}t \tag{22}$$

which can be solved to have

$$B_i^{\mathrm{NS}}(t) = C \exp\left[P(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \mathrm{d}t\right]. \tag{23}$$

Here

$$P(x) = \frac{2}{3}\{3 + 4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{\mathrm{d}\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\} \tag{24}$$

and C is a constant originated due to integration. Equation (23) gives the Q^2-dependent part of the structure functions, which is obtained by solving DGLAP equation analytically. Substituting the expression for $B_i^{\mathrm{NS}}(t)$ in eqs (5) and (6) we get

$$F_i^{\mathrm{NS}}(x,t) = C \exp\left[P(x)\int\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \mathrm{d}t\right] x^{0.5}. \tag{25}$$

Now, defining an input point

$$F_i^{\mathrm{NS}}(x,t_0) = C \exp\left[P(x)\int\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \mathrm{d}t\right] x^{0.5}\Bigg|_{t=t_0}, \tag{26}$$

at any $t = t_0$ and dividing eq. (25) by eq. (26) and rearranging a bit we obtain the t evolution of $F_i^{\mathrm{NS}}(x,t)$ with respect to the input point $F_i^{\mathrm{NS}}(x,t_0)$ as

$$F_i^{\mathrm{NS}}(x,t) = F_i^{\mathrm{NS}}(x,t_0) \exp\left[P(x)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{LO}} \mathrm{d}t\right] x^{0.5}. \tag{27}$$

Proceeding in a similar way, the DGLAP eqs (13) and (14), in NLO and NNLO can be solved to have the solutions representing t evolution of $F_i^{\mathrm{NS}}(x,t)$ structure functions as

$$\begin{aligned} F_i^{\mathrm{NS}}(x,t) = {} & F_i^{\mathrm{NS}}(x,t_0) \exp\left[P(x)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NLO}} \mathrm{d}t\right. \\ & \left. + Q(x)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{\mathrm{NLO}} \mathrm{d}t\right] \end{aligned} \tag{28}$$

and

$$\begin{aligned} F_i^{\mathrm{NS}}(x,t) = {} & F_i^{\mathrm{NS}}(x,t_0) \exp\left[P(x)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{\mathrm{NNLO}} \mathrm{d}t\right. \\ & \left. + Q(x)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{\mathrm{NNLO}} \mathrm{d}t + R(x)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^3_{\mathrm{NNLO}} \mathrm{d}t\right], \end{aligned} \tag{29}$$

respectively. Here,

$$Q(x) = \int_x^1 \frac{\mathrm{d}\omega}{\omega} P^2(\omega)\omega^{-0.5} \quad \text{and} \quad R(x) = \int_x^1 \frac{\mathrm{d}\omega}{\omega} P^2(\omega)\omega^{-0.5}.$$

Substituting the expressions for running coupling constant in LO, NLO and NNLO, respectively and then performing the corresponding integrations, we shall obtain the Q^2 evolution of structure functions from eqs (27), (28) and (29).

The non-singlet structure functions $F_2^{\mathrm{NS}}(x, Q^2)$ and $xF_3(x, Q^2)$ evolved in accordance with (27)–(29) are depicted in figures 1, 2 and 3 in comparison with the experimental and parametrization results taken from NMC, NNPDF, CCFR, NuTeV, CHORUS, CDHSW and MSTW Collaborations. As NuTeV, CHORUS, CDHSW and MSTW data for $xF_3(x, Q^2)$ have similar x bins (within the range $x \leq 0.045$ of consideration), they are combined in figure 3, and the CCFR data at different x, are plotted separately in figure 2. In order to obtain the evolution of structure functions, a suitable input point

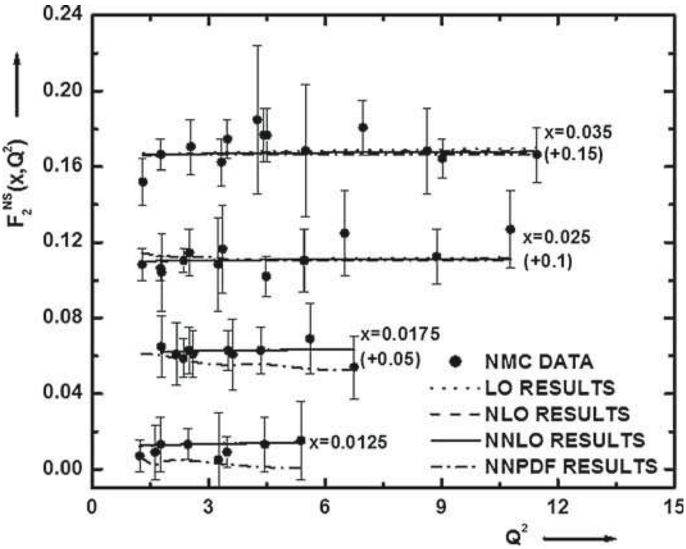

Figure 1. Q^2 evolution of $F_2^{\mathrm{NS}}(x, Q^2)$ structure functions in accordance with eqs (27)–(29) in comparison with NMC [29] and NNPDF [34] results. For clarity, the points are offset by the amount given in parenthesis.

$F_i^{\mathrm{NS}}(x, t_0)$ is required. The input point can be considered from the experimental data. If the input point is more accurate and precise, better results can be expected. Therefore, a point with minimum experimental errors is considered as input point at each value of x

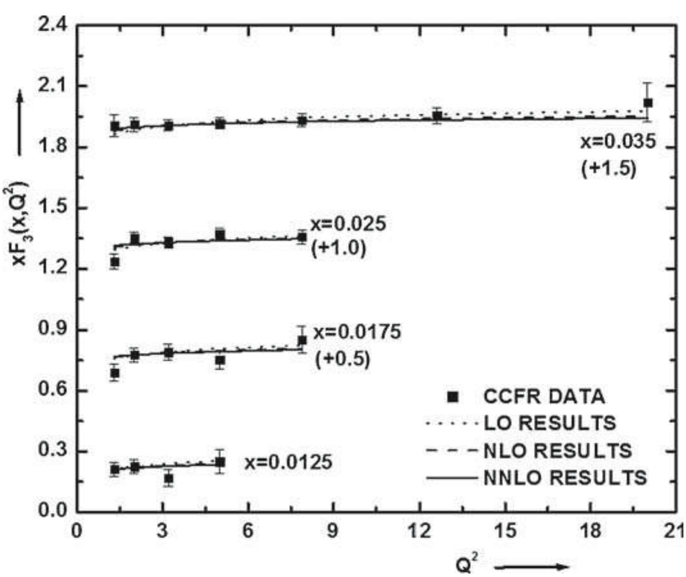

Figure 2. Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accordance with eqs (27)–(29) in comparison with CCFR [30] results. For clarity, the points are offset by the amount given in parenthesis.

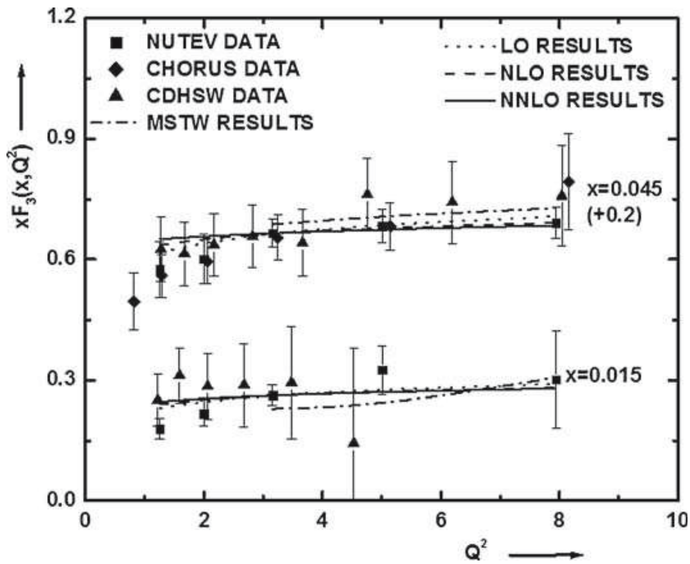

Figure 3. Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accordance with eqs (27)–(29) in comparison with NuTeV [31], CHORUS [32], CDHSW [33] and MSTW [35] results. For clarity, the points are offset by the amount given in parenthesis.

to evolve the structure functions with respect to that point against Q^2. The figures show a very good agreement between the theoretical and experimental results.

4. Conclusion

In this paper, an attempt was made to investigate the efficiency of a Regge-inspired ansatz as the initial input to DGLAP equations in evolving non-singlet structure functions at small x. The investigation was performed by solving DGLAP equations up to NNLO, applying the ansatz as the initial input, and performing phenomenological analysis of the solutions in comparison with the experimental and parametrization results taken from NMC, CCFR, NuTeV, CHORUS, CDHSW, NNPDF and MSTW Collaborations. A very good agreement between the theoretical and experimental results within the considered kinematical range $x \leq 0.045$ and $Q^2 \leq 20.0\,\text{GeV}^2$ was observed. The initial parametrization did not require any arbitrary parameter to be fitted and the unknown Q^2-dependent functions $B_{i=2,3}^{NS}(t)$ were determined analytically. Therefore, obtaining inaccurate results was unfeasible. Thus, in accordance with the phenomenological success achieved in this study, it is concluded that the simple QCD-featured Regge-behaved ansatz is capable of evolving non-singlet structure functions with Q^2 in accordance with DGLAP equation at small x.

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