PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 85, No. 4
October 2015
pp. 593–603

The first integral method to study the (2+1)-dimensional Jaulent–Miodek equations

M MATINFAR, M ESLAMI* and S ROSHANDEL

Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran

*Corresponding author. E-mail: mostafa.eslami@umz.ac.ir

MS received 10 April 2014; revised 29 May 2014; accepted 21 August 2014

DOI: 10.1007/s12043-014-0916-1; ***e*Publication:** 9 July 2015

Abstract. In this paper, we have presented the applicability of the first integral method for constructing exact solutions of (2+1)-dimensional Jaulent–Miodek equations. The first integral method is a powerful and effective method for solving nonlinear partial differential equations which can be applied to nonintegrable as well as integrable equations. The present paper confirms the significant features of the method employed and exact kink and soliton solutions are constructed through the established first integrals.

Keywords. (2+1)-dimensional Jaulent–Miodek equation; the first integral method; kinks; solitons.

PACS Nos 02.30.Jr; 05.45.Yv

1. Introduction

In nonlinear sciences, it is well known that the nonlinear partial differential equations (NLPDEs) are widely used to describe complex phenomena in various fields of sciences particularly, in physics. So the study of exact solutions of these equations are very important. Nonlinear evolution equations involve more than one of dispersion, dissipation, diffusion, reaction and convection. The investigation of the exact solutions for nonlinear evolution equations plays an important role in nonlinear physical science because these solutions may well describe various natural phenomena, such as vibrations, solitons and propagation with a finite speed. Solitons are generated by the balance between nonlinearity and linear dispersion. However, the balance between nonlinearity and genuinely nonlinear dispersion gives the so-called compactons (another kind of solitons that is free of exponential wings). Researchers aim to study other phenomena arising in solitary waves theory to highlight the structures of the obtained waves solutions.

In recent years, many new approaches, such as the inverse scattering transformation method [1], Hirota direct method [2,3], tanh method [4,5], extended tanh method [6–8], modified simplest equation method [9–12], multiple exp-function method [13], transformed rational function method [14], first integral method [15], G'/G-expansion method [16], and so on have been proposed to find exact solutions of nonlinear evolution equations.

One of the most effective and direct methods for constructing exact solutions of nonlinear equations is the first integral method. Based on the ring theory of commutative algebra, Feng [15] first proposed the first integral method for solving Burgers-KdV equation. The basic idea of this method is to construct a first integral with polynomial coefficients of an explicit form to an equivalent autonomous planar system by using the division theorem. Recently, this method was widely used by many ([17–23] and by the references therein). Lu [17] used division theorem combined with the Riccati equation $U'(\xi) = b_1 + b_2U^2(\xi)$ to obtain travelling wave solutions of some class of NLPDEs. Geng *et al* [24] developed four (2+1)-dimensional models from the Jaulent–Miodek hierarchy [25],

$$\begin{aligned}
w_t &= -(w_{xx} - 2w^3)_x - \frac{3}{2}\left(w_x\partial_x^{-1}w_y + ww_y\right), \\
w_t &= \frac{1}{2}(w_{xx} - 2w^3)_x + \frac{3}{2}\left(-\frac{1}{4}\partial_x^{-1}w_{yy} + ww_y\right), \\
w_t &= -\frac{1}{4}(w_{xx} - 2w^3)_x - \frac{3}{4}\left(\frac{1}{4}\partial_x^{-1}w_{yy} + w_x\partial_x^{-1}w_y\right), \\
w_t &= 2(w_{xx} - 2w^3)_x - \frac{3}{4}\left(\partial_x^{-1}w_{yy} - 2w_x\partial_x^{-1}w_y - 6ww_y\right),
\end{aligned}$$

where ∂_x^{-1} denotes an inverse operator of $\partial_x = (\partial/\partial x)$ with the condition $\partial_x\partial_x^{-1} = \partial_x^{-1}\partial_x = 1$, which can be defined as $\partial_x^{-1}f(x) = \int_{-\infty}^{x} f(t)\mathrm{d}t$ under the decaying condition at infinity. Liu and Yan [26] discussed the bifurcation and exact travelling wave solutions of the third (2+1)-dimensional model. Wazwaz [27] obtained the multiple kink solutions and multiple singular kink solutions for the same. Furthermore, Wazwaz [28] generalized these four models to (3+1)-dimensional and obtained some multiple soliton solutions by using Hereman–Nuseir method. The aim of this paper is to find exact solutions of (2+1)-dimensional Jaulent–Miodek equations by using the division theorem combined with the Riccati equation $U'(\xi) = b_0 + b_1U(\xi) + b_2U^2(\xi)$.

Theorem. *Let (u, v) be a compatible solution of the first two Jaulent–Miodek equations*:

$$\begin{cases} u_t - \frac{1}{4}v_{xxx} + uv_x + \frac{1}{2}vu_x = 0, \\ v_t + u_x + \frac{3}{2}vv_x = 0, \end{cases} \tag{1}$$

$$\begin{cases} u_t + u_{xxx} + \frac{3}{2}vv_{xxx} + \frac{9}{2}v_xv_{xx} - 6uu_x - 6uvv_x - \frac{3}{2}u_xv^2 = 0, \\ v_t + v_{xxx} - 6u_xv - 6uv_x - \frac{15}{2}v_xv^2 = 0. \end{cases} \tag{2}$$

Then w, *x*, *y*, *t*, *solves any of the* (2+1)-*dimensional equation in the following list*:

$$w_t = -(w_{xx} - 2w^3)_x - \frac{3}{2}(w_x\partial_x^{-1}w_y + ww_y), \tag{3}$$

$$w_t = \frac{1}{2}(w_{xx} - 2w^3)_x + \frac{3}{2}\left(-\frac{1}{4}\partial_x^{-1}w_{yy} + ww_y\right), \tag{4}$$

$$w_t = -\frac{1}{4}(w_{xx} - 2w^3)_x - \frac{3}{4}\left(\frac{1}{4}\partial_x^{-1}w_{yy} + w_x\partial_x^{-1}w_y\right), \tag{5}$$

$$w_t = 2(w_{xx} - 2w^3)_x - \frac{3}{4}(\partial_x^{-1}w_{yy} - 2w_x\partial_x^{-1}w_y - 6ww_y), \tag{6}$$

where ∂_x^{-1} *denotes an inverse operator of* $\partial_x = (\partial/\partial x)$ *with the condition* $\partial_x \partial_x^{-1} = \partial_x^{-1}\partial_x = 1$, *which can be defined as* $\partial_x^{-1} f(x) = \int_{-\infty}^{x} f(t)\mathrm{d}t$ *under the decaying condition at infinity.*

2. Division theorem combined with the Riccati equation

Consider the nonlinear partial differential equation in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, \ldots) = 0. \tag{7}$$

Using travelling wave

$$u(x, y, t) = U(\xi), \quad \xi = k(x + ly - \lambda t),$$

from eq. (7) we obtain the ordinary differential equation (ODE):

$$Q(U, U', U'', U''', ...) = 0, \tag{8}$$

where the prime denotes the derivation with respect to ξ. Suppose that the solution of ODE (8) can be written as follows:

$$u(x, y, t) = U(\xi) = f(\xi). \tag{9}$$

Next, we introduce a new independent variable

$$X(\xi) = f(\xi), \quad Y(\xi) = f_\xi(\xi), \tag{10}$$

which leads a system of nonlinear ODEs

$$\begin{aligned} &X_\xi(\xi) = Y(\xi), \\ &Y_\xi(\xi) = F_1(X(\xi), Y(\xi)). \end{aligned} \tag{11}$$

By using the division theorem, one can obtain the first integral to eq. (11) which can reduce eq. (8) to the following Riccati equation:

$$U'(\xi) = a_0 + a_1 U(\xi) + a_2 U^2(\xi), \quad a_i \in R, \quad a_2 \neq 0, \tag{12}$$

then periodic and solitary wave solutions to eq. (7) are obtained by using the solutions of Riccati eq. (12) directly. We have the following significant special solutions [29] of (12):

Type 1. When $\Delta = a_1^2 - 4a_0 a_2 > 0$, the solutions of eq. (12) are

$$\begin{aligned} U_1(\xi) &= -\frac{\sqrt{\Delta}}{2a_2} \tanh\left(\frac{\sqrt{\Delta}}{2}(\xi + \xi_0)\right) - \frac{a_1}{2a_2}, \\ U_2(\xi) &= -\frac{\sqrt{\Delta}}{2a_2} \coth\left(\frac{\sqrt{\Delta}}{2}(\xi + \xi_0)\right) - \frac{a_1}{2a_2}. \end{aligned} \tag{13}$$

Type 2. When $\Delta = a_1^2 - 4a_0 a_2 < 0$, the solutions of eq. (12) are

$$\begin{aligned} U_3(\xi) &= \frac{\sqrt{-\Delta}}{2a_2} \tan\left(\frac{\sqrt{-\Delta}}{2}(\xi + \xi_0)\right) - \frac{a_1}{2a_2}, \\ U_4(\xi) &= -\frac{\sqrt{-\Delta}}{2a_2} \cot\left(\frac{\sqrt{-\Delta}}{2}(\xi + \xi_0)\right) - \frac{a_1}{2a_2}. \end{aligned} \tag{14}$$

Type 3. When $\Delta = a_1^2 - 4a_0a_2 = 0$, the solution of eq. (12) is

$$U_5(\xi) = -\frac{1}{a_2(\xi + \xi_0)} - \frac{a_1}{2a_2}. \tag{15}$$

Division theorem. *Suppose that $P(w, z)$, $Q(w, z)$ are polynomials in $C(w, z)$ and $P(w, z)$ is irreducible in $C(w, z)$. If $Q(w, z)$ vanishes at all zero points of $P(w, z)$, then there exists a polynomial $G(w, z)$ in $C(w, z)$ such that*

$$Q(w, z) = P(w, z)G(w, z).$$

3. Application

Based on the above description, the first integral method can be used for eq. (3)

$$u_t + u_{xxx} - 6u^2u_x + \frac{3}{2}(uu_y + u_x\partial_x^{-1}u_y) = 0. \tag{16}$$

Now, we assume that eq. (16) admits a solution of the form

$$U(x, y, t) = U(\xi), \quad \xi = k(x + ly - \lambda t),$$

then we have

$$-\lambda U' + K^2U''' - 6U^2U' + \frac{3}{2}lUU' = 0, \tag{17}$$

where the prime denotes the derivation with respect to ξ.

Integrating eq. (17) with respect to ξ yields

$$-\lambda U + k^2U'' - 2U^3 + \frac{3}{4}lU^2 = R, \tag{18}$$

where R is the integration constant.

Using (10) we get

$$X'(\xi) = Y(\xi), \tag{19a}$$

$$Y'(\xi) = \frac{2}{k^2}X^3(\xi) - \frac{3l}{4k^2}X^2(\xi) + \frac{\lambda}{k^2}X(\xi) + \frac{R}{k^2}. \tag{19b}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of eqs (19a), (19b), and $Q(X, Y) = \sum_{i=0}^{m} a_i(X)Y^i = 0$ is an irreducible polynomial in the complex domain $C[X, Y]$ such that

$$Q(X(\xi), Y(\xi)) = \sum_{i=0}^{m} a_i(X(\xi))Y^i(\xi) = 0, \tag{20}$$

where $a_i(X)$ $(i = 0, 1, \ldots, m)$ are polynomials of X and $a_m(X) \neq 0$. Equation (20) is called the first integral to (19). We note that $\mathrm{d}Q/\mathrm{d}\xi$ is a polynomial of X and Y, and $Q(X(\xi), Y(\xi)) = 0$ implies that $(\mathrm{d}Q/\mathrm{d}\xi)\big|_{(19)} = 0$. According to the division theorem, there exists a polynomial $g(X) + h(X)Y$, in the complex domain $C[X, Y]$ such that

$$\frac{\mathrm{d}Q}{\mathrm{d}\xi} = \frac{\mathrm{d}Q}{\mathrm{d}X}\frac{\mathrm{d}X}{\mathrm{d}\xi} + \frac{\mathrm{d}Q}{\mathrm{d}Y}\frac{\mathrm{d}Y}{\mathrm{d}\xi} = (g(X) + h(X)Y)\sum_{i=0}^{m} a_i(X)Y^i. \tag{21}$$

In this example, we take two different cases, by assuming $m = 1$ and $m = 2$ in eq. (20).

Case I. Suppose $m = 1$. By comparing with the coefficients of $Y^i (i = 2, 1, 0)$ on both sides of (21), we have

$$a_1'(X) = a_1(X)h(X), \tag{22}$$

$$a_0'(X) = a_0(X)h(X) + a_1(X)g(X), \tag{23}$$

$$a_1(X)\left[\frac{2}{k^2}X^3 - \frac{3l}{4k^2}X^2 + \frac{\lambda}{k^2}X + \frac{R}{k^2}\right] = a_0(X)g(X). \tag{24}$$

As a_i(X)($i = 0$, 1, 2) are polynomials, then from (22) we deduce that $a_1(X)$ is constant and $h(X) = 0$. For simplicity, take $a_1(X) = 1$. Balancing the degrees of $g(X)$ and $a_0(X)$, we conclude that $\deg(g(X)) = 1$ only. Suppose that $g(X) = A_1 X + B_0$, then we find $a_0(X)$ as

$$a_0(X) = A_0 + B_0 X + \frac{1}{2}A_1 X^2, \tag{25}$$

where A_0 is an arbitrary integration constant.

Substituting $a_0(X)$ and $g(X)$ in (24) and setting all the coefficients of powers X to zero, we then obtain a system of nonlinear algebraic equations and on solving them, we obtain

$$R = \frac{l^3 - 16\lambda}{128}, \quad A_0 = \frac{16\lambda - l^2}{32k}, \quad A_1 = \frac{2}{k}, \quad B_0 = -\frac{l}{4k}, \tag{26}$$

$$R = \frac{l^3 - 16\lambda}{128}, \quad A_0 = -\frac{16\lambda - l^2}{32k}, \quad A_1 = -\frac{2}{k}, \quad B_0 = \frac{l}{4k}, \tag{27}$$

where k, l and λ are arbitrary constants.

Using the conditions (26) in eq. (20), we obtain

$$Y_1(\xi) = \frac{l^2 - 16\lambda}{32k} + \frac{l}{4k}X - \frac{1}{k}X^2. \tag{28}$$

Combining (28) with (19a), (19b), we can reduce eq. (18) to Riccati equation as follows:

$$U'(\xi) = \frac{l^2 - 16\lambda}{32k} + \frac{l}{4k}U - \frac{1}{k}U^2. \tag{29}$$

By substituting the solution of eq. (12) in (29), the significant special solutions to (2+1)-dimensional nonlinear evolution eq. (16) can be written as:

Type 1. When $\Delta > 0$, $3l^2 - 32\lambda > 0$, we have

(1) The kink-shaped solution

$$u_1(x, y, t) = \frac{l}{8} + \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \tanh\left[\frac{\sqrt{3l^2 - 32\lambda}}{8k}(k(x + ly - \lambda t) + \xi_0)\right], \tag{30}$$

(2) The singular soliton solution

$$u_2(x, y, t) = \frac{l}{8} + \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \coth\left[\frac{\sqrt{3l^2 - 32\lambda}}{8k}(k(x + ly - \lambda t) + \xi_0)\right], \tag{31}$$

Type 2. When $32\lambda - 3l^2 > 0$, we have

$$u_3(x, y, t) = \frac{l}{8} - \frac{\sqrt{32\lambda - 3l^2}}{8} \times \tan\left[\frac{\sqrt{32\lambda - 3l^2}}{8k}(k(x + ly - \lambda t) + \xi_0)\right], \tag{32}$$

$$u_4(x, y, t) = \frac{l}{8} + \frac{\sqrt{32\lambda - 3l^2}}{8} \times \cot\left[\frac{\sqrt{32\lambda - 3l^2}}{8k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{33}$$

Type 3. When $32\lambda = 3l^2$, we have

$$u_5(x, y, t) = \frac{l}{8} + \frac{k}{k(x + ly - \lambda t) + \xi_0}. \tag{34}$$

Similarly, in the case of (27), from (20), we obtain

$$Y_2(\xi) = \frac{16\lambda - l^2}{32k} - \frac{l}{4k}X + \frac{1}{k}X^2. \tag{35}$$

Combining (35) with (19a), (19b), we can reduce eq. (18) to Riccati equation as follows:

$$U'(\xi) = \frac{16\lambda - l^2}{32k} - \frac{l}{4k}U + \frac{1}{k}U^2. \tag{36}$$

Substituting the solution of eq. (12) in (36), the exact solutions to (2+1)-dimensional nonlinear evolution eq. (16) can be written as:

Type 1. When $3l^2 - 32\lambda > 0$, we have

(1) The kink-shaped solution

$$u_6(x, y, t) = \frac{l}{8} - \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \tanh\left[\frac{\sqrt{3l^2 - 32\lambda}}{8k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{37}$$

(2) The singular soliton solution

$$u_7(x, y, t) = \frac{l}{8} - \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \coth\left[\frac{\sqrt{3l^2 - 32\lambda}}{8k}(k(x + ly - \lambda t) + \xi_0)\right]. \quad (38)$$

Type 2. When $32\lambda - 3l^2 > 0$, we have

$$u_8(x, y, t) = \frac{l}{8} + \frac{\sqrt{32\lambda - 3l^2}}{8} \times \tan\left[\frac{\sqrt{32\lambda - 3l^2}}{8k}(k(x + ly - \lambda t) + \xi_0)\right], \quad (39)$$

$$u_9(x, y, t) = \frac{l}{8} - \frac{\sqrt{32\lambda - 3l^2}}{8} \times \cot\left[\frac{\sqrt{32\lambda - 3l^2}}{8k}(k(x + ly - \lambda t) + \xi_0)\right]. \quad (40)$$

Type 3. When $3l^2 = 32\lambda$, we have

$$u_{10}(x, y, t) = \frac{l}{8} - \frac{k}{k(x + ly - \lambda t) + \xi_0}. \quad (41)$$

Case II. Suppose $m = 2$. By comparing with the coefficients of $Y^i (i = 3, 2, 1, 0)$ on both sides of (21), we have

$$a_2'(X) = a_2(X)h(X), \quad (42)$$

$$a_1'(X) = a_1(X)h(X) + a_2(X)g(X), \quad (43)$$

$$a_0'(X) = -a_2(X)\left[\frac{2}{k^2}X^3 - \frac{3l}{4k^2}X^2 + \frac{\lambda}{k^2}X + \frac{R}{k^2}\right] + a_0(X)h(X) + a_1(X)g(X), \quad (44)$$

$$a_1(X)\left[\frac{2}{k^2}X^3 - \frac{3l}{4k^2}X^2 + \frac{\lambda}{k^2}X + \frac{R}{k^2}\right] = a_0(X)g(X). \quad (45)$$

As $a_i(X) (i = 0, 1, 2)$ are polynomials, then from (42) we deduce that $a_2(X)$ is constant and $h(X) = 0$. For simplicity, take $a_2(X) = 1$. Balancing the degrees of $g(X)$, $a_0(X)$ and $a_1(X)$, we conclude that $\deg(g(X)) = 1$ only. Suppose that $g(X) = A_1X + B_0$, then we find $a_1(X)$ and $a_0(X)$ as follows:

$$a_1(X) = A_0 + B_0X + \frac{1}{2}A_1X^2, \quad (46)$$

$$a_0(X) = d + \left(A_0B_0 - \frac{R}{k^2}\right)X + \frac{1}{2}\left(A_0A_1 + B_0^2 - \frac{\lambda}{k^2}\right)X^2 + \frac{1}{3}\left(\frac{3}{2}B_0A_1 + \frac{3l}{4k^2}\right)X^3 + \frac{1}{4}\left(\frac{1}{2}A_1^2 - \frac{2}{k^2}\right)X^4. \quad (47)$$

Substituting $a_1(X)$, $a_0(X)$ and $g(X)$ in (45) and setting all the coefficient of powers X to zero, we obtain a system of nonlinear algebraic equations and by solving them, we obtain

$$B_0 = -\frac{\sqrt{3}l}{4k}, \quad A_0 = \frac{16\sqrt{3}\lambda - \sqrt{3}l^2}{32k}, \quad A_1 = \frac{2\sqrt{3}}{k},$$
$$R = \frac{3l^5 - 64\lambda l^3 + 256\lambda^2 l}{128(3l^2 - 16\lambda)}, \quad d = \frac{l^2 - 16\lambda}{8k^2}\left(\frac{3l^4 - 64\lambda l^2 + 256\lambda^2}{128(3l^2 - 16\lambda)}\right), \tag{48}$$

$$B_0 = \frac{\sqrt{3}l}{4k}, \quad A_0 = -\frac{16\sqrt{3}\lambda - \sqrt{3}l^2}{32k}, \quad A_1 = -\frac{2\sqrt{3}}{k},$$
$$R = \frac{3l^5 - 64\lambda l^3 + 256\lambda^2 l}{128(3l^2 - 16\lambda)}, \quad d = \frac{l^2 - 16\lambda}{8k^2}\left(\frac{3l^4 - 64\lambda l^2 + 256\lambda^2}{128(3l^2 - 16\lambda)}\right), \tag{49}$$

where λ, l and k are arbitrary constants.

Using conditions (48) in eq. (20), we obtain

$$Y_1(\xi) = \frac{\sqrt{3}l^2 - 16\sqrt{3}\lambda}{64k} + \frac{\sqrt{3}l}{8k}X - \frac{\sqrt{3}}{2k}X^2. \tag{50}$$

Combining (50) with (19a), (19b), we can reduce eq. (18) to Riccati equation as follows:

$$U'(\xi) = \frac{\sqrt{3}l^2 - 16\sqrt{3}\lambda}{64k} + \frac{\sqrt{3}l}{8k}U - \frac{\sqrt{3}}{2k}U^2. \tag{51}$$

Substituting the solution of eq. (12) in (51), the exact solutions to (2+1)-dimensional nonlinear evolution eq. (16) can be written as:

Type 1. When $\Delta = \left[\left(9l^2 - 96\lambda\right)/64k^2\right] > 0$, $3l^2 - 32\lambda > 0$, we have

(1) The kink-shaped solution

$$u_{11}(x, y, t) = \frac{l}{8} + \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \tanh\left[\frac{\sqrt{9l^2 - 96\lambda}}{16k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{52}$$

(2) The singular soliton solution

$$u_{12}(x, y, t) = \frac{l}{8} + \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \coth\left[\frac{\sqrt{9l^2 - 96\lambda}}{16k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{53}$$

Type 2. When $32\lambda - 3l^2 > 0$, we have

$$u_{13}(x, y, t) = \frac{l}{8} - \frac{\sqrt{32\lambda - 3l^2}}{8} \times \tan\left[\frac{\sqrt{96\lambda - 9l^2}}{16k}(k(x + ly - \lambda t) + \xi_0)\right], \tag{54}$$

$$u_{14}(x, y, t) = \frac{l}{8} + \frac{\sqrt{32\lambda - 3l^2}}{8} \times \cot\left[\frac{\sqrt{96\lambda - 9l^2}}{16k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{55}$$

Type 3. When $3l^2 = 32\lambda$, we have

$$u_{15}(x, y, t) = \frac{l}{8} + \frac{2\sqrt{3}k}{3(k(x + ly - \lambda t) + \xi_0)}. \tag{56}$$

Similarly, in the case of (49), from (20), we obtain

$$Y_2(\xi) = \frac{16\sqrt{3}\lambda - \sqrt{3}l^2}{64k} - \frac{\sqrt{3}l}{8k}X + \frac{\sqrt{3}}{2k}X^2. \tag{57}$$

Combining (57) with (19a), (19b), we can reduce eq. (18) to Riccati equation as follows:

$$U'(\xi) = \frac{16\sqrt{3}\lambda - \sqrt{3}l^2}{64k} - \frac{\sqrt{3}l}{8k}U + \frac{\sqrt{3}}{2k}U^2. \tag{58}$$

Substituting the solution of eq. (12) in (58), the exact solutions to (2+1)-dimensional nonlinear evolution eq. (16) can be written as:

Type 1. When $3l^2 - 32\lambda > 0$, we have

(1) The kink-shaped solution

$$u_{16}(x, y, t) = \frac{l}{8} - \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \tanh\left[\frac{\sqrt{9l^2 - 96\lambda}}{16k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{59}$$

(2) The singular soliton solution

$$u_{17}(x, y, t) = \frac{l}{8} - \frac{\sqrt{3l^2 - 32\lambda}}{8} \times \coth\left[\frac{\sqrt{9l^2 - 96\lambda}}{16k}(k(x + ly - \lambda t) + \xi_0)\right]. \tag{60}$$

Type 2. When $32\lambda - 3l^2 > 0$, we have

$$u_{18}(x, y, t) = \frac{l}{8} + \frac{\sqrt{32\lambda - 3l^2}}{8} \times \tan\left[\frac{\sqrt{96\lambda - 9l^2}}{16k}(k(x + ly - \lambda t) + \xi_0)\right], \tag{61}$$

$$u_{19}(x, y, t) = \frac{l}{8} - \frac{\sqrt{32\lambda - 3l^2}}{8} \times \cot\left[\frac{\sqrt{96\lambda - 9l^2}}{16k}(k(x + ly - \lambda t) + \xi_0)\right]. \quad (62)$$

Type 3. When $3l^2 = 32\lambda$, we have

$$u_{20}(x, y, t) = \frac{l}{8} - \frac{2\sqrt{3}k}{3(k(x + ly - \lambda t) + \xi_0)}. \quad (63)$$

4. Conclusion

In this paper, the first integral method was applied successfully for constructing exact travelling solutions of a (2+1)-dimensional Jaulent–Miodek equation [24–30]. This method is found to be reliable, effective and we have successfully obtained some new solutions which may be useful for describing certain nonlinear phenomena. Thus, we conclude that the proposed method can be extended to solve the nonlinear problems which arise in the theory of solitons and other areas. We anticipate that our results can be found potentially useful for applications in mathematical physics.

Acknowledgement

The authors are grateful to the anonymous referee for the careful checking of the details and for helpful comments that improved this paper.

References

[1] V O Vakhnenko, E J Parkes and A J Morrison, *Chaos, Solitons and Fractals* **17**(**4**), 683 (2003)
[2] R Hirota, *Phys. Rev. Lett.* **27**, 1192 (1971)
[3] R Hirota, *The direct method in soliton theory* (Cambridge University Press, 2004)
[4] W X Ma, *Phys. Lett. A* **180**, 221 (1993)
[5] W Malfliet, *Am. J. Phys.* **60**(**7**), 650 (1992)
[6] S A El-Wakil and M A Abdou, *Chaos, Solitons and Fractals* **31**(**4**), 840 (2007)
[7] E Fan, *Phys. Lett. A* **277**, 212 (2000)
[8] A M Wazwaz, *Appl. Math. Comput.* **187**, 1131 (2007)
[9] N A Kudryashov, *Phys. Lett. A* **342**(**12**), 99 (2005)
[10] N A Kudryashov, *Chaos, Solitons and Fractals* **24**(**5**), 1217 (2005)
[11] M Eslami, M Mirzazadeh and A Biswas, *J. Mod. Opt.* **60**(**19**), 1627 (2013)
[12] N K Vitanov, Z I Dimitrova and H Kantz, *Appl. Math. Comput.* **216**(**9**), 2587 (2010)
[13] W X Ma, T W Huang and Y Zhang, *Phys. Scr.* **82**, 065003 (2010)
[14] W X Ma and J-H Lee, *Chaos, Solitons and Fractals* **42**, 1356 (2009)
[15] M Eslami, A Neyrame and M Ebrahimi, *J. King Saudi Univ. Sci.* **24**(**1**), 69 (2012)
[16] Z S Feng and X H Wang, *Phys. Lett. A* **308**, 173 (2003)
[17] B Lu, *Commun. Non. Sci. Numer. Simul.* **17**, 4626 (2012)
[18] B Lu, *J. Math. Anal. Appl.* **395**(**2**), 684 (2012)

[19] B Lu, H Q Zhang and F D Xie, *Appl. Math. Comput.* **216**, 1329 (2010)
[20] A Biswas, M Mirzazadeh and M Eslami, *Acta Phys. Polon. B* **45**(**4**), 849 (2014)
[21] M Eslami, B Fathi Vajargah, M Mirzazadeh and A Biswas, *Indian J. Phys.* **88**(**2**), 177 (2014)
[22] M F El-Sabbagh and S I El-Ganaini, *Appl. Math. Sci.* **6**(**78**), 3893 (2012)
[23] H Jafari, A Sooraki, Y Talebi and A Biswas, *Nonlinear Analysis: Modelling and Control*. **17**(**2**), 182 (2012)
[24] X G Geng, C W Cao and H H Dai, *J. Phys. A: Math. Gen.* **34**, 989 (2001)
[25] M Jaulent and I Miodek, *Lett. Math. Phys.* **1**(**3**), 243 (1976)
[26] H Liu and F Yan, *Int. J. Nonlinear Sci.* **11**, 200 (2011)
[27] A M Wazwaz, *Phys. Lett. A* **373**, 1844 (2009)
[28] A M Wazwaz, *Appl. Math. Lett.* **25**(**11**), 1936 (2012)
[29] W X Ma and B Fuchssteiner, *Int. J. Non-Linear Mech.* **31**, 329 (1996)
[30] A Biswas and A H Kara, *Appl. Math. Comput.* **217**(**2**), 944 (2010)