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Ternary fission

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Abstract. We present the ternary fission of ^{252}Cf and ^{236}U within a three-cluster model as well as in a level density approach. The competition between collinear and equatorial geometry is studied by calculating the ternary fragmentation potential as a function of the angle between the lines joining the stationary middle fragment and the two end fragments. The obtained results for the ^{16}O accompanying ternary fission indicate that collinear configuration is preferred to equatorial configuration. Further, for all the possible third fragments, the potential energy surface (PES) is calculated corresponding to an arrangement in which the heaviest and the lightest fragments are considered at the end in a collinear configuration. The PES reveals several possible ternary modes including true ternary modes where the three fragments are of similar size. The complete mass distributions of Si and Ca which accompanied ternary fission of ^{236}U is studied within a level density picture. The obtained results favour several possible ternary combinations.

Keywords. Ternary fission; three-cluster model; level density; single-particle energies.

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1. Introduction

Ternary fission is defined as a nuclear breakup into three fragments with usually the third fragment being an α-particle. Though several other third fragments have been reported, the break up into three equal-mass fragments has not yet been detected (normally referred to as true ternary fission, TTF). However, theoretically, it has been envisaged that such heavy third fragment accompanied fission is energetically a viable process. Further, this ternary fission process carries nuclear structural information as is evident from the experimental signature. The favourable ternary breakups are found to be associated with either spherical or deformed closed shells. Thus, the study of this exotic decay mode, the ternary fission, will give structural information of nuclear species.

The branching ratio of ternary fission and/or light charged particle accompanied fission with respect to binary fission is small and being an exotic process, theoretical as well as experimental understanding of this process is not completely exploited. Hence the study on this process is important for understanding fission mechanism. Such a study would help us to understand the conditions of the nucleus at scission, for example, velocity of the fragments, dynamics, trajectories, etc.

2. Methodology

Recently, we have studied the various aspects associated with the ternary fission process. A model, called the three-cluster model (TCM) [1–6] has been put forth. This accounts for the energy minimization of all possible ternary breakups of a heavy radioactive nucleus. Further, within the TCM we have analysed the competition between different geometries as well as different positioning of the fragments. Also, an attempt was made to calculate the mass distribution of ternary fission process within the statistical approach. The TCM and the statistical approach used to calculate the mass distribution are discussed as follows.

2.1 *The three-cluster model*

The TCM is based on the dynamical or quantum mechanical fragmentation theory of cold fusion phenomenon in heavy-ion reactions and fission dynamics, including the prediction of cluster radioactivity (CR) [7–11].

In TCM the available Q-value to the kinetic energies E_i of the three fragments, i.e., $Q = E_1 + E_2 + E_3$, is defined as

$$Q = M - \sum_{i=1}^{3} m_i, \tag{1}$$

where M and m_i are the mass excesses of the decaying nucleus and the product nuclei, respectively, which are expressed in MeV and taken from [12].

The ternary fragmentation potential between the three (spherical) fragments (referred to as PES), within the TCM [1–6], is defined as the sum of the total Coulomb potential, total nuclear potential, ℓ-dependent potential and the sum of the mass excesses of ternary fragments. It can be written as

$$V_{\mathrm{tot}}(\theta) = \sum_{i=1}^{3} \sum_{j>i}^{3} (m_i + V_{ij}(\theta)) + V_l(\theta), \tag{2}$$

$$V_{ij}(\theta) = V_{\mathrm{C}ij}(\theta) + V_{\mathrm{N}ij}(\theta). \tag{3}$$

The Coulomb interaction energy $V_{\mathrm{C}ij}$ describes the force of repulsion between the two interacting charges and we consider the pairwise interaction of the three fragments. The Coulomb energy expression [13] is defined as

$$V_{\mathrm{C}ij}(\theta) = \begin{cases} \dfrac{Z_i Z_j e^2}{R_{ij}}, & R_{ij} \geq R_i + R_j, \\[2ex] \dfrac{Z_i Z_j e^2}{R_{ij}} \Delta, & R_i - R_j \leq R_{ij} \leq R_i + R_j, \\[2ex] Z_i Z_j e^2 \Upsilon, & 0 \leq R_{ij} \leq R_i - R_j, \end{cases} \tag{4}$$

with

$$\Delta = \left[1 - \frac{(R_i + R_j - R_{ij})^4 \Delta_1}{160 R_i^3 R_j^3}\right],$$

$$\Delta_1 = (R_{ij}^2 + 4R_{ij}(R_i + R_j) + 20R_i R_j - 5R_i^2 - 5R_j^2),$$

$$\Upsilon = \frac{15R_i^2 - 3R_j^2 - 5R_{ij}^2}{10R_i^3},$$

where R_{ij} is the centre-to-centre distance between the fragments i and j. For the nuclear part of the potential $V_{\mathrm{N}ij}$, one can use either the proximity potential $V_{\mathrm{P}ij}$ or the short-range Yukawa plus exponential nuclear attractive potential $V_{\mathrm{Y}ij}$ amongst the three fragments.

The proximity potential $V_{\mathrm{P}ij}$ is defined as

$$V_{\mathrm{P}ij} = 4\pi \bar{R} \gamma b \phi(\xi). \tag{5}$$

The universal function $\phi(\xi)$ depends only on the distance between two nuclei and is independent of the atomic numbers of the two nuclei. This is given as

$$\phi(\xi) = \begin{cases} -\frac{1}{2}(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3, & \xi < 1.2511, \\ -3.437 \exp(-\xi/0.75), & \xi \geq 1.2511. \end{cases} \tag{6}$$

Here $\xi = s/b$. The function $\phi(\xi)$ is defined for negative (the overlap region), zero (touching configuration) and positive (separated configuration) values of the surface separation $s = R_{ij} - (R_i + R_j)$ and $b \approx 1$ fm is the diffusivity parameter.

The specific nuclear surface tension γ is given by

$$\gamma = 0.9517 \left[1 - 1.7826 \left(\frac{N - Z}{A}\right)^2\right] \mathrm{MeV\ fm}^{-2}, \tag{7}$$

where $\bar{R}$ is the mean curvature radius. The potential as defined in eq. (2) can be calculated as a function of angle θ between the line passing through the centre of the fixed fragment along the fission direction denoted as x-axis and the line passing through the centres of the fixed fragment and the other two fragments. The value of the angle $\theta = 0$ refers to a collinear arrangement as shown in figure 1a. Varying the angle of the end fragments would lead to a triangular configuration as shown in figure 1b where all the surfaces of the three fragments touch each other. The distance between the centres of the interacting fragments R_{ij} can be denoted as

$$R_{32} = R_3 + R_2; \qquad R_{31} = R_1 + R_3, \tag{8}$$

$$R_{12} = \sqrt{R_{23}^2 + R_{13}^2 + 2R_{23}R_{13}\cos 2\theta}. \tag{9}$$

The radius of each fragment is defined as $R_x = r_0 A_i^{1/3}$, with $r_0 = 1.16$ fm (i taking the values of 1, 2 and 3 corresponding to fragments A_1, A_2 and A_3).

The distance vector $\mathbf{R}_{ij}$ given as the angle components between ith and jth nuclei are given as

$$\mathbf{R}_{31} = (-R_{31}\cos\theta)\hat{i} + (R_{31}\sin\theta)\hat{j}, \tag{10}$$

$$\mathbf{R}_{32} = R_{32}\cos\theta\hat{i} + R_{32}\sin\theta\hat{j}. \tag{11}$$

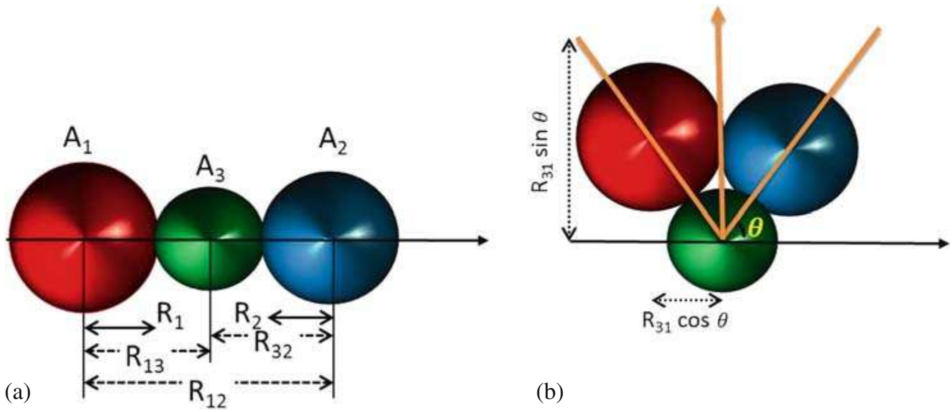


Figure 1. Schematic configurations of three nuclei in (**a**) a collinear arrangement and (**b**) triangular arrangement, with the parameters as labelled.

2.2 *A level density approach to ternary fission*

In the work of Rajasekaran and Devanathan [14], the mass distribution for the binary fission of a parent nucleus was obtained by considering the charge-to-mass ratios of the fissioning fragments. The same idea is extended for the ternary fission as

$$\frac{Z_1}{A_1} \approx \frac{Z_2}{A_2} \approx \frac{Z_3}{A_3} \approx \frac{Z_{\mathrm{p}}}{A_{\mathrm{p}}}, \tag{12}$$

where A_{p}, Z_{p} and A_i, Z_i ($i = 1$, 2 and 3) correspond to the mass and charge numbers of the parent nucleus and the three fission fragments. Further, we impose a condition $A_1 \geq A_2 \geq A_3$ to avoid repetition in fragment combinations.

The ternary fission yield, the ratio between the probability of a given ternary fragmentation and the sum of the probabilities of all the possible ternary fragmentations, is given by

$$Y(A_j, Z_j) = \frac{P(A_j, Z_j)}{\sum P(A_j, Z_j)}. \tag{13}$$

Here A_j and Z_j refer to a ternary fragmentation involving three fragments with mass and charge numbers as A_1, A_2, A_3 and Z_1, Z_2, Z_3 which are obtained from eq. (12).

In the statistical theory of Fong [15], the fission probability at the scissioning stage is considered to be proportional to the product of nuclear level densities of the fission fragments as

$$P(A_j, Z_j) \propto \rho_1(A_1, Z_1)\rho_2(A_2, Z_2)\rho_3(A_3, Z_3). \tag{14}$$

The nuclear level density [16] is

$$\rho = \frac{1}{12}(\pi^2/a)^{1/4} E^{-5/4} \exp(2\sqrt{aE}), \tag{15}$$

where the level density parameter $a = E/T^2$ and the excitation energy $E = E_{\mathrm{tot}} - E_0$. The total energy of the system $E_{\mathrm{tot}} = \sum_k n_k^Z \epsilon_k^Z + \sum_k n_k^N \epsilon_k^N$, where n_k^Z and n_k^N are the occupation probabilities of Z protons and N neutrons of a particular fragment and the summation is for all the single-particle energies considered. The ground-state energy $E_0 = \sum_{k=1}^{Z} \epsilon_k^Z + \sum_{k=1}^{N} \epsilon_k^N$. The particle number equations

$$Z = \sum_k n_k^Z = \frac{1}{1 + \exp(-\alpha^Z + \beta\epsilon_k^Z)}, \tag{16}$$

$$N = \sum_k n_k^N = \frac{1}{1 + \exp(-\alpha^N + \beta\epsilon_k^N)} \tag{17}$$

are numerically solved to determine the Lagrangian multipliers α^Z and α^N at a given temperature, $T = 1/\beta$. For our calculations, the single-particle energies of protons ϵ_k^Z and neutrons ϵ_k^N are retrieved from reference input parameter library (RIPL-3) by suitably modifying the code *spl-retrieve.for* given in the website [17]. These single-particle energies are calculated using the finite-range droplet model of Möller *et al* [18]. The physics and data included in the RIPL-3 are described in [19].

3. Results and discussion

The competition between collinear (figure 1a) and equatorial configuration (figure 1b) is studied for various third fragments of different parent nuclei. We present here a representative case by considering the third fragment as $A_3 = {}^{16}$O for the parent nucleus $A = {}^{252}$Cf. For a given third fragment, several possible binary combinations are possible. From our earlier studies based on the potential energy minimization we consider $A_1 = {}^{132}$Sn and $A_2 = {}^{104}$Zr for this study. Figure 2a presents the results for the arrangement of fragments in the order of A_1, A_2 and A_3. In figure 2b the lightest fragment A_3 is considered at the middle of the two other fragments. It is evident from figure 2a that the

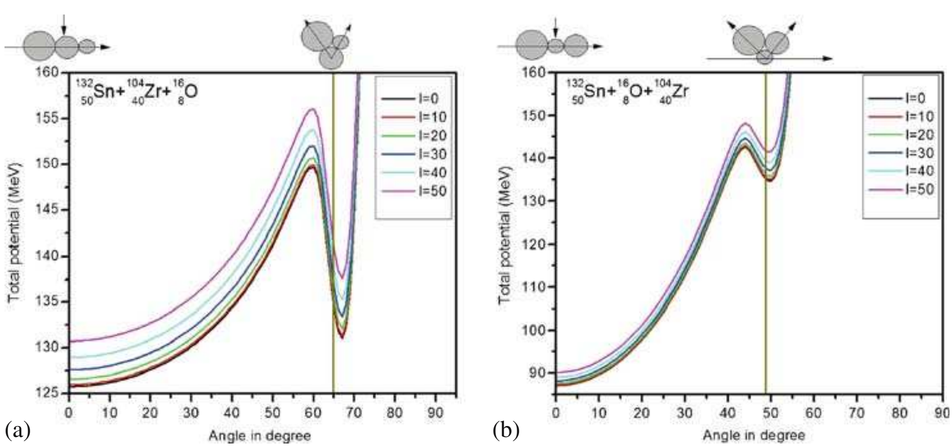
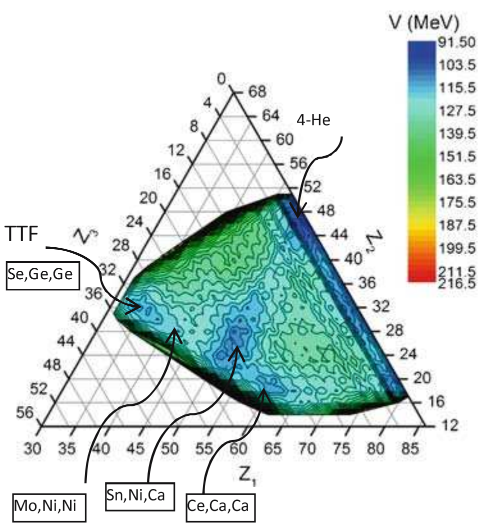


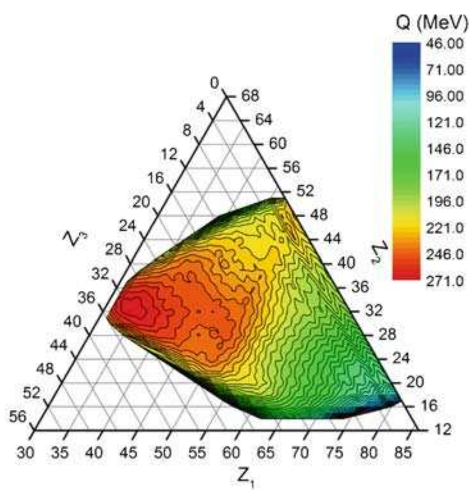
Figure 2. Ternary fragmentation potential as a function of angle and angular momentum (the effect of angular momentum is not significant) for (**a**) $A_1 + A_2 + A_3$ (^{132}Sn + ^{104}Zr + ^{16}O) and (**b**) $A_1 + A_3 + A_2$ (^{132}Sn + ^{16}O + ^{104}Zr) arrangements.

collinear and equatorial arrangements more or less compete with each other (the potential energy differs by about 5 MeV only) if one considers fragment arrangement in the order of A_1, A_2 and A_3. However, if the lightest fragment is considered at the middle, the collinear configuration is found to be favourable because the potential energy for the equatorial configuration is more by about 50 MeV. The underlying result is that if the lightest third fragment (heavier than α) is considered at the middle, collinear fissioning is more probable.

The potential energy landscape of ternary fission study requires complete minimization of potential energy of all the mass and charge asymmetries involved. To minimize PES in TCM, we impose a condition on mass numbers of fission fragments such as $A_1 \geq A_2 \geq A_3$ to avoid repetition of combinations. The potential energies of the thus identified favourable combinations are plotted in figure 3a and their corresponding Q-values are plotted in figure 3b. The PES can be computed for three different arrangements of the fragments. Figure 3a corresponds to an arrangement in which the heaviest (A_1) and the lightest (A_3) fragments are placed at the end. PES results indicate different favourable ternary breakups as labelled. The ^{4}He-accompanied ternary breakup remains the most favourable split which is also supported by a larger Q-value around (Sn, Pd, He). Apart from this, PES reveals three distinct regions where the potential energy is minimum as labelled. Among these, the deepest minimum corresponds to (Sn, Ni, Ca). However, a large Q-value and a minimum in PES are seen for TTF splitting (Se, Ge, Ge). The PES and Q-value systematics clearly indicate the possibility of heavy charged particle accompanied fission including the TTF mode.



(a)



(b)

Figure 3. (**a**) The PES shown in the ternary graph is plotted as a function of charge number of the three fragments for all the possible ternary split-ups of the parent nucleus ^{252}Cf corresponding to an arrangement in which the heaviest and lightest fragments are considered at either end in a collinear configuration. Most probable ternary valleys and true ternary fission modes are labelled. (**b**) The Q-values corresponding to the fragment combinations as presented in PES.

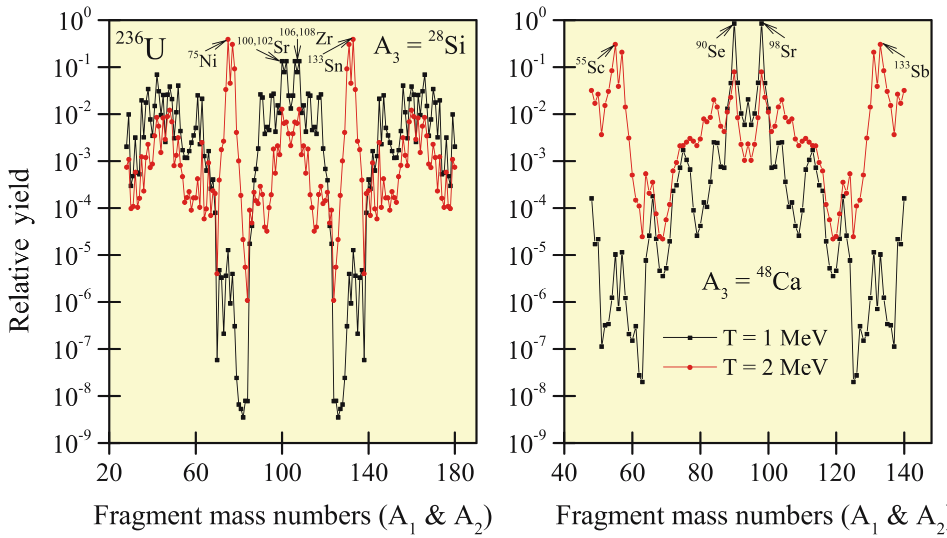


Figure 4. Ternary fission yields and/or mass distributions of ^{236}U nucleus for two different third fragments $A_3 = {}^{28}$Si and ^{48}Ca are plotted as a function of fragment mass numbers A_1 and A_2 at temperatures, $T = 1$ and 2 MeV.

To study ternary fission mass distribution, we recently adopted a level density-based approach. This method allows to calculate mass distribution of ternary fission of a parent nucleus for a particular third fragment. We present here, the ternary fission mass distribution of ^{236}U at two different temperatures $T = 1$ and 2 MeV for two different third fragments $A_3 = {}^{28}$Si and ^{48}Ca. The nuclear level densities ρ_i are evaluated using eq. (15) and employed in eq. (14) to compute probability of a particular ternary fragmentation. The results are presented in figure 4. The ternary mass distribution of ^{236}U with $A_3 = {}^{28}$Si show prominent peaks for ternary fragmentation of ^{106}Zr + ^{100}Sr + ^{28}Si and ^{133}Sn + ^{75}Ni + ^{28}Si at $T = 1$ and 2 MeV, respectively. For $A_3 = {}^{48}$Ca the favourable ternary configurations are found to be ^{98}Sr + ^{80}Se + ^{48}Ca and ^{133}Sb + ^{55}Sc + ^{48}Ca at $T = 1$ and 2 MeV, respectively. It is to be mentioned that these yield values of different ternary fragmentations refer only to ^{28}Si and ^{48}Ca accompanied ternary fission channels. Except for Sn, Ni, Ca all the other fragments mentioned here as favourable breakups are not associated with any shell closures either in proton or in neutron numbers. Hence the larger yield values for these combinations are not due to any shell effects. Further, it is noted that all these fragment combinations presented have larger deformation values. The level density approach helps one to find the most probable ternary breakup of a parent nucleus.

4. Summary

The potential energy surface as a function of angle leading from collinear configuration to equatorial configuration is studied for various third-particle accompanied fission. The ^{16}O-accompanied ternary fission corresponding to two different arrangements are presented. Of the two different arrangements, the one in which the lightest fragment is positioned at the middle is found to be more favourable. The PES reveals that the collinear configuration is favourable over the equatorial configuration. For all the possible third fragments of the parent nucleus ^{252}Cf, the ternary PES is calculated in which the fragments are considered to be arranged in increasing order of mass numbers with the heaviest and the lightest on either ends. The PES reveals several ternary fission modes with a strong valley for ^{4}He-accompanied fission also supported by the Q-value systematics. Further, several other ternary fission valleys are seen with a clear indication for true ternary fission modes. To calculate complete mass distribution of ternary events we adopted a level density approach study. The obtained results indicate several possible ternary split ups. It is also shown that with increase in temperature the mass distribution gets changed. The results pertaining to level density calculations are qualitative in nature. The model ingredients considered can be improved and the fragment combinations can be obtained by proper minimization.

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