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Fission approach to cluster radioactivity

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Abstract. Fission theory is used to explain α decay. Also, the analytical superasymmetric fission (ASAF) model is successfully employed to make a systematic search and to predict, with other models, cluster radioactivity. The macroscopic–microscopic method is illustrated for the superheavy nucleus ^{286}Fl. Then a few results of the theoretical approach of α decay (ASAF, UNIV and semFIS models), cluster decay (ASAF and UNIV) and spontaneous fission dynamics are described with Werner–Wheeler and cranking inertia. UNIV denotes universal curve and semFIS the fission-based semiempirical formula.

Keywords. Cluster decay; alpha decay; spontaneous fission; lifetimes.

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1. Introduction

At the 75th anniversary of nuclear fission we should remind ourselves that induced fission was discovered in 1939 by Otto Hahn (Nobel prize 1944), Lise Meitner and Fritz Strassmann (E Fermi Award 1966). Spontaneous fission was discovered in 1940 by Flerov and Petrzhak. Nuclear decay modes were explained by tunnelling through the potential barrier developed in 1928 by Gamow for α decay where quantum mechanics was first applied in nuclear physics.

Since 1979, we used fission theory to explain α decay (αD). The analytical superasymmetric fission (ASAF) model was successfully employed to carry out a systematic search and to predict, with other models, cluster decay (CD) [1,2]. This is the reason why we have not missed the conferences [3–6] in Berlin, Gaussig near Dresden, Leningrad and Gaithersburg dedicated to the 50th anniversary of Nuclear Fission.

In the region of heavy nuclei with atomic number $Z = 87$–96, the measured ^{14}C, ^{20}O, ^{23}F, $^{22,24-26}$Ne, ^{28,30}Mg and ^{32,34}Si cluster radioactivities [7] confirmed our predictions of 1980 [1,8]. CD shows up as a rare phenomenon in a huge background of α-particles.

Many Indian scientists, including Prof. Raj K Gupta, are or have been active in studying cluster decay, e.g. S K Arun, M Balasubramaniam, A Bandyopadhyay, D N Basu, A Bhagwat, M Bhattacharya, R K Biju, P R Chowdhury, A Devi, N K Dhiman, Y K Gambhir, K K Girija, A K Jain, A Joseph, B Kamalaharan, P V Kunhikrishnan, R Kumar, S Kumar, S S Malik, K Manimaran, I Mehrotra, R K Puri, T R Routray, B Sahu, C Samanta, K P Santhosh, G Sawhney, G Shanmugam, M K Sharma, B Singh, S Singh, G S K Swamy, etc.

Neutron-deficient superheavy (SH) nuclei with $Z = 104$–118 synthesized by fusion reactions [9–11] decay mainly by α emission or in a few cases by spontaneous fission (SF). Production of SHs closer to β-stability line could be possible in the future [12]. Usually, SH nuclei are defined as elements with $Z \geq 104$ which exist due to nuclear shell effects. Rf, Db, Sg, Bh, Hs, Mt, Ds, Rg, Cn, 113, Fl, 115, Lv, 117, 118 are examples.

Our calculations [13,14] have shown a trend towards branching ratio $b = T_\alpha/T_c$ relative to αD larger than unity for heavier SHs with $Z > 121$. Despite the good agreement between theory and experiment for αD [15] and CD [16,17], in the regions of nuclear chart which are not experimentally achieved, there is a large uncertainty of the calculated half-lives as a consequence of the differences in calculated atomic masses by different models. We continue our systematic search by using new mass tables: experimental AME12 [18] available for neutron-deficient SHs up to $Z = 118$ and theoretical WS-10 and WS3-11 [19,20] well extended up to the neutron drip line. There are many other mass models which do not cover these neutron-rich regions of SHs.

We illustrate the macroscopic–microscopic method with an example of PES for ^{286}Fl. Then we describe a few results of our theoretical approach of αD (ASAF, UNIV and semFIS models), CD (ASAF and UNIV) and SF dynamics with Werner–Wheeler and cranking inertia. UNIV denotes universal curve and semFIS, the fission-based semiempirical formula.

2. Macroscopic–microscopic method

One of the simplest parametrization of the nuclear surface of a binary system is that of two intersected spheres. It is axially symmetric and is characterized by a single deformation parameter: separation distance R which is initially $R_i = R_0 - R_e$ and at the touching point $R_t = R_e + R_d$. Here R_0 is the radius of the parent nucleus, R_e and R_d are the radii

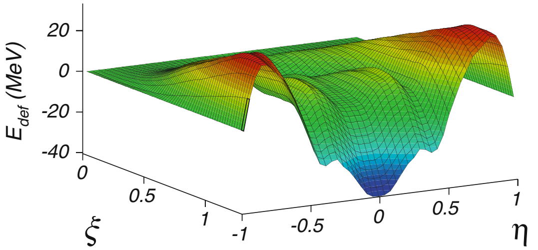

Figure 1. PES of ^{286}Fl vs. separation (ξ) and mass asymmetry (η). The two deepest valleys are due to the magicity of the daughter ^{208}Pb for ^{78}Ge radioactivity at about $\eta = 0.45$ and of the light fragment ^{132}Sn at $\eta \simeq 0.07$.

of the emitted cluster and daughter, respectively. The corresponding normalized variable is $\xi = (R - R_\mathrm{i})/(R_\mathrm{t} - R_\mathrm{i})$. By adding Strutinsky's shell and BCS pairing corrections [17] calculated with microscopic two-centre shell model [21] to the macroscopic Yukawa-plus-exponential model (Y+EM) deformation energy [22] for binary systems with different charge densities [23] we obtain the result plotted in figure 1. The figure clearly shows two deep valleys due to the magicity of the daughter ^{208}Pb for ^{78}Ge radioactivity at about $\eta = 0.45$ and of the light fragment ^{132}Sn at $\eta \simeq 0.07$. The mass asymmetry is defined as $\eta = (A_\mathrm{d} - A_\mathrm{e})/A$.

3. Alpha decay and cluster decay

For αD and CD we use our analytical superasymmetric fission (ASAF) model [24] employing Myers–Swiatecki's liquid-drop model [25] adjusted with a phenomenological correction and the universal curve (UNIV) [16]. For αD we also have the semFIS model [17] based on fission theory. Unlike the majority of other models exhibiting large deviations from experimental values in the vicinity of the magic number of neutrons (e.g., $N = 126$) semFIS model behaves well around. Detailed presentations of our models as well as many references may be found in [2,17,26–29]. Comprehensive tables have also been published [24,30].

A parent nucleus decays into an emitted ion and a daughter nucleus

$$^{A}\mathrm{Z} \rightarrow {}^{A_\mathrm{e}}\mathrm{Z}_\mathrm{e} + {}^{A_\mathrm{d}}\mathrm{Z}_\mathrm{d}. \tag{1}$$

The measurable quantities are: (1) kinetic energy of the emitted cluster $E_k = QA_\mathrm{d}/A$ or the released energy $Q = M - (M_\mathrm{e} + M_\mathrm{d})$, (2) decay constant $\lambda = \ln 2/T$ or half-life ($T < 10^{32}$ s) or branching ratio $b_\alpha = T_\alpha/T$ ($b_\alpha > 10^{-17}$).

In theory the decay constant $\lambda = \nu S P_s$ is a function of three model-dependent quantities: ν is the frequency of assaults or $E_v = h\nu/2$, S is the preformation probability and P_s is the penetrability of the external barrier. The method to estimate preformation as penetrability of internal barrier, $S = \exp(-K_\mathrm{ov})$, was presented in [31]. The action integral is calculated within the Wentzel–Kramers–Brillouin (WKB) quasiclassical approximation

$$K_\mathrm{ov} = \frac{2}{\hbar}\int_{R_\mathrm{i}}^{R_\mathrm{t}} \sqrt{2B(R)E(R)}\,\mathrm{d}R, \tag{2}$$

where E is the potential energy of deformation, $B = \mu$ is the nuclear inertia, equal to the reduced mass for $R \geq R_\mathrm{t}$.

Experiments [7] were performed in various universities and research institutes situated in Oxford, Moscow, Orsay, Berkeley, Dubna, Argonne, Livermore, Geneva, Milano, Vienna and Beijing. The standard r.m.s. deviations of $\log T$ values for 580 α emitters (largest set available) with $Z = 52$–118 are presented in table 1. Within ASAF, UNIV and semFIS models, the deviations for 512 (88%), 527 (91%) and 555 (96%) α emitters out of the total of 580, are under one order of magnitude.

ASAF and UNIV may reproduce 23 (85%) and 24 (89%) experimental data from the total of 27 cluster emissions with standard deviations under one order of magnitude

Table 1. The standard r.m.s. deviations of log T values for 580 α emitters.

Group	n	σ_{ASAF}	σ_{UNIV}	σ_{semFIS}
e–e	188	0.415	0.354	0.221
e–o	147	0.713	0.640	0.527
o–e	131	0.637	0.562	0.441
o–o	114	0.876	0.810	0.605

Table 2. The standard r.m.s. deviations of log T values calculated with ASAF and UNIV models for CD.

n	Parity	σ_{ASAF}	σ_{UNIV}
27	All	1.156	0.797
16	e–e	0.971	0.564
6	e–o	2.011	0.859
5	o–e	0.305	0.677

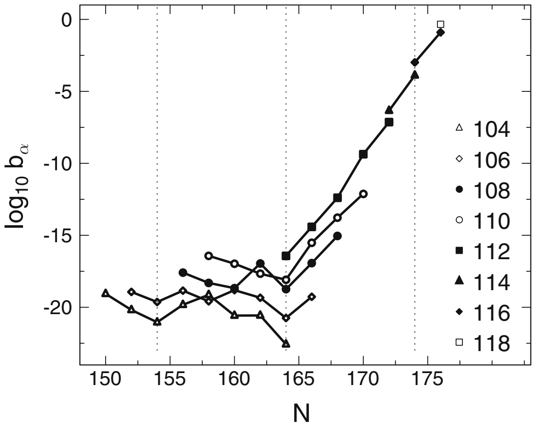

Figure 2. Branching ratio vs. the neutron number of the superheavy even–even parent nuclei. Calculations are carried out with Q-values obtained using atomic masses AME12 from [18].

(see table 2). There is no successful experiment found on o–o cluster emitter. The r.m.s. standard deviation is calculated as

$$\sigma = \left\{ \sum_{i=1}^{n} [\log(T_{\text{i}}/T_{\text{exp}})]^2/(n-1) \right\}^{1/2}. \tag{3}$$

For SHs the concept of CD is changed to allow emitted particles with $Z_{\text{e}} > 28$ from parents with $Z > 110$ and daughters around ^{208}Pb. From the typical example given in figure 2 the trend of increasing b_α for heavier nuclei may be clearly seen, $b_\alpha = T_\alpha/T_{\text{c}}$.

It is interesting to know the major decay mode leading to the shortest half-life, hence the calculated CD half-life, T_c, is found after the minimization of the half-lives obtained by considering all the possible combinations of the emitted cluster–daughter nucleus for a given parent.

For even–even parent SHs αD is dominant (shortest T) for neutron-deficient $Z = 118, 120$ isotopes. For a few isotopes of 118, 120 and 122 SF may compete with αD. CD is the most important decay mode of the element 124. It could also be important for several neutron-rich isotopes of 118, 120 and 122. Experimentally T should be longer than 1 μs due to the flight time through the recoil separator. Any $T < 1$ μs is not detectable.

4. Spontaneous fission

For spontaneous fission calculations we refer to Hartree–Fock–Bogoliubov approach with finite-range and density-dependent Gogny force [32] and self-consistent symmetry-unrestricted nuclear density functional with Skyrme energy density functional and cranking inertia [33]. The value closest to the experimental one was obtained by using a dynamical approach [34,35]. Simple relationships [36–38] have also been used.

We have already shown [39] that the calculated α decay half-lives are in agreement with the experimental data within one order of magnitude, while the discrepancy between the theory and the experiment can be as high as 10 orders of magnitude for spontaneous fission. It is clear that Werner–Wheeler approximation [40,41] for the nuclear inertia leads to too small values to explain the measured spontaneous fission half-life. We would like to try a better method based on the microscopic cranking inertia [42–44] introduced by Inglis [45]. The deformation energy and the inertia tensor were calculated to determine the half-life. Potential barrier was computed by the macroscopic–microscopic method [46] with a two-centre shell model [21]. To increase the calculated half-life, a smaller penetrability, hence a larger action integral, or for a given potential barrier, a larger nuclear inertia calculated with the cranking model, is required. The inertia tensor components are given by [47]

$$B_{ij} = 2\hbar^2 \sum_{\nu\mu} \frac{\langle\nu|\partial H/\partial\beta_i|\mu\rangle\langle\mu|\partial H/\partial\beta_j|\nu\rangle}{(E_\nu + E_\mu)^3}(u_\nu v_\mu + u_\mu v_\nu)^2, \tag{4}$$

where $\beta_1, \beta_2, \ldots, \beta_n$ are the deformation parameters, H is the single-particle Hamiltonian to determine the energy levels and the wave functions $|\nu\rangle$; u_ν, v_ν are the BCS occupation probabilities and E_ν is the quasiparticle energy. By choosing the distance between fragments, R, as a deformation coordinate, the inertia at the touching point of the two fragments is equal to the reduced mass $\mu = (A_1A_2/A)m$ in a binary system. When only one deformation coordinate, R, is considered the inertia tensor becomes a scalar, B. By using the cranking inertia it is possible to reproduce the spontaneous fission half-life of ^{282}Cn, $\log_{10} T_f^{\exp}(s) = -3.086$, with a reasonable value of the zero-point vibration energy ($E_v = 0.437$ MeV).

In conclusion, we would like to stress the need to make more reliable calculations for spontaneous fission half-lives of heavy and SH nuclei and to extend these calculations for superheavy nuclei closer to the line of β-stability and neutron-rich nuclei.

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