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Fission barrier heights in the $A \sim 200$ mass region

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Abstract. Statistical model analysis is carried out for p- and α-induced fission reactions using a consistent description for fission barrier and level density in $A \sim 200$ mass region. A continuous damping of shell correction with excitation energy is considered. Extracted fission barriers agree well with the recent microscopic–macroscopic model. The shell corrections at the saddle point were found to be insignificant.

Keywords. Fission barrier; damping of shell correction; statistical model.

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1. Introduction

Experimental determination of fission barrier height in the mass region $A \sim 200$ continues to be a challenging problem. Accurate knowledge of fission barrier height is vital not only to understand the heavy-ion-induced fusion–fission dynamics and prediction of superheavy elements, but also other areas, such as stellar nucleosynthesis and nuclear energy applications. In the actinide region, the fission barrier heights are comparable to the neutron separation energies and could be determined accurately from the measured fission excitation functions, which exhibit a characteristic rise in the barrier energy followed by a flat plateau. In the $A \sim 200$ mass region, fission barrier heights are much higher than the neutron separation energies. Most of the measurement of fission cross-sections in this mass region are performed at energies much higher than the fission barrier, where there are other open channels and a statistical description is essential.

Although a number of studies have been carried out, there are still ambiguities in choosing various input parameters for the statistical model analysis. According to the statistical model of compound nucleus (CN) decay, the probabilities of decay to different channels are governed by the transmission coefficient and relative density of states (phase-space). The nuclear level density depends on the level density parameter (a) related to the single-particle density near the Fermi surface and the available thermal energy (U). The ground-state shell corrections in the nuclei around the doubly closed shell nucleus

^{208}Pb ($Z = 82$, $N = 126$) are large and its damping with excitation energy has to be incorporated accurately in the statistical model analysis. The nuclear level density of a shell-closed nucleus shows the same energy dependence at high excitation energy (>40 MeV) as that of nuclei away from shell closure, if the excitation energy is measured from the liquid-drop surface, indicating the complete washing out of the shell corrections at those energies [1]. At intermediate energy, the dependence is phenomenologically described in terms of energy-dependent level density parameter approaching asymptotically to the liquid-drop value [2]. The phenomenological description of gradual damping of the ground-state shell correction with excitation energy was obtained by examining the density of neutron resonances situated near the neutron threshold (∼8 MeV). Recently, it has also been studied by measuring evaporation spectra in the ^{208}Pb region [3]. The knowledge about the shell corrections at the saddle point in $A \sim 200$ mass region is ambiguous.

The heavy-ion-induced fission excitation functions are not sensitive to the correlated variation of the fission barrier height and the ratio of the level density parameter at the saddle point to that at the equilibrium deformation ($\tilde{a}_f/\tilde{a}_n$) [4–7]. However, the pre-fission neutron multiplicity (ν_{pre}) data are sensitive to this correlated variation and hence it can be used to constrain the statistical model parameters. However, the measured ν_{pre} can have dynamical contribution, which should be taken care of. Analysis [7] of the fission and evaporation residue cross-sections along with pre-fission neutron multiplicity data for the ^{12}C+^{198}Pt system required large shell corrections at the saddle point and yielded fission barriers much smaller (13.4 MeV) than those (∼22 MeV) obtained for the same compound nuclei from the analysis of light-ion-induced reactions. In the analysis, the ν_{pre} data were corrected for a dynamical emission corresponding to the fission delay of 30×10^{-21} s.

In this article, we discuss the statistical model analysis for p+^{209}Bi, α+^{184}W, ^{206,208}Pb, ^{209}Bi systems using the same prescription for level density and fission barrier as in [7]. The experimental fission excitation functions are taken from [8–12].

2. Statistical model analysis

The statistical model analyses have been carried out using the code PACE [13] with a modified prescription for fission barrier and level density [7]. The fission barrier is expressed as

$$B_f(J) = c_f \times B_F^{\text{RFRM}}(J) - \Delta_n + \Delta_f, \tag{1}$$

where c_f, Δ_n and Δ_f are the scaling factors to the rotating finite-range model (RFRM) fission barrier [14], shell correction at the equilibrium and shell correction at the saddle deformation, respectively. The shell corrections at the equilibrium deformations are taken from [15]. Fermi gas level density formula was used to calculate the level densities at the equilibrium and saddle-point deformations. The excitation energy of the compound nucleus (CN) is calculated as

$$U_n = E_{\text{c.m.}} + Q - E_{\text{rot}}(J) - \delta_p, \tag{2}$$

where $E_{\text{c.m.}}$, Q, $E_{\text{rot}}(J)$ and δ_p are the energy in the centre-of-mass (c.m.), Q-value for fusion, rotational energy and pairing energy, respectively. The Q-value for fusion as well as the particle separation energies for subsequent decays are calculated using the experimental masses [16]. The excitation energy available at the saddle-point deformation is taken as $U_f = U_n - B_f(J)$. The damping of the shell correction with excitation energy is realized by assuming energy-dependent level density parameter as

$$a_x(U) = \tilde{a}_x[1 + (\Delta_x/U_x)(1 - e^{-\eta U_x})] \quad (3)$$

with $x = n$ and f corresponding to the equilibrium deformation and saddle-point deformation, respectively. The asymptotic liquid-drop value of the level density parameter at the equilibrium deformation is taken as $\tilde{a}_n = A/9$ MeV^{-1}. The asymptotic liquid-drop value of the level density parameter at the saddle-point deformation ($\tilde{a}_f$) may be different from that of $\tilde{a}_n$ due to the difference in nuclear shapes at these two cases. Hence, the ratio $\tilde{a}_f/\tilde{a}_n$ was kept as a free parameter to be decided by the fit to the data.

The spin distribution of CN is taken as

$$\sigma(\ell) = \frac{\pi(2\ell + 1)}{k^2[1 + \exp((\ell - \ell_{\max})/\delta\ell)]}. \quad (4)$$

The value of $\ell_{\max}$ and $\delta\ell$ are determined from the experimental fusion cross-section (σ_{fus}) and fission fragment angular anisotropy data. The sum of the xn cross-sections available in [17] was taken as fusion cross-section for the $p+^{209}$Bi system. For $\alpha+^{206}$Pb system, xn cross-sections are not available. However, xn cross-sections for the $\alpha+^{209}$Bi system are available [18]. Statistical model analysis of $\alpha+^{209}$Bi system considering fusion cross-section from the Bass systematics [19] reproduces the experimental xn cross-sections well. Hence, fusion cross-sections for α-induced reactions were estimated using the Bass systematics. The values of $\delta\ell = 2$ and 3 reproduce the experimental fission fragment angular anisotropy data for p- and α-induced reactions, respectively.

In figures 1 and 2, the statistical model predictions are compared with the experimental xn cross-sections for p- and α-induced reaction on ^{209}Bi target. Fusion process leads to bell-shaped excitation functions for xn channels and the presence of pre-equilibrium particle emission gives rise to high-energy tails to these distributions. A significant contribution from pre-equilibrium particle emission to the xn cross-sections will lead to

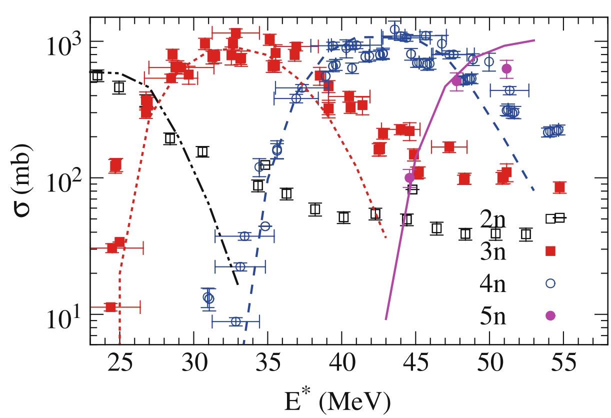

Figure 1. Statistical model predictions of xn cross-sections for the $p+^{209}$Bi system are compared with the experimental data.

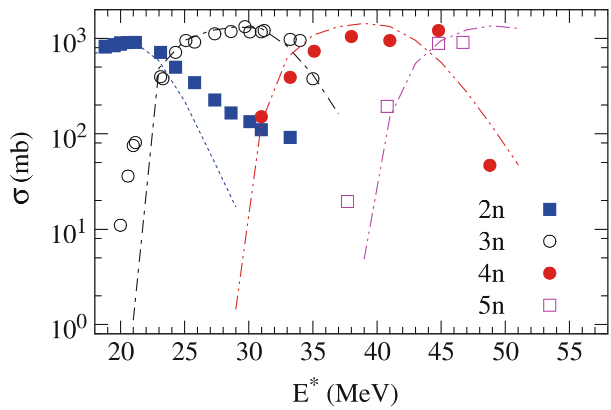

Figure 2. Same as figure 1 for $\alpha+^{209}$Bi system.

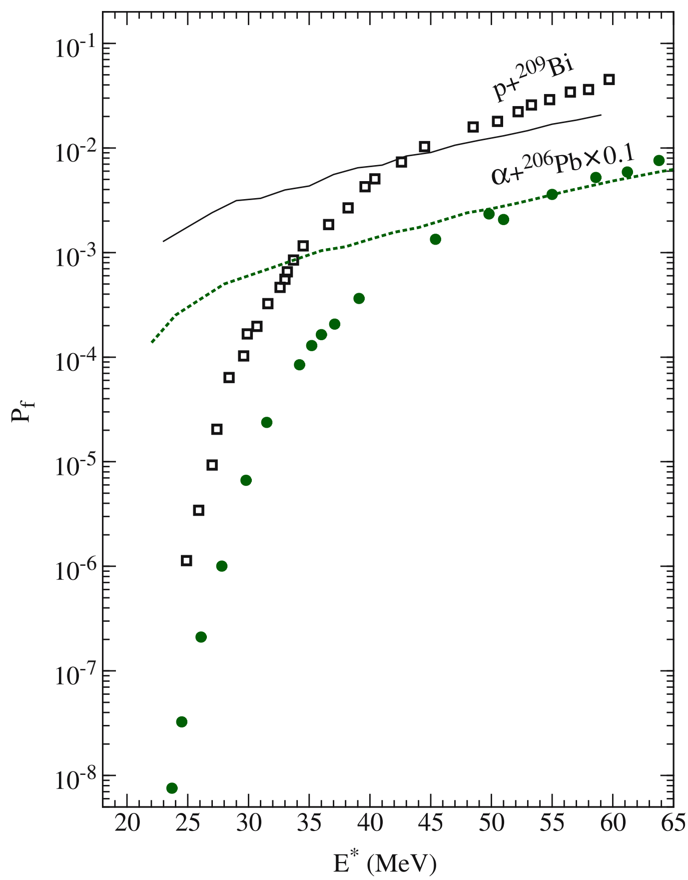

Figure 3. Experimental fission probabilities for $p+^{209}$Bi and $\alpha+^{206}$Pb systems are compared with the statistical model calculations using parameters from [7] ($B_f(0) =$ 13.4 MeV) obtained from the fit to the experimental ER and fission excitation functions along with the pre-fission neutron multiplicity data for the ^{12}C+^{198}Pt system.

overestimation of the fusion cross-section. As can be seen from the figures, the distribution of the xn cross-sections could be well reproduced, except for the high-energy tails in the experimental distributions. We have estimated the contribution of the pre-equilibrium emission to xn cross-sections from these high-energy tails. It was found that the pre-equilibrium contribution to the xn cross-sections is of the order of 10% for excitation energies below 40 MeV and it is of the order of 20% at 50 MeV for p-induced reaction. Fission following pre-equilibrium particle emission will be negligible because of the population of the target-like nuclei with much lower excitation energies than that of the CN. For the α-induced reaction, the pre-equilibrium contribution to xn cross-section is found to be insignificant in the energy range considered in the present analysis.

In figure 3, the experimental fission probabilities ($P_f = \sigma_{\text{fis}}/\sigma_{\text{fus}}$) for p+^{209}Bi and α+^{206}Pb systems are compared with the predictions of the statistical model using the parameters ($B_f = 13.4$ MeV) of [7], which fits the fission excitation functions and the ν_{pre} data for ^{12}C+^{198}Pt system simultaneously. The statistical model calculation using the parameters of [7] fails to reproduce the shape of the excitation functions. Attempts to fit the excitation functions by varying the values of Δ_f and $\tilde{a}_f/\tilde{a}_n$ result in $\Delta_f \sim 0.5$ MeV. As the resulted shell correction at the saddle point (Δ_f) is within the uncertainty of the RFRM prediction and the damping of shell corrections at the saddle point is also not well known, the value of Δ_f was assumed to be zero and the values of c_f and $\tilde{a}_f/\tilde{a}_n$ were varied to fit the excitation functions.

Initially, we attempted to fit the excitation functions using an energy-independent damping factor $\eta = 0.054$ MeV^{-1} [2] for damping of the shell correction at the equilibrium deformation. It could not fit the excitation functions for the entire excitation energy range

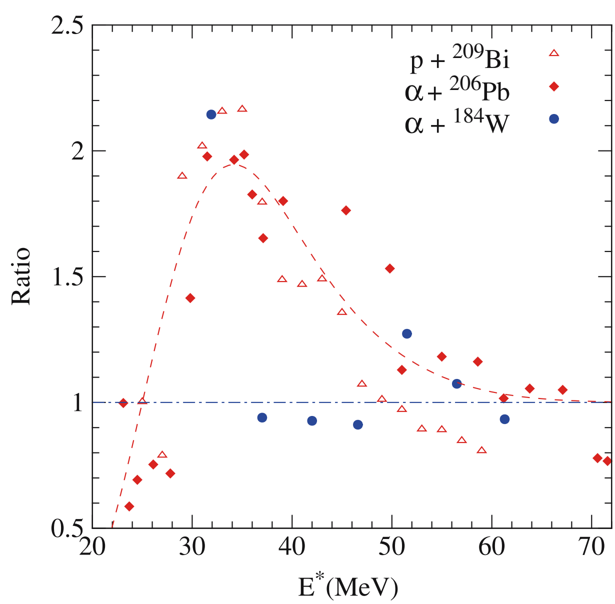

Figure 4. Ratio of the experimental fission probability to the statistical model prediction using energy-independent damping factor $\eta = 0.054$ MeV^{-1}. The lines are to guide the eye.

considered in the analysis. The ratio of the experimental fission probabilities to the predictions of the statistical model using energy-independent damping factor for $p+^{209}$Bi, $\alpha+^{184}$W, ^{206}Pb systems are shown in figure 4. While the ratio shows a systematic deviation

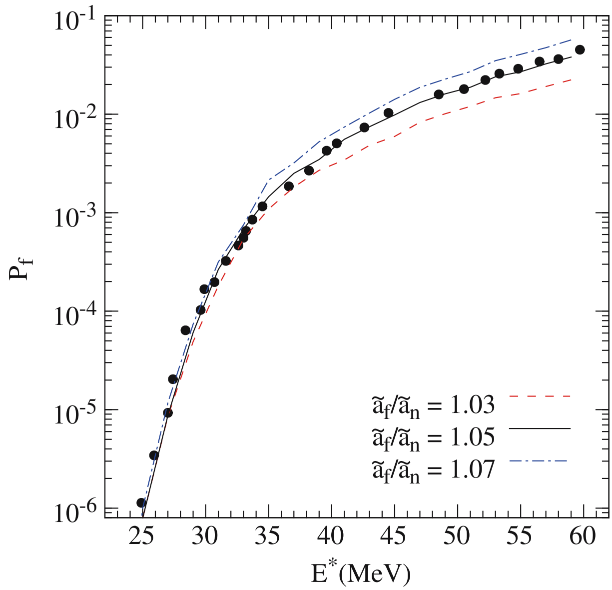

Figure 5. Statistical model predictions of fission probabilities for the $p+^{209}$Bi system with different values of $\tilde{a}_f/\tilde{a}_n$ for a fixed value of B_f.

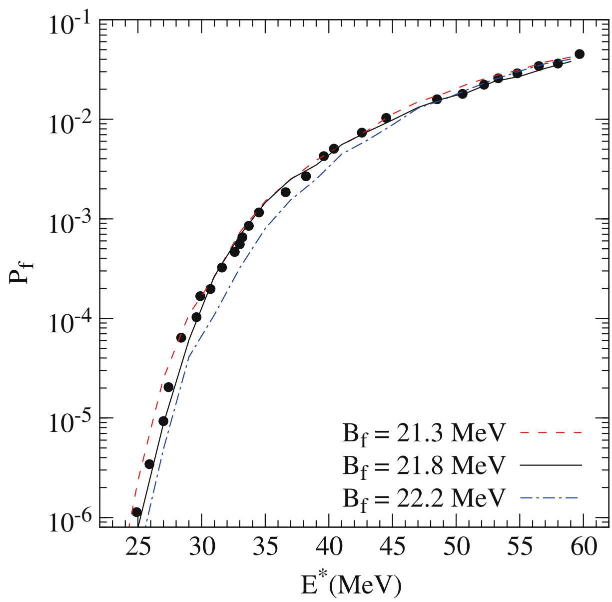

Figure 6. Statistical model predictions of fission probabilities with different values of B_f for the $p+^{209}$Bi system. The values of $\tilde{a}_f/\tilde{a}_n$ are varied to get the best-fit.

from unity for the $p+^{209}$Bi, $\alpha+^{206}$Pb ($\Delta_n = -10.6$ MeV) systems, no such deviation was observed for the $\alpha+^{184}$W ($\Delta_n = -1.9$ MeV) system. From this observation it was concluded that the deviation is due to the improper damping of the shell correction with excitation energy. An energy-dependent shell damping factor $\eta = 0.054 + 0.002 \times U_n$ is found to give better agreement with the experimental data.

We have also studied the sensitivity of the parameters to the excitation functions. Predictions of the statistical model with different values of $\tilde{a}_f/\tilde{a}_n$ for $B_f = 21.8$ MeV are compared with experimental fission probabilities for the $p+^{209}$Bi systems in figure 5. It is found that the lower part of the excitation function is not sensitive to the values of $\tilde{a}_f/\tilde{a}_n$. The predictions of the statistical model with different values of B_f for $p+^{209}$Bi system are shown in figure 6. For each value of fission barrier, the value of $\tilde{a}_f/\tilde{a}_n$ was varied to obtain the best-fit. The low-energy part of the fission excitation functions are found to be very sensitive to the variation of B_f. It should be mentioned here that pre-equilibrium particle emission will not have significant effect at the lower part of the excitation function and hence on the extracted fission barrier height.

The results of the statistical model calculations are shown in figure 7. Best-fit parameters are listed in table 1.

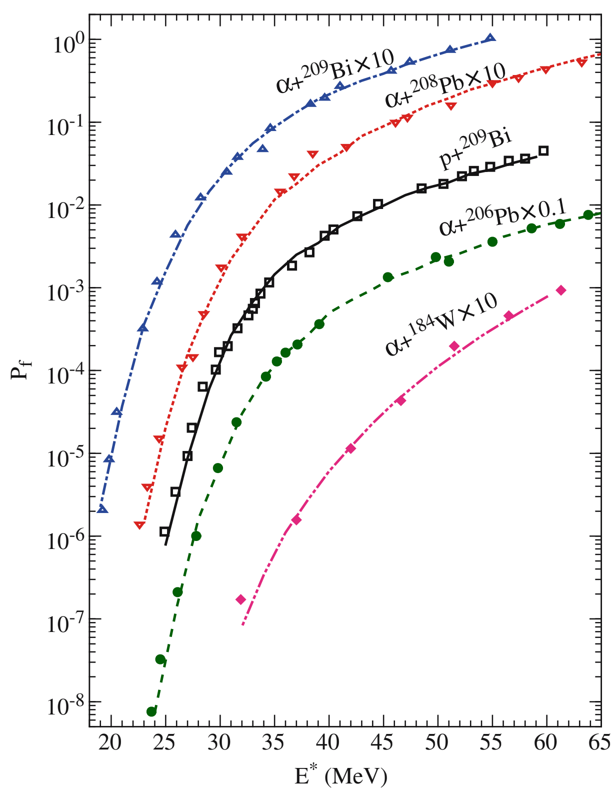

Figure 7. Experimental fission probabilities for p- and α-induced reactions are compared with the statistical model calculations using the parameters given in table 1.

Table 1. Relevant statistical model parameters corresponding to best-fits. Fission barriers extracted from the present analysis ($B_f^{\rm exp}(0)$) are also compared with the macroscopic–microscopic finite-range liquid-drop model (LDM) [20] fission barrier ($B_f^{\rm cal}$).

System	$\tilde{a}_f/\tilde{a}_n$	c_f	Δ_n (MeV)	$B_f^{\rm exp}(0)$ (MeV)	$B_f^{\rm cal}$ [20]
$\alpha+^{209}$Bi $\rightarrow^{213}$At	1.034	1.04	−7.51	17.8±0.2	18.56
$\alpha+^{208}$Pb $\rightarrow^{212}$Po	1.035	1.06	−8.45	20.3±0.3	20.27
$p+^{209}$Bi $\rightarrow^{210}$Po	1.042	1.04	−10.62	21.8±0.2	22.14
$\alpha+^{206}$Pb $\rightarrow^{210}$Po	1.051	1.07	−10.62	22.1±0.3	22.14
$\alpha+^{184}$W $\rightarrow^{188}$Os	1.082	1.18	−1.89	24.3±0.6	–

3. Summary and conclusion

We have carried out the statistical model calculations for p- and α-induced reactions to determine the fission barrier heights in the $A \sim 200$ mass region. Sensitivity of the relevant parameters was studied. While the low-energy part of the excitation functions was found to be sensitive to the height of the fission barrier, high-energy part of the excitation functions was found to be sensitive to the value of $\tilde{a}_f/\tilde{a}_n$. Effect of pre-equilibrium particle emission on the extracted fission barrier was estimated to be insignificant. The shell correction at the saddle point was also found to be insignificant. However, the statistical model calculation without shell correction at the saddle point substantially underpredicted the measured pre-fission neutron multiplicity data in heavy-ion fusion–fission reactions [7,21]. Further investigation is required to study the statistical nature of these pre-fission neutrons and contributions of other sources (e.g., dynamical emission and near-scission emission).

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