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Effects of resonator input power on Kerr lens mode-locked lasers

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Abstract. Using the ABCD matrix method, the common stability region between the sagittal and tangential planes of a four-mirror Kerr lens mode-locked (KLM) laser cavity is obtained for different ranges of input power. In addition, the effect of the input power on the Kerr lens sensitivity is investigated. Optimal input power and position for highest Kerr lens sensitivity in the stability region are presented and self-starting regime has been achieved. Results show that the resonator input power has a great influence on designing the KLM lasers which can be used in fabricating an optimal femtosecond laser.

Keywords. Femtosecond pulses; Kerr lens sensitivity; Kerr lens mode-locked laser.

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1. Introduction

Kerr lens mode-locking (KLM) has many applications in generating ultrashort pulses from a variety of solid-state lasers [1–6]. Applying this technique causes a phase-lock in the longitudinal modes of the laser [7]. In addition, due to the self-focussing effect, the transverse mode size is power-dependent and results in a loss or gain modulation. This is provided by either a proper intracavity aperture (hard-aperture KLM) [8] or the transverse gain profile of the active material (soft-aperture KLM) [9,10]. The first-order loss variation caused by an aperture, which is also called the Kerr lens sensitivity, is given by [7,11]

$$\delta = \left(\frac{1}{w} \frac{\mathrm{d}w}{\mathrm{d}(P/P_{\mathrm{cr}})} \right)_{P/P_{\mathrm{cr}}=0}. \tag{1}$$

In this equation, w is the spot size at the aperture plane and P_{cr} is the critical power for self-focussing. For a negative δ, an aperture leads to KLM because in this situation the losses decrease as the power increases [4,7,11]. So, to establish a KLM system, δ must be negative. Furthermore, it must be noted that the value of $|\delta| \geq 0.5$ is necessary for a reliable mode locking. In this case, initiating KLM usually needs a cavity perturbation [7]. Moreover, a sufficiently large $|\delta|$ leads to a self-starting KLM [12]. Therefore, a stable mode-locking regime depends on a careful adjustment of the resonator and properly controlling the focussing condition of the Kerr medium [13]. To the best of the authors' knowledge, a study considering the effects of the resonator input power in designing KLM laser has not yet been reported in the literature.

In this paper, the effects of the resonator input power on the resonator parameters such as the stability region and Kerr lens sensitivity are investigated. This input power can be adjusted by changing the transmission coefficient of the output mirror. This paper is organized as follows. In §2, the resonator configuration is presented and its astigmatism compensation method is described. In addition, the method of calculating the Kerr lens sensitivity is introduced. In §3, the common stability region between the sagittal and tangential planes is obtained for different ranges of input power. Also, the effect of input power on the Kerr lens sensitivity is studied and optimum values for the self-started KLM laser is calculated. Finally, conclusion is given in §4.

2. Theory

A four-mirror cavity is an usual astigmatism-compensated resonator for KLM laser [14]. In this resonator, the beam propagates over many rounds in continuous wave (cw) operation until a steady state is reached. Then, the nearest slit to the output mirror is progressively closed until pure KLM pulses, without any cw component in the spectrum, were achieved [12]. Therefore, calculations in this paper are carried out for the cw steady-state operation. This resonator configuration is shown in figure 1. In this configuration, slits S_1 and S_2 are placed close to the output (M_1) and the back (M_2) mirrors, respectively. Slit S_1 is in sagittal plane and provides the KLM. Slit S_2 is in tangential plane and avoids wavelength instabilities. Additionally, in order to simplify the article, a Gaussian beam is considered.

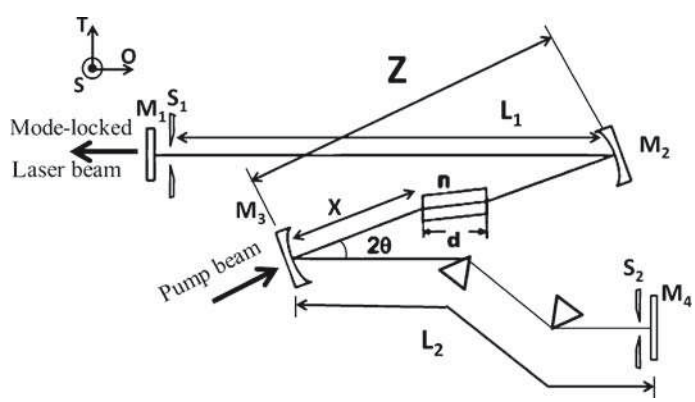

Figure 1. Resonator configuration for KLM laser: tilted mirrors M_2 and M_3 are focussing; output mirror M_1 and back mirror M_4 are flat; S_1 and S_2 are the slits; L_1 and L_2 are the arms. The Kerr medium is placed between the mirrors M_2 and M_3 in the central resonator.

2.1 *Astigmatism compensation method*

A Ti:sapphire rod is considered as the Kerr medium. This rod has also provided the required gain for KLM lasers [7,13]. In order to compensate the astigmatism of the Brewster-cut Ti:sapphire rod, the focussing mirrors are tilted at an angle θ [15,16]. Therefore, the focal lengths of these mirrors are different in sagittal and tangential planes, which are given by [17]

$$f_{\mathrm{S}} = \frac{f}{\cos(\theta)},$$
$$f_{\mathrm{T}} = f \times \cos(\theta). \tag{2}$$

The optical lengths of Kerr medium with thickness d and refractive index n under Brewster angle in the sagittal and tangential planes are [18]

$$d_{\mathrm{S}} = d\sqrt{n^2+1}/n^2,$$
$$d_{\mathrm{T}} = d\sqrt{n^2+1}/n^4. \tag{3}$$

To compensate for astigmatism, different optical lengths of eq. (3) have to correspond for the different focal lengths of eq. (2) [16]. Assuming the radius of curvature of tilted mirrors M_2 and M_3 are equal, $R_1 = R_2 = R = 2f$, leads to

$$d_{\mathrm{S}} - 2f_{\mathrm{S}} = d_{\mathrm{T}} - 2f_{\mathrm{T}}. \tag{4}$$

Substituting eqs (2) and (3) in eq. (4) gives a quadric equation for $\cos\theta$. Solving this equation leads to

$$\theta = \arccos\left[\sqrt{1+\left(\frac{Nd}{2R}\right)^2} - \frac{Nd}{2R}\right], \tag{5}$$

where

$$N = \sqrt{n^2+1}\,\frac{n^2-1}{n^4}.$$

Therefore, if the resonator of figure 1 is tilted to the angle given by eq. (5), the astigmatism is compensated [18].

2.2 *Transfer matrix of KLM lasers*

For high intensity, the nonlinear Kerr effect modifies the refractive index according to [19]

$$n = n_0 + n_{2E}|E|^2, \tag{6}$$

where E is the electric field, n_0 is the refractive index of the medium in the absence of the light field and n_{2E} is the Kerr constant. Under Brewster angle of incidence, one has to distinguish between the beam size of the sagittal and tangential planes according to

$$w_{\mathrm{S}} = w,$$
$$w_{\mathrm{T}} = w \times n_0, \tag{7}$$

where w is the half width at e^{-1} maximum intensity of incident beam. The transfer matrix of the Kerr medium is given as [20]

$$T_{\mathrm{K}}(z) = \begin{bmatrix} \cos(\gamma z) & \frac{1}{\gamma}\sin(\gamma z) \\ -\gamma\sin(\gamma z) & \cos(\gamma z) \end{bmatrix}, \tag{8}$$

where the parameter γ is [18]

$$\gamma = 2\sqrt{\frac{n_{2E}}{w_{\mathrm{S}} w_{\mathrm{T}} n_0}} E_0. \tag{9}$$

Self-focussing of laser radiation occurs if the focussing effect due to the intensity dependence of refractive index increases beyond the spreading of the beam by diffraction. For Gaussian beam, the critical power that leads to self-focussing is [7]

$$P_{\mathrm{cr}} = \frac{\pi c \varepsilon_0 \lambda_0^2}{64 n_{2E}}, \tag{10}$$

where c, ε_0 and λ_0 are the speed of light, electric susceptibility and the wavelength in vacuum, respectively. The self-focussing at the critical power of eq. (10) just compensates for diffraction spreading. But, if the laser input power significantly exceeds the critical power, the beam focussing due to Kerr effect can cause catastrophic self-focus within the medium. In this case, the beam breaks up to filaments which is called beam collapse power. For Gaussian beam, this power level is defined by [7]

$$P_{\mathrm{collapse}} = \frac{a c \varepsilon_0 \lambda_0^2}{16 \pi n_{2E}}, \tag{11}$$

where the factor a can take values between 3.77 and 4 depending on the environmental conditions [21]. The pair of prisms which are shown in figure 1, just compensate the dispersion. So, they do not cause a change in the system parameters used in this study. Therefore, as in [14], they can be neglected. In order to consider the difference between the focal lengths of tilted focussing mirrors in sagittal and tangential planes and the optical lengths of Brewster-cut Kerr medium in these two planes, the resonator is separated into two equivalent linear resonators corresponding to the sagittal and tangential planes, which is shown in figure 2.

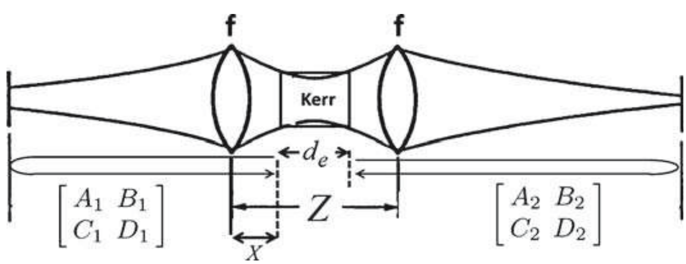

Figure 2. Linear resonator configuration of KLM laser. The matrices represent the propagation from the face of the Kerr rod to the corresponding end mirror and back to the face of the rod.

The equality of the arms ($L_1 = L_2$) and the radius of curvature of the focussing mirrors ($R_1 = R_2$) are generally essential for increasing the laser stability, which is considered in this paper. Due to the difference of the optical length of the Kerr medium in the tangential and sagittal planes, the optical length of the centre resonator is given as

$$Z_i = Z + d_i - d. \tag{12}$$

The subscript $i = \mathrm{S}, \mathrm{T}$ represents the components of sagittal and tangential planes. In addition, the transfer matrices of the lens and free propagation are

$$T_{\text{lens}}(f) = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$T_{\text{trans}}(z) = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}. \tag{13}$$

Therefore, the round trip ABCD matrix of the KLM laser is given by

$$\begin{aligned} T_{R.T.i} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} &= T_{\text{trans}}(L) T_{\text{lens}}(f_i) \\ &\times T_{\text{trans}}\left(\frac{Z_i - d_i}{2}\right) T_{\text{Kerr}.i}(d_i) T_{\text{trans}}\left(\frac{Z_i - d_i}{2}\right) \\ &\times T_{\text{lens}}(f_i) T_{\text{trans}}(2L) T_{\text{lens}}(f_i) \\ &\times T_{\text{trans}}\left(\frac{Z_i - d_i}{2}\right) T_{\text{Kerr}.i}(d_i) T_{\text{trans}}\left(\frac{Z_i - d_i}{2}\right) \\ &\times T_{\text{lens}}(f_i) T_{\text{trans}}(L), \end{aligned} \tag{14}$$

where f_i and d_i are obtained from eqs (2) and (3), respectively.

2.3 *Kerr lens sensitivity*

Due to the difficulty of calculating Kerr lens sensitivity at the plane of an end mirror M_1 by eq. (1), the parameters can be obtained by [11,12]

$$\delta_i = \frac{\alpha_1 + \alpha_2 S}{2(\alpha_1 + \alpha_2 + 2\alpha_1 \alpha_2 S)}, \tag{15}$$

where

$$S = \frac{A_0 + D_0}{2}$$

$$\alpha_j = B_j / d_e + A_j, \qquad j = 1, 2. \tag{16}$$

In these equations d_e is the equivalent length of the Kerr medium, A_j and B_j are the matrix elements defined in figure 2 and A_0 and D_0 are the elements of the round-trip matrix, which are defined in eq. (14). If $-1 < S < 1$, the resonator is optically stable. According to the introduction and eq. (1), KLM is obtained only when $\delta < 0$. Considering the slit S_1 oriented to cut the beam in sagittal plane, qualitatively for $|\delta_S| > 0.5$, KLM starts only with a small shock to the mirror M_1 and will be stable for several hours, whereas for smaller values of $|\delta_S|$, KLM needs to fine-tune the aperture size and is very sensitive to the abnormalities. On the other hand, for large values of $|\delta_S|$, KLM will be self-started.

3. Results and discussion

In this paper, parameters for KLM configuration, which is depicted in figure 1, are similar to those used in the experiment of [12]. These parameters are reported in table 1. Accordingly, using eqs (5), (10) and (11), the astigmatism compensation angle, the critical power and the collapse power are

$$\theta = 18.877^\circ,$$
$$P_{cr} = 0.334 \text{ MW},$$
$$P_{collapse} = 1.636 \text{ MW}. \quad (17)$$

Equation (17) indicates that the laser input power must be lower than $4.77 \times P_{cr}$ to prevent the catastrophic beam collapse. Using eqs (2), (3) and (17), the focal lengths of resonator focussing mirror and the optical length of Kerr medium are modified for sagittal and tangential planes, which are presented in table 2.

3.1 *The stability region*

Using the values in table 2, the round-trip matrix for sagittal and tangential planes is achieved from eq. (15). Using these matrix elements, the common stability region between the sagittal and tangential planes vs. the resonator length (Z) and the distance between mirror M_3 and the Ti:sapphire rod (X) for different values of the input power are obtained and illustrated in figure 3. Comparing this figure with the experimental values [12,22] reveals that although this figure covers the stability region of the articles, yet by increasing the laser power, a small part of the region becomes unstable. This unstable region is shown with white colour in figure 3.

3.2 *Evaluating the optimal KLM resonator*

The main quantity to enhance the KLM laser operation is the Kerr lens sensitivity, which is expressed in eq. (15). Therefore, in order to design an optimal resonator that best exploits

Table 1. The parameters for KLM configuration depicted in figure 1.

Parameters	Values
Central wavelength	$\lambda_0 = 800$ nm
Length of the arms	$L_1 = L_2 = 400$ mm
Radius of curvature of the focussing mirrors	$R_1 = R_2 = 2f = 100$ mm
Thickness of the Kerr medium	$d = 25$ mm
Refractive index of the Kerr medium	$n_0 = 1.76$
Kerr constant	$n_{2E} = 8.06 \times 10^{-23}$ m^2/V^2
Environmental factor of Ti:sapphire	$a = 3.8$

Table 2. The modified values of some parameters of table 1 for sagittal and tangential planes.

Parameters	Values
Radius of curvature of the focussing mirrors in sagittal plane	$f_S = 50.02$ mm
Radius of curvature of the focussing mirrors in tangential plane	$f_T = 49.98$ mm
Optical length of Kerr medium in sagittal plane	$d_S = 16.337$ mm
Optical length of Kerr medium in tangential plane	$d_T = 5.274$ mm
Optical length of central resonator in sagittal plane	$Z_S = Z + d_S - d$ $= Z - 8.663$ mm
Optical length of central resonator in tangential plane	$Z_T = Z + d_T - d$ $= Z - 19.726$ mm

the nonlinearity for KLM, the values of X, Z and input power that satisfy the following conditions must be found.

(1) The values must be located in the stability region.
(2) The Kerr lens sensitivity must be negative in both sagittal and tangential planes.
(3) According to the explanation in §2.3, the Kerr lens sensitivity in the sagittal plane, that includes slit S_1, should be minimized.

The optimal value, which is provided by the conditions of table 3 is $\delta_S = -18.54$.

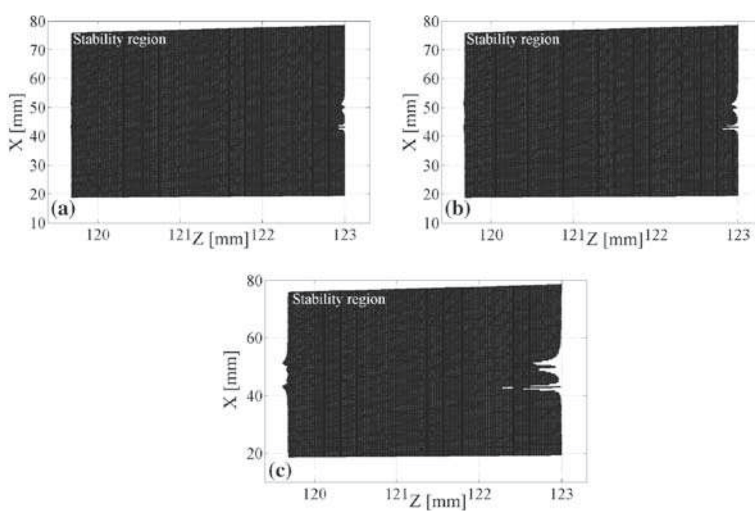

Figure 3. The black colour indicates the common stability region between the sagittal and tangential planes vs. the central resonator length (Z) and the rod position (X) for (**a**) $P = 0.5P_{cr}$, (**b**) $P = P_{cr}$ and (**c**) $P = 4P_{cr}$.

Table 3. The values for optimum KLM operation.

Parameters	Values
Central resonator length	$Z = 121.03$ mm
Rod position	$X = 48.02$ mm
Ratio of the input power to the critical power	$P/P_{cr} = 1.56$

3.3 *Effects of input power on Kerr lens sensitivity*

In order to study the effects of input power, the Kerr lens sensitivity for the optimum situation of table 3 vs. resonator input power is depicted in figure 4. In addition, comparing the optimum condition with a normal one, the Kerr lens sensitivity for the normal condition, which is described in table 4, is plotted in figure 5.

Figure 4 reveals that Kerr lens sensitivity in the optimum condition is strongly dependent on the resonator input power. In addition, this figure also shows that:

(1) By increasing the laser power upto $P = 1.56P_{cr}$, the absolute value of Kerr lens sensitivity ($|\delta|$) is largely increased.
(2) Note that this is applicable for beam power upto $P = 1.56P_{cr}$. At higher power the Kerr lens sensitivity becomes positive which is detrimental for the KLM process.

Therefore, in order to achieve the optimal KLM application, one has to increase the resonator power upto $P = 1.56P_{cr}$ but not more.

Figure 5 indicates that in non-optimum condition, the Kerr lens sensitivity is almost independent of input power. In addition, in this regime, $|\delta|$ is less than 1.5 for sagittal and tangential planes, which causes difficultly in initiating the KLM process. Furthermore, the maximum absolute value of the Kerr lens sensitivity reported in [12,22] is less than 2.5. This is true for a normal operation mode that is presented in figure 4, while in this study the amount of the quantity is significantly improved by optimizing the effect of resonator input power on the Kerr lens sensitivity.

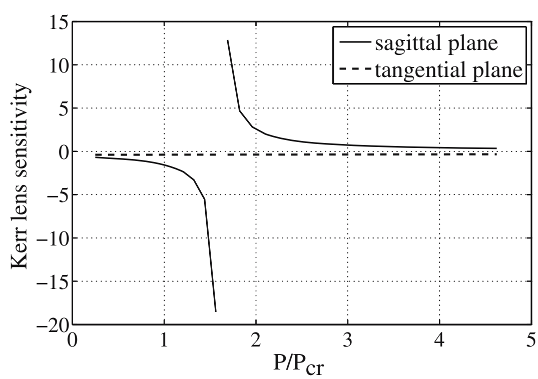

Figure 4. Kerr lens sensitivity vs. resonator input power in optimum operation.

Table 4. The values for normal KLM operation.

Parameters	Values
Central resonator length	$Z = 121.4$ mm
Rod position	$X = 57.8$ mm
The ratio of the input power to the critical power	$P/P_{cr} = 1.38$

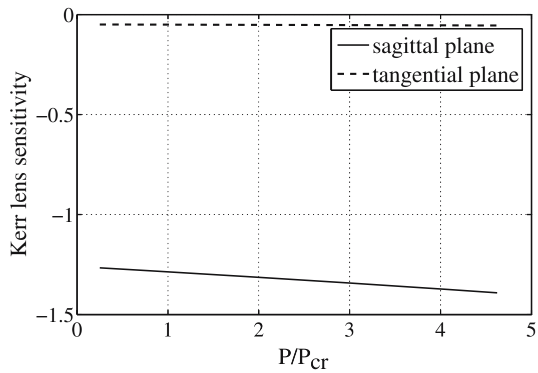

Figure 5. Kerr lens sensitivity vs. resonator input power for normal operation.

4. Conclusion

In conclusion, we have studied the effect of input power on the stability region and Kerr lens sensitivity, which are effective parameters for designing self-started KLM lasers. Considering a four-mirror optical system as one of the best resonators for KLM operation, the common stability region for sagittal and tangential planes was calculated. Also, the astigmatism was compensated by the tilted focussing mirror under the necessary astigmatism compensation angle. In addition, the following corrections were performed:

(1) As the resonator beam is not perpendicular to the focussing mirrors, the focal lengths of sagittal and tangential planes were calculated and adjusted.
(2) As the nonlinear Kerr medium is placed at the Brewster angle, its optical length would be different for sagittal and tangential planes. These differences were considered in this study.
(3) The effect of laser input power was also considered in this research.

Furthermore, calculating the stability region for various input powers indicates that by increasing the laser power, a small part of this region becomes unstable.

Resonator parameters and the proper input power in the stability region, which has negative Kerr lens sensitivity for sagittal and tangential planes, were selected. To design an optimal resonator, the absolute Kerr lens sensitivity in the sagittal plane of slit S_1 was maximized. The computed optimal values provide the possibility to significantly improve the amount of Kerr lens sensitivity so that the KLM laser was substantially developed.

Finally, the effects of input power on Kerr lens sensitivity for the optimum operation and a normal operation were investigated. It showed that although the Kerr lens sensitivity is strongly dependent on the input power for the optimum operation, in the non-optimum situation, it is almost independent of the input power and its absolute value is less than 1.5 in both sagittal and tangential planes. In addition, in optimum operation, by increasing the input power upto $P = 1.56P_{\rm cr}$, the Kerr lens sensitivity increases tremendously. However, for higher power the Kerr lens sensitivity becomes positive which is harmful for the KLM process. So, in order to achieve optimal system performance, one has to increase the resonator power close to $P = 1.56P_{\rm cr}$ but not more.

In this paper, the strong dependence of stability region and the Kerr lens sensitivity on the input power was expressed. Considering this issue provides the possibility to build reliable self-started KLM laser. Our results suggest some resonator design guidelines which are useful in manufacturing femtosecond lasers.

References

[1] D E Spence, P N Kean and W Sibbett, *Opt. Lett.* **16**, 42 (1991)
[2] P K Datta, C Basu, S Mukhopadhyay, S K Das, G K Samanta and A Agnesi, *Pramana – J. Phys.* **63**, 1003 (2004)
[3] T Sudmeyer, C Krankel, C R E Baer, O H Heckl, C J Saraceno, M Golling, R Peters, K Petermann, G Huber and U Keller, *Appl. Phys. B* **97**, 281 (2009)
[4] C R E Baer, C Krankel, C J Saraceno, O H Heckl, M Golling, R Peters, K Petermann, T Sudmeyer, G Huber and U Keller, *Opt. Lett.* **35**, 2302 (2010)
[5] W D Tan, F Chen, R J Knize, J Zhang, D Tang and L J Li, *Opt. Mater.* **33**, 679 (2011)
[6] A V Ramamurthi and K P J Reddy, *Pramana – J. Phys.* **52**, 19 (1999)
[7] W Koechner, *Solid-state laser engineering* (Springer, New York, 2005)
[8] T Brabec, P F Curley, Ch Spielmann, E Wintner and A J Schmidt, *J. Opt. Soc. Am. B* **10**, 1029 (1993)
[9] A A Lagatsky, C T A Brown and W Sibbett, *J. Opt. Express* **12**, 3928 (2004)
[10] J H Lin, W F Hsieh and H H Wub, *J. Opt. Commun.* **212**, 149 (2002)
[11] V Magni, G Cerullo and S D Silvestri, *J. Opt. Commun.* **101**, 365 (1993)
[12] G Cerullo, S D Silvestri, V Magni and L Pallaro, *Opt. Lett.* **19**, 807 (1994)
[13] U Keller, *Appl. Phys. B* **100**, 8 (2010)
[14] V L Kalashnikov, V P Kalosha, I G Poloyko and V P Mikhailov, *J. Opt. Soc. Am. B* **14**, 964 (1997)
[15] K H Lin, Y Lai and W F Hsieh, *J. Opt. Soc. Am. B* **12**, 468 (1995)
[16] V Magni, G Cerullo, S D Silvestri and A Monguzzi, *J. Opt. Soc. Am. B* **12**, 476 (1995)
[17] O Svelto, *Principle of laser* (Springer, New York, 1998)
[18] H Kogelnik, E P Ippen, A Dienes and C V Shank, *J. Quantum Electron* **8**, 373 (1972)
[19] R Boyd, *Nonlinear optics* (Academic Press, California, 2008)
[20] J T Verdeyen, *Laser electronics* (Prentice Hall, New Jersey, 1995)
[21] R E Bridges, R Boyd and G P Agrawal, *J. Opt. Lett.* **18**, 2026 (1993)
[22] G Cerullo, S Silvestri and V Magni, *J. Opt. Lett.* **19**, 1040 (1994)