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Planar dust-acoustic waves in electron–positron–ion–dust plasmas with dust-size distribution under higher-order transverse perturbations

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Abstract. Propagation of small but finite nonlinear dust-acoustic solitary waves are investigated in a planar unmagnetized dusty plasma, which consists of electrons, positrons, ions and negatively charged dust particles with different sizes and masses. A Kadomtsev–Petviashvili (KP) equation is obtained by using reductive perturbation method. The effect of positron density and positron–electron temperature ratio on dust-acoustic solitary structures are studied. Numerical results show that the increase in positron number density increases the amplitude of hump-like solitons but decreases the dip-like solitary waves. Furthermore, increase in the positron–electron temperature ratio results in the decrease of the amplitude of dip-like solitary waves. It seems that both the dip and hump-like solitary waves can exist in this system. Our results also suggest that the dust-size distribution has a significant role on the amplitude of the solitary waves.

Keywords. Electron–positron–ion–dust plasma; dust-acoustic wave; solitons; dust-size distribution.

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1. Introduction

At present, there has been a great deal of interest in understanding wave propagation in electron–positron–ion–dust plasma, which is believed to exist in spatial and laboratory environment, such as interstellar clouds, pulsar magnetospheres, active galactic nuclei, supernova environments as well as in laboratory experiments of cluster explosions by intense laser beams [1–6]. It is obvious that the characteristics of collective behaviour should be different from those in three-component electron–ion–dust plasma and electron–positron–ion plasmas. For example, the highly charged dust grains can increase the phase velocity and modify the dispersion relation for ion-acoustic waves in electron–ion plasma [7–9]. Recently, Ghosh and Bharuthram [10] studied ion-acoustic solitons and double layers in electron–positron–ion plasmas with dust particulates. The large-amplitude dust-acoustic solitary waves are investigated in electron–positron–ion plasma with dust grains by using Sagdeev potential approaches in ref. [11]. Jehan *et al* [12] analysed planar and nonplanar dust-acoustic solitary waves in electron–positron–ion–dust plasmas. In their investigation, the mass and size of the charged dust grains were taken to be constant. In fact, dust grains have different masses and sizes [13–17]. Results showed that the dust-size distribution satisfied power-law distribution in space plasma and Gaussian distribution in laboratory plasma [13–15,18]. Also, Ghosh and Bharuthram [10] and Jehan *et al* [12] studied the nonlinear waves in one-dimensional geometry. However, Xue [19] and Tarsem *et al* [20] proved that the observed wave phenomena in the low-altitude and higher-altitude auroral regions could not be explained by a purely one-dimensional picture. At least, some transverse perturbations would always exist in the higher-dimensional system. Therefore, in this work we use reductive perturbation method to study the small but finite nonlinear dust-acoustic waves in electron–positron–ion–dust plasmas, under higher-order transverse perturbations, in the presence of dust-size distribution. The paper is organized as follows. Basic equations for different species are given in §2. The nonlinear Kadomtsev-Petviashvili (KP) equation and its solitary wave solution are derived in §3. In §4, the continuous power law dust-size distribution is applied to our results. Numerical results are presented in §5. Section 6 gives a conclusion of the present work.

2. Governing equations

Here, we consider an unmagnetized dusty plasma containing negatively-charged dust grains, positrons, ions and Boltzmann-distributed electrons. The sizes of dust grains are assumed to be much smaller than the electron Debye length λ_{Dd} as well as the inter-grain distance. We assumed that there are N species of different dust grains with masses $m_{\mathrm{d}j}$ and radius $a_j (j = 1, 2, \ldots, N)$. The charge neutrality condition at equilibrium is $n_{\mathrm{i}0} + n_{\mathrm{p}0} - n_{\mathrm{e}0} - \sum_{j=1}^{N} Z_{\mathrm{d}0j} n_{\mathrm{d}0j} = 0$, where $n_{\mathrm{i}0}, n_{\mathrm{p}0}, n_{\mathrm{e}0}$ and $n_{\mathrm{d}0j}$ refer to the number densities of unperturbed ions, positrons, electrons and jth dust grain, respectively. $Z_{\mathrm{d}0j}$ is the unperturbed number of charges of the jth dust grain. The normalized equations for the dust grains are given by

$$\frac{\partial n_{\mathrm{d}j}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{d}j} u_{\mathrm{d}j}) + \frac{\partial}{\partial y}(n_{\mathrm{d}j} v_{\mathrm{d}j}) = 0, \tag{1}$$

$$\frac{\partial u_{\mathrm{d}j}}{\partial t} + u_{\mathrm{d}j}\frac{\partial u_{\mathrm{d}j}}{\partial x} + v_{\mathrm{d}j}\frac{\partial u_{\mathrm{d}j}}{\partial y} = \frac{1}{\alpha}\frac{Z_{\mathrm{d}j}}{m_{\mathrm{d}j}}\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial v_{\mathrm{d}j}}{\partial t} + u_{\mathrm{d}j}\frac{\partial v_{\mathrm{d}j}}{\partial x} + v_{\mathrm{d}j}\frac{\partial v_{\mathrm{d}j}}{\partial y} = \frac{1}{\alpha}\frac{Z_{\mathrm{d}j}}{m_{\mathrm{d}j}}\frac{\partial \phi}{\partial y}, \tag{3}$$

$$\frac{1}{\alpha}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = \sum_{j=1}^{N} n_{\mathrm{d}j} Z_{\mathrm{d}j} + n_{\mathrm{e}} - n_{\mathrm{p}} - n_{\mathrm{i}}, \tag{4}$$

$$n_{\mathrm{e}} = \mu_{\mathrm{e}} \exp(\phi), \tag{5}$$

$$n_{\rm p} = \mu_{\rm p} \exp(-\phi/\sigma_{\rm p}), \tag{6}$$

$$n_{\rm i} = \mu_{\rm i} \exp(-\phi/\sigma_{\rm i}). \tag{7}$$

Here $n_{{\rm d}j}$, $u_{{\rm d}j}$, $v_{{\rm d}j}$, $m_{{\rm d}j}$ and $Z_{{\rm d}j}$ refer to the perturbed number densities, velocity, mass and the charge of the jth dust grain; $n_{\rm e}$, $n_{\rm p}$ and $n_{\rm i}$ are the perturbed number densities of electrons, positrons and ions, respectively. The other physical parameters mentioned above are defined as follows:

$$\mu_{\rm e} = \frac{n_{\rm e0}}{\bar{Z}_{\rm d0} N_{\rm tot}}; \quad \mu_{\rm p} = \frac{n_{\rm p0}}{\bar{Z}_{\rm d0} N_{\rm tot}}; \quad \mu_{\rm i} = \frac{n_{\rm i0}}{\bar{Z}_{\rm d0} N_{\rm tot}};$$

$$\sigma_{\rm p} = \frac{T_{\rm p}}{T_{\rm e}}; \quad \sigma_{\rm i} = \frac{T_{\rm i}}{T_{\rm e}}; \tag{8}$$

$$\lambda_{\rm Dd} = \lambda_{\rm De}^{-2} + \lambda_{\rm Dp}^{-2} + \lambda_{\rm Di}^{-2} = \frac{\lambda_{\rm Da}}{\sqrt{\sigma}}; \quad \lambda_{\rm Da} = \sqrt{\frac{T_{\rm a}}{4\pi n_{\rm a0} e^2}}; \quad {\rm a} \equiv {\rm e, p, i}; \tag{9}$$

$$\sigma = 1 + \frac{\delta_{\rm i}}{\sigma_{\rm i}} + \frac{\delta_{\rm p}}{\sigma_{\rm p}}; \quad \delta_{\rm i} = \frac{n_{\rm i0}}{n_{\rm e0}}; \quad \delta_{\rm p} = \frac{n_{\rm p0}}{n_{\rm e0}};$$

$$\delta_{\rm d} = \frac{1}{\mu_{\rm e}}; \quad \alpha = \frac{\delta_{\rm d}}{\sigma}; \tag{10}$$

$$N_{\rm tot} = \sum_{j=1}^{N} n_{{\rm d0}j}; \quad \bar{Z}_{\rm d0} = \frac{\sum_{j=1}^{N} n_{{\rm d0}j} Z_{{\rm d0}j}}{N_{\rm tot}}; \quad \bar{m}_{\rm d} = \frac{\sum_{j=1}^{N} n_{{\rm d0}j} m_{{\rm d}j}}{N_{\rm tot}};$$

$$\bar{a} = \frac{\sum_{j=1}^{N} n_{{\rm d0}j} a_j}{N_{\rm tot}}. \tag{11}$$

Here $N_{\rm tot}$ is the total number density of all dust grains, $\bar{a}$ is the average radius of the dust, $\bar{m}_{\rm d}$ is the average mass of the dust grains, $\bar{Z}_{\rm d0}$ is the average unperturbed charge numbers residing on the dust grain. $T_{\rm d}$, $T_{\rm e}$, $T_{\rm p}$ and $T_{\rm i}$ are the temperature of the dust grains, electrons, positrons and ions, respectively. The normalized quantities are given as follows: $n_{{\rm d}j}$, $m_{{\rm d}j}$ and $Z_{{\rm d}j}$ are normalized by $N_{\rm tot}$, $\bar{m}_{\rm d}$ and $\bar{Z}_{\rm d0}$, respectively. Velocities $u_{{\rm d}j}$ and $v_{{\rm d}j}$ are normalized by the effective dust-acoustic speed $C_{\rm d} = \sqrt{\bar{Z}_{\rm d0} T_{\rm e} \alpha / \bar{m}_{\rm d}}$, space coordinate x and y are normalized by the effective Debye length $\lambda_{\rm Dd} = \lambda_{\rm De}/\sqrt{\sigma}$, electrostatic potential ϕ and time t are normalized by $T_{\rm e}/e$ and the inverse of effective dust plasma frequency $\omega_{\rm pd}^{-1} = (\bar{m}_{\rm d}/4\pi \bar{Z}_{\rm d0}^2 N_{\rm tot} e^2)^{1/2}$, respectively.

3. Derivation of the KP equation

To construct a weak nonlinear theory for the low-frequency dust-acoustic wave, the stretched space-time coordinates taken are $\xi = \epsilon(x - v_0 t)$, $\eta = \epsilon^2 y$ and $\tau = \epsilon^3 t$, where v_0 is the soliton phase velocity of the wave and ϵ is a small parameter measuring the weakness of the nonlinearity. The depended variables $n_{{\rm d}j}$, $u_{{\rm d}j}$, $v_{{\rm d}j}$ and ϕ are expanded as powers series of ϵ, as

$$n_{{\rm d}j} = n_{{\rm d0}j} + \epsilon^2 n_{{\rm d1}j} + \epsilon^4 n_{{\rm d2}j} + \cdots, \tag{12}$$

$$u_{\mathrm{d}j} = \epsilon^2 u_{\mathrm{d}1j} + \epsilon^4 u_{\mathrm{d}2j} + \cdots , \tag{13}$$

$$v_{\mathrm{d}j} = \epsilon^3 v_{\mathrm{d}1j} + \epsilon^5 v_{\mathrm{d}2j} + \cdots , \tag{14}$$

$$\phi = \epsilon^2 \phi_1 + \epsilon^4 \phi_2 + \cdots . \tag{15}$$

Substituting eqs (12)–(15) to eqs (1)–(7), the lowest-order in ϵ yields

$$n_{\mathrm{d}1j} = -\frac{n_{\mathrm{d}0j} Z_{\mathrm{d}j}}{\alpha v_0^2 m_{\mathrm{d}j}} \phi_1 \tag{16}$$

$$u_{\mathrm{d}1j} = -\frac{Z_{\mathrm{d}j}}{\alpha v_0 m_{\mathrm{d}j}} \phi_1 \tag{17}$$

$$v_0^2 = \sum_{j=1}^{N} \frac{n_{\mathrm{d}0j} Z_{\mathrm{d}j}^2}{m_{\mathrm{d}j}} \tag{18}$$

and

$$\frac{\partial v_{\mathrm{d}1j}}{\partial \xi} = -\frac{1}{\alpha v_0} \frac{Z_{\mathrm{d}j}}{m_{\mathrm{d}j}} \frac{\partial \phi_1}{\partial \eta}. \tag{19}$$

One can obtain the following equations at the next higher order in ϵ:

$$-v_0 \frac{\partial n_{\mathrm{d}2j}}{\partial \xi} + \frac{\partial n_{\mathrm{d}1j}}{\partial \tau} + \frac{\partial}{\partial \xi}(n_{\mathrm{d}0j} u_{\mathrm{d}2j}) + \frac{\partial}{\partial \xi}(n_{\mathrm{d}1j} u_{\mathrm{d}1j}) + \frac{\partial}{\partial \eta}(n_{\mathrm{d}0j} v_{\mathrm{d}1j}) = 0, \tag{20}$$

$$-v_0 \frac{\partial u_{\mathrm{d}2j}}{\partial \xi} + \frac{\partial u_{\mathrm{d}1j}}{\partial \tau} + u_{\mathrm{d}1j} \frac{\partial u_{\mathrm{d}1j}}{\partial \xi} = \frac{1}{\alpha} \frac{Z_{\mathrm{d}j}}{m_{\mathrm{d}j}} \frac{\partial \phi_2}{\partial \xi}, \tag{21}$$

$$-v_0 \frac{\partial v_{\mathrm{d}2j}}{\partial \xi} + \frac{\partial v_{\mathrm{d}1j}}{\partial \tau} + u_{\mathrm{d}1j} \frac{\partial v_{\mathrm{d}1j}}{\partial \xi} = \frac{1}{\alpha} \frac{Z_{\mathrm{d}j}}{m_{\mathrm{d}j}} \frac{\partial \phi_2}{\partial \eta}, \tag{22}$$

$$\frac{\partial^2 \phi_2}{\partial \xi^2} = \alpha \sum_{j=1}^{N} n_{\mathrm{d}2j} Z_{\mathrm{d}j} + \phi_2 + \frac{\alpha}{2} \left(1 - \frac{\mu_\mathrm{p}}{\sigma_\mathrm{p}^2} - \frac{\mu_\mathrm{i}}{\sigma_\mathrm{i}^2}\right) \phi_1^2. \tag{23}$$

Eliminating $n_{\mathrm{d}2j}$, $u_{\mathrm{d}2j}$, $v_{\mathrm{d}2j}$ and ϕ_2 using eqs (16)–(19), we get the KP equation as

$$\frac{\partial}{\partial \xi} \left(\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} \right) + C \frac{\partial^2 \phi_1^2}{\partial \eta^2} = 0, \tag{24}$$

where

$$A = -\frac{v_0}{2} \left[\alpha \left(\mu_\mathrm{e} - \frac{\mu_\mathrm{p}}{\sigma_\mathrm{p}^2} - \frac{\mu_\mathrm{i}}{\sigma_\mathrm{i}^2} \right) + \frac{3}{\alpha v_0^4} \sum_{j=1}^{N} \frac{n_{\mathrm{d}0j} Z_{\mathrm{d}j}^3}{m_{\mathrm{d}j}^2} \right], \quad B = \frac{v_0}{2}, \quad C = \frac{v_0}{2}. \tag{25}$$

The stationary solution of the KP equation is

$$\phi_1 = \phi_m \operatorname{sech}^2\left(\frac{\xi + \eta - U_0\tau}{\omega}\right). \tag{26}$$

Here $\phi_m = 3(U_0 - C)/A$ is the amplitude of the solitary wave and its width is $w = 2\sqrt{B/(U_0 - C)}$.

4. Continuous power-law dust-size distribution

In space plasmas, the dust size distribution is given by a power-law distribution. Now we consider a power-law distribution with radius r in a given range $a_{\min} < a < a_{\max}$. The expressed distribution function takes the form within the range:

$$n(a)\mathrm{d}a = Ka^{-\beta}\mathrm{d}a. \tag{27}$$

Here, K is a constant which can be derived using the equation $K = N_{\mathrm{tot}}^{-1}\int_{a_{\min}}^{a_{\max}} a^{-\beta}\mathrm{d}a$, β is the power-law index. $n(a) = 0$ when $a < a_{\min}$ or $a > a_{\max}$.

If the dust grain size $a < \lambda_{\mathrm{Dd}}$, the mass of the dust grain can be given as $m_{\mathrm{d}j} = k_m a_j^3$, where $k_m \approx \frac{4}{3}\pi\rho_{\mathrm{d}}$ (ρ_{d} is the mass density). The charge of the dust particles $Z_{\mathrm{d}j} = k_z a_j^l$ [15,18], $k_z \approx 4\pi\varepsilon_0 V/e$, k_m and k_z are approximately constants. l is a constant which depends on some dust parameters and other plasma condition, $l \geq 1$.

For power law of dust grain distribution

$$N_{\mathrm{tot}} = \int_{a_{\min}}^{a_{\max}} n(a)\mathrm{d}a \tag{28}$$

we can obtain the following equations:

$$N_{\mathrm{tot}} = \frac{Ka_{\min}^{1-\beta}}{1-\beta}(r^{1-\beta} - 1), \tag{29}$$

$$v_0^2 = \frac{Kk_z^2 a_{\min}^{2l-\beta-2}}{k_m(2l-\beta-2)}(r^{2l-\beta-2} - 1), \tag{30}$$

$$A = -\frac{Kk_z^3}{2\alpha v_0^3 k_m^2}\frac{a_{\min}^{3l-\beta-5}}{3l-\beta-5}(r^{3l-\beta-5} - 1) - \frac{\alpha v_0}{2}\left(\mu_{\mathrm{e}} - \frac{\mu_{\mathrm{p}}}{\sigma_{\mathrm{p}}^2} - \frac{\mu_{\mathrm{i}}}{\sigma_{\mathrm{i}}^2}\right), \tag{31}$$

where r is the ratio of radius of the dust grain with maximum size and minimum size, i.e., $r = a_{\max}/a_{\min}$.

5. Numerical results

In our numerical simulations, the system parameters selected based on the typical values of dust-laden plasmas in interstellar clouds are [17]: $n_{\mathrm{e}} = 10^{-3}$–10^{-4} cm^{-3}, $T_{\mathrm{e}} \approx 12$ K, $n_{\mathrm{d}} \approx 10^{-7}$cm^{-3}, $r_{\mathrm{d}} \approx 0.2$ μm and $n_{\mathrm{i}} \approx 10^{-4}$ cm^{-3}. Other parameters were not given, so these values are assumed. As $U_0 > 0$, $B > 0$ and $C > 0$, eq. (26) reveals that the type of solitary structure, whether dip or hump, depends on the sign of the coefficient A. The dip-like solitary waves can exist only when $A < 0$. Then we get

$$0 < \mu_{\rm p} < \frac{K k_z^3 \sigma_{\rm p}^2 a_{\rm min}^{3l-\beta-5}(r^{3l-\beta-5}-1)}{\alpha^2 v_0^4 k_m^2} + \sigma_{\rm p}^2\left(\mu_{\rm e} - \frac{\mu_{\rm i}}{\sigma_{\rm i}^2}\right) \tag{32}$$

or

$$\sigma_{\rm p} > \frac{\alpha^2 v_0^4 k_m^2 \mu_{\rm p}}{K k_z^3 a_{\rm min}^{3l-\beta-5}(r^{3l-\beta-5}-1) + \alpha^2 v_0^4 k_m^2 \sigma_{\rm p}^2(\mu_{\rm e} - \frac{\mu_{\rm i}}{\sigma_{\rm i}^2})}, \tag{33}$$

otherwise, one can get the hump-like waves as $A > 0$. Therefore, for a fixed electron density, there exists a small range of positron number density and its temperature, where the sign of coefficient A can change and become positive. The critical value of positron to electron temperature ratio $\sigma_{\rm pc}$ and positron density $\mu_{\rm pc}$ for $A = 0$ are shown in figure 1, where the other parameters are: $\sigma_{\rm i} = 0.2$, $\alpha = 1/13$, $\beta = 3$, $l = 1$, $a = 0.02$ and $r = 10$. One can find from figure 1 that if $\mu_{\rm p} > \mu_{\rm pc}$ or $\sigma_{\rm p} < \sigma_{\rm pc}$, the hump-like solitary waves can exist, otherwise, the dip-like solitary structures exist. For example, for $\mu_{\rm e} = 2.0$ and $\mu_{\rm p} = 1.0$, the formation of the hump soliton requires $\sigma_{\rm p} < \sigma_{\rm pc} \approx 0.71$, for $\mu_{\rm e} = 1.5$, if $\sigma_{\rm p} = 0.58$, the hump solitary waves exist only when $\mu_{\rm p} > \mu_{\rm pc} \approx 0.5$. Furthermore, the critical values of $\mu_{\rm pc}$ and $\sigma_{\rm pc}$ decreases with the increase in the electron density $\mu_{\rm e}$.

Figure 2 shows the variation of the amplitude of dip-like solitary waves ϕ_m vs. the minimum radius of dust grain $a_{\rm min}$ for different $\mu_{\rm p}$. We can observe from figure 2 that for a given $a_{\rm min}$, the amplitude of the dip-like solitary waves ϕ_m decreases with the increase in positron density $\mu_{\rm p}$. On the other hand, the amplitude of dip-like solitons increases with increase in $a_{\rm min}$.

The variations of the amplitude of dip-like solitary waves ϕ_m vs. r for different $\sigma_{\rm p}$ are shown in figure 3. It is clear that ϕ_m, the dip-like soliton amplitude decreases with the increase of $\sigma_{\rm p}$. Figure 3 also shows that the increase in r increases the amplitude of

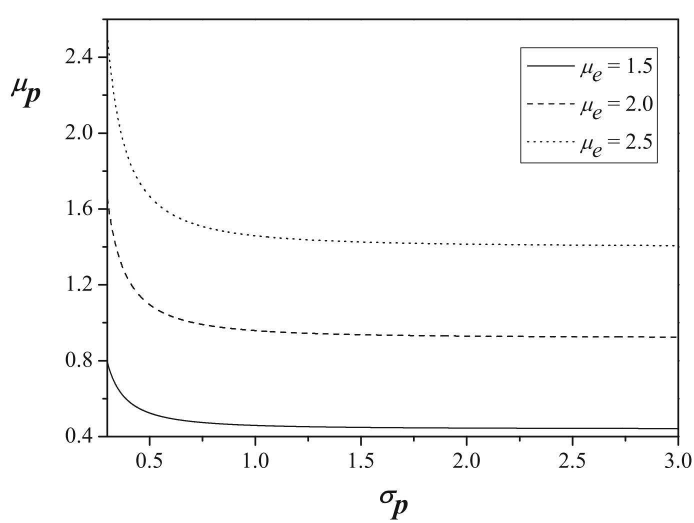

Figure 1. The coefficient $A = 0$ in the $(\mu_{\rm p}, \sigma_{\rm p})$ plane for $\mu_{\rm e} = 1.5, 2.0, 2.5$.

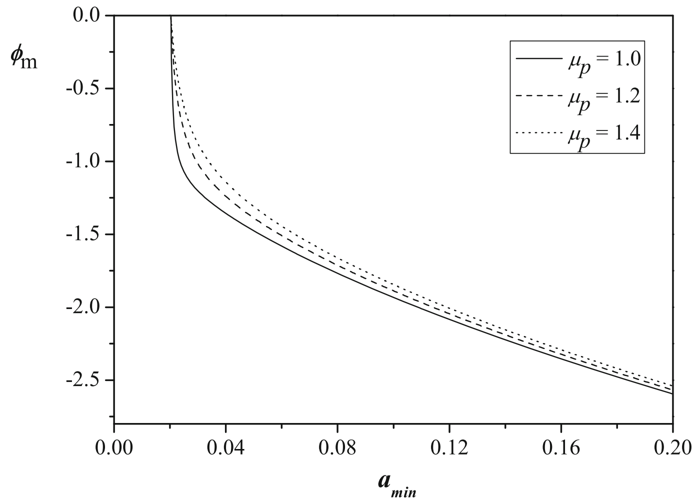

Figure 2. Variation of the amplitude of dip-like solitary waves ϕ_m vs. $a_{\min}$ for $\mu_\mathrm{p} =$ 1.0, 1.2, 1.4. Other parameters are the same as in figure 1.

the dip-like negative potential solitons ϕ_m in electron–positron–ion–dust plasma. We can conclude that if the difference between the maximum and minimum size is large enough, the dust-size distribution effect will increase the dip-like soliton amplitude significantly.

Figure 4 displays variation of the amplitude of hump-like solitary waves on the system parameters σ_i for different system parameters μ_p. It suggests that as σ_i increases, ϕ_m increases. Furthermore, ϕ_m increases with the increase of μ_p.

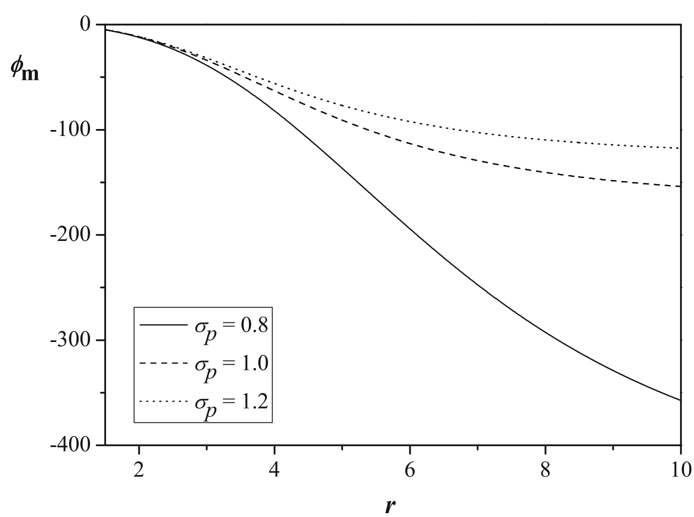

Figure 3. Variation of the amplitude of dip-like solitary waves ϕ_m vs. r for $\sigma_\mathrm{p} =$ 0.8, 1.0, 1.2. Other parameters are the same as in figure 1.

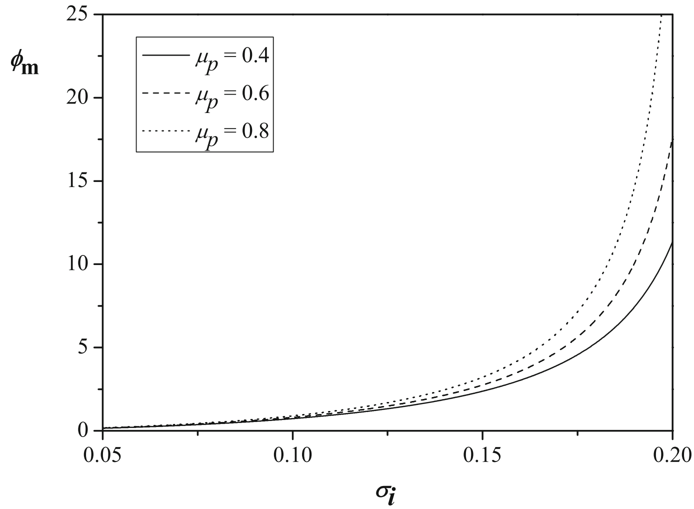

Figure 4. Variation of the amplitude of hump-like solitary waves ϕ_m vs. σ_i for $\mu_p =$ 0.4, 0.6, 0.8. Other parameters are the same as in figure 1.

6. Summary

In this paper, nonlinear dust-acoustic waves in a planar unmagnetized electron–positron–ion–dust plasma with different size grains which satisfied power-law distribution was investigated. The KP equation describing the nonlinear dust-acoustic waves under transverse perturbations was derived and the stationary solution was obtained by using the reductive perturbation method. The number density of positron and the ratio of positron–electron temperature on the soliton amplitude were studied. It is noted that both dip and hump-like solitary waves can exist. The increase in positron density increases the amplitude of hump-like solitons but decreases the amplitude of dip-like solitary waves. Furthermore, increase in the positron–electron temperature ratio results in the decrease of the amplitude of dip-like solitary waves and the amplitude of hump-like soliton increases with the increase in ion–electron temperature ratio. Our results also indicated that the dust-size distribution has a great impact on the amplitude of the solitary waves in electron–positron–ion–dust plasma.

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