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The influence of atomic coherence and dipole–dipole interaction on entanglement of two qubits with nondegenerate two-photon transitions

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Abstract. Considering two artificial identical atoms interacting with two-mode thermal field through non-degenerate two-photon transitions, this paper studies the influence of atomic coherence and dipole–dipole interaction on the entanglement of two qubits. It is found that the entanglement is greatly enhanced by these mechanisms.

Keywords. Entanglement; nondegenerate two-photon interaction; two-mode thermal field; dipole–dipole interaction; atomic coherence.

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1. Introduction

Entanglement is an essential feature of quantum mechanics. It is a major resource for many fundamental applications in quantum information science such as quantum teleportation, quantum dense coding, quantum cryptography, and quantum computing [1]. It is important to implement reliable methods to generate entangled states between systems. In order to function optimally, these applications require maximally entangled states. The interaction between the environment and quantum systems usually leads to the disappearance of quantum correlations. The interaction between the environment and quantum systems may result in decoherence. It also can induce entanglement [2]. Recently, Bose *et al* [3] have shown that entanglement can be generated by the interaction of an arbitrarily large system in any mixed state with a single qubit in pure state, and illustrated this using the Jaynes–Cummings interaction of a two-level atom in pure state with a field in thermal state at an arbitrarily high temperature. Kim *et al* [4] have investigated the atom–atom entanglement in a system of two identical two-level atoms with one-photon transition induced by a single-mode thermal field. They showed that a chaotic field can entangle atoms which were initially in a separable state. Zhou *et al* [5] have considered the same problem for nonidentical atoms with different couplings. Zhou *et al* [6] studied the entanglement between two identical two-level atoms through nonlinear two-photon interaction with one-mode thermal field. They showed that atom–atom entanglement induced by nonlinear interaction is more than the entanglement induced by linear interaction. Bashkirov [7] has discovered that two atoms can be entangled also through nonlinear nondegenerate two-photon interaction with two-mode thermal field. The influence of dipole–dipole interaction on entanglement between two qubits in incoherent states has been investigated in [8].

Hu *et al* [9,10] have shown that the problem of creating or controlling the entanglement is interlinked to the atomic coherence of population between different levels. They also have proved that the entanglement between two atoms induced by one-mode thermal field can be manipulated by changing the initial parameters of the atoms, such as the superposition coefficients and the relative phases of the initial atomic coherent states. Later, Hu and Fang [11] have explored the effect of atomic coherence on the entanglement of two atoms interacting with a two-mode thermal field through a nondegenerate two-photon process. They have found that for some atomic initial states, the entanglement induced by the nonlinear interaction may be larger than that induced by the linear interaction. They have also found that atomic coherence can dramatically enhance entanglement. But the influence of dipole–dipole interaction on entanglement has not been considered in [11].

We intend to generalize the results of Hu and Fang [11] by taking into account the direct dipole–dipole interaction between qubits. For natural two-level atoms, the processes of two-photon emission and absorption transition take place between two atomic levels with the same parity, while the matrix elements of dipole moment for atomic states with the same parity are equal to zero. To investigate the role of dipole–dipole interaction on entanglement behaviour we consider the Δ-type artificial atoms with three dipole-allowed transitions among the lowest three levels such as the superconducting flux qubits [12]. The dynamics of atom–field entangled states for such atoms have been investigated in [13].

2. Model and its exact dynamics

The model we consider here consists of two different cavities strongly coupled to a pair of identical Δ-type artificial atoms. Let us designate the lowest three level of atoms as $|e\rangle$, $|i\rangle$ and $|g\rangle$. Here $|e\rangle$ is the excited, $|g\rangle$ is the ground and $|i\rangle$ is the intermediate state in both qubits. Let us suppose that a cascade of the atomic transitions $|e\rangle \to |i\rangle \to g\rangle$ is resonant with the sum of field frequencies $\omega_{ge} = \omega_1 + \omega_2$, where the intermediate transition frequencies ω_{ei} and ω_{ig} are strongly detuned from ω_1 and ω_2, where ω_1 and ω_2 are cavity frequencies. After adiabatically eliminating the intermediate state as in [14], one arrives at the effective qubits-field interaction Hamiltonian

$$H_1 = \hbar g \sum_{i=1}^{2} (a_1^+ a_2^+ \sigma_i^- + \sigma_i^+ a_1 a_2),$$

where σ_i^+ and σ_i^- are the raising and the lowering operators for the ith qubit ($i = 1, 2$) and g is the effective atom–field coupling constant. The dipole–dipole coupling between the qubits is determined by an extra term in the Hamiltonian of a pseudospin system

$$H_2 = J(\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-),$$

where J is the strength of the dipole–dipole interaction. As a result, one can obtain the effective interaction Hamiltonian of the considered model in the following form:

$$H = \hbar g \sum_{i=1}^{2} (a_1^+ a_2^+ \sigma_i^- + \sigma_i^+ a_1 a_2) + J(\sigma_1^+ \sigma_2^- + \sigma_2^+ \sigma_1^-). \tag{1}$$

The two-atom wave function can be expressed as a combination of state vectors of the form $|v_1, v_2\rangle = |v_1||v_2\rangle$, where $v_1, v_2 = e, g$.

The density operator for the atom–field system follows a unitary time evolution generated by the evolution operator $U_I(t)$. On the two-atom basis, $|e, e\rangle,\ |e, g\rangle,\ |g, e\rangle,\ |g, g\rangle$, the analytical form of the evolution operator $U_I(t)$ is given by [15]

$$U_I(t) = \begin{pmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{31} & U_{32} & U_{33} & U_{34} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{pmatrix}, \tag{2}$$

where

$$U_{11} = 1 + 2a_1 a_2 \frac{A}{\lambda} a_1^+ a_2^+, \quad U_{14} = 2a_1 a_2 \frac{A}{\lambda} a_1 a_2, \quad U_{41} = 2a_1^+ a_2^+ \frac{A}{\lambda} a_1^+ a_2^+,$$

$$U_{44} = 1 + 2a_1^+ a_2^+ \frac{A}{\lambda} a_1 a_2, \quad U_{12} = U_{13} = a_1 a_2 \frac{B}{\theta}, \quad U_{21} = U_{31} = \frac{B}{\theta} a_1^+ a_2^+,$$

$$U_{24} = U_{34} = \frac{B}{\theta} a_1 a_2, \quad U_{42} = U_{43} = a_1^+ a_2^+ \frac{B}{\theta},$$

$$U_{22} = U_{33} = \frac{\exp\left[-\imath (g/2)(\alpha + \theta)t\right]}{4\theta} \left\{[1 - \exp(\imath g\theta t)]\alpha \right.$$
$$\left. + 2\theta \exp[\imath (g/2)(3\alpha + \theta)t] + \theta[1 + \exp(\imath g\theta t)]\right\},$$

$$U_{23} = U_{32} = \frac{\exp\left[-\imath (g/2)(\alpha + \theta)t\right]}{4\theta} \left\{[1 - \exp(\imath g\theta t)]\alpha \right.$$
$$\left. - 2\,\theta \exp[\imath (g/2)(3\alpha + \theta)t] + \theta[1 + \exp(\imath g\theta t)]\right\},$$

and

$$A = \exp\left[-\imath \frac{g\alpha}{2} t\right] \left\{\cos\left(\frac{g\theta}{2} t\right) + \imath \frac{\alpha}{\theta} \sin\left(\frac{g\theta}{2} t\right)\right\} - 1,$$

$$B = \exp\left[-\imath \frac{g}{2}(\alpha + \theta)t\right] \left[1 - \exp(\imath g\theta t)\right],$$

$$\alpha = \frac{J}{g}, \quad \lambda = 2(a_1 a_2 a_1^+ a_2^+ + a_1^+ a_2^+ a_1 a_2),$$

$$\theta = \sqrt{8(a_1 a_2 a_1^+ a_2^+ + a_1^+ a_2^+ a_1 a_2) + \alpha^2}.$$

The initial cavity mode state is assumed to be the thermal two-mode state

$$\rho_F(0) = \sum_{n_1} \sum_{n_2} p_1(n_1) p_2(n_2) |n_1, n_2\rangle\langle n_1, n_2|. \tag{3}$$

The weight functions are

$$p_i(n_i) = \frac{\bar{n}_i^{n_i}}{(1+\bar{n}_i)^{n_i+1}},$$

where $\bar{n}_i$ is the mean quasiparticle or photon number in the ith cavity mode, $\bar{n}_i = (\exp[\hbar\omega_i/k_B T] - 1]^{-1}$, k_B is the Boltzmann constant and T is the equilibrium cavity temperature.

We consider also the initial state of each artificial atom to be in a coherent superposition of the two levels, i.e.

$$|\Psi_1(0)\rangle = \cos\theta_1|+\rangle + e^{\imath\varphi_1}\sin\theta_1|-\rangle, \quad |\Psi_2(0)\rangle = \cos\theta_2|+\rangle + e^{\imath\varphi_2}\sin\theta_2|-\rangle. \tag{4}$$

Here θ_1 and θ_2 denote the amplitudes of the artificial polarized atoms, and φ_1 and φ_2 are relative phases, respectively.

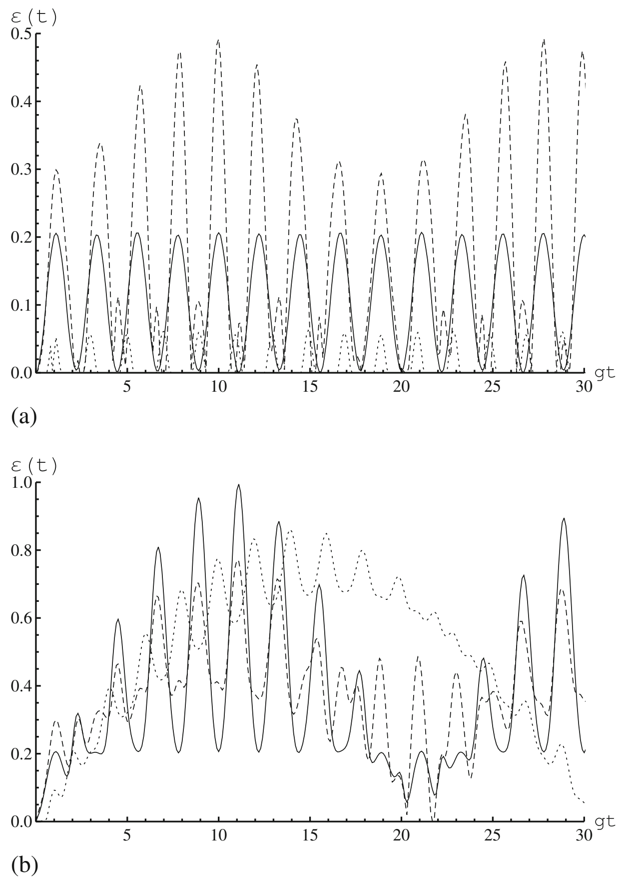

Figure 1. The negativity as a function of gt for the model with $\bar{n}_1 = \bar{n}_2 = 0.01$ and $\theta_1 = \pi/2, \theta_2 = 0$ (solid line), $\theta_1 = \pi/4, \theta_2 = \pi/4$ (dashed line), $\theta_1 = \pi/4, \theta_2 = -\pi/4$ (dotted line). The dipole strengths are (**a**) $\alpha = 0$ and (**b**) $\alpha = 0.1$. The phases of the atomic states are $\varphi_1 = \varphi_2 = 0$.

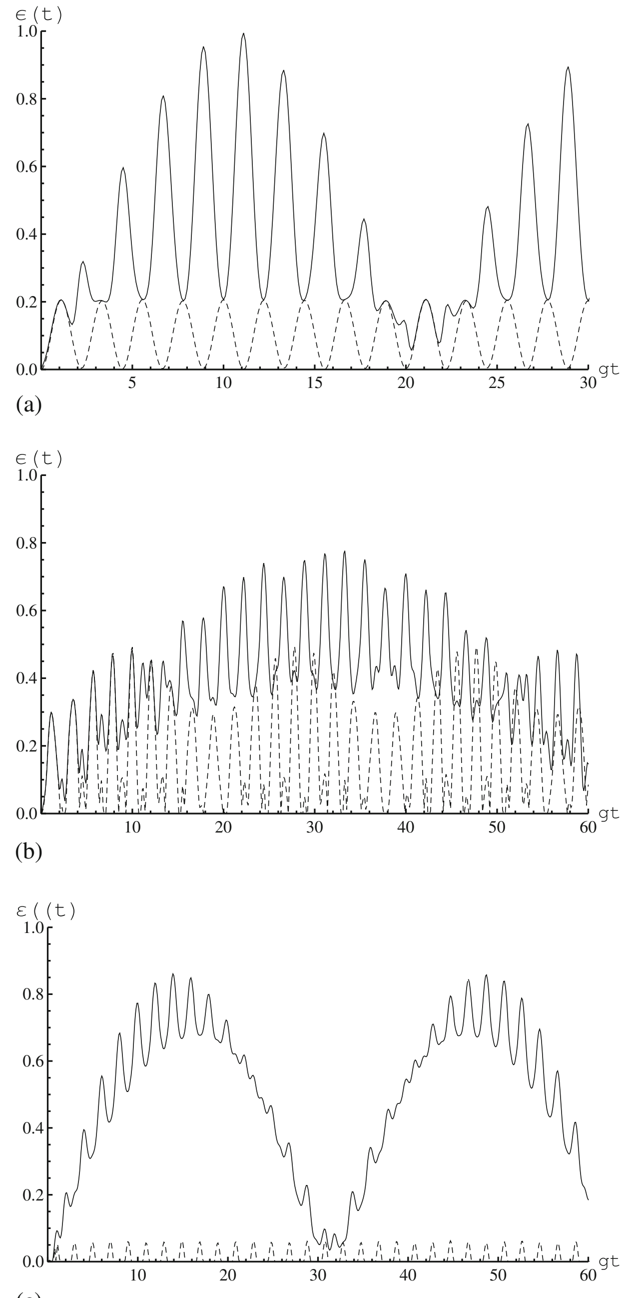

Figure 2. The negativity as a function of gt for the model with $\bar{n}_1 = \bar{n}_2 = 0.01$ and $\alpha = 0$ (dashed line), $\alpha = 0.1$ (solid line). The amplitudes of the artificial polarized atoms are (**a**) $\theta_1 = \pi/2,\ \theta_2 = 0$, (**b**) $\theta_1 = \pi/4,\ \theta_2 = \pi/4$ and (**c**) $\theta_1 = \pi/4,\ \theta_2 = -\pi/4$. The phases of the atomic states are $\varphi_1 = \varphi_2 = 0$.

To investigate the entanglement between qubits one has to obtain the time-dependent reduced density operator by tracing the combined atoms–field density operator over the field variables:

$$\rho_A(t) = \mathrm{Tr}_{\mathrm{F}} U(t)\rho_{\mathrm{F}}(0) \otimes \rho_A(0) U^+(t). \tag{5}$$

For two-qubit system described by the density operator $\rho_A(t)$, a measure of entanglement or negativity can be defined in terms of the negative eigenvalues μ_i^- of partial transpose of the reduced density matrix [16,17]

$$\varepsilon = -2 \sum \mu_i^-. \tag{6}$$

$\varepsilon = 0$ indicates that two qubits are separable, $\varepsilon > 0$ indicates the atom–atom entanglement and $\varepsilon = 1$ indicates maximum entanglement.

Using eqs (2)–(5) one can obtain the reduced density matrix at time t. Accordingly, we can write down the partial transpose matrix as

$$\rho_A(t)^{T_1} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13}^* & \rho_{23}^* \\ \rho_{12}^* & \rho_{22} & \rho_{14}^* & \rho_{24}^* \\ \rho_{13} & \rho_{14} & \rho_{33} & \rho_{34} \\ \rho_{23} & \rho_{24} & \rho_{34}^* & \rho_{44} \end{pmatrix}. \tag{7}$$

The evident expressions of the matrix elements are complex and so are not presented here.

3. Results

In figures 1–3 we plot the negativity (6) as a function of gt for small values of mean photon numbers in the cavity modes with $\bar{n}_1 = \bar{n}_2 = 0.01$. In figure 1 the time evolution of negativity is shown by the dashed ($\theta_1 = \pi/4, \theta_2 = \pi/4$) or the dotted ($\theta_1 = \pi/4, \theta_2 = -\pi/4$) lines for two artificial atoms initially in different coherent states and by the solid line for

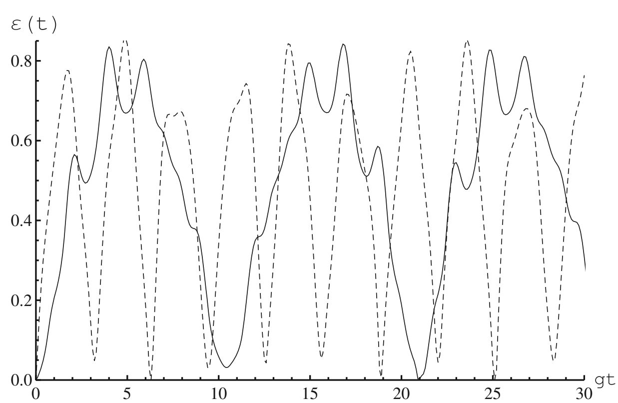

Figure 3. The negativity as a function of gt for the model with $\bar{n}_1 = \bar{n}_2 = 0.01$ and $\theta_1 = \pi/4,\ \theta_2 = \pi/4,\ \varphi_1 = \varphi_2 = 0$. The dipole strengths are $\alpha = 0.3$ (solid line) and $\alpha = 1$ (dashed line).

atoms initially in the incoherent state. In figure 1a we represent these curves without taking into account the dipole–dipole interaction. One can see that the atomic coherence may considerably enhance the entanglement. The curves plotted in figures 1a and 1b are clearly different. For incoherent atomic state the entanglement is greater than that for all coherent atomic states. In figure 2 we show the time dependence of negativity for different values of dipole strength. The dashed and solid curves correspond to the model with dipole strength $\alpha = 0$ and 0.1, respectively. The curves in figure 2a correspond to a case when artificial atoms are in incoherent initial states. The curves in figures 2b and 2c are plotted when qubits are initially in different coherent states. One can see that larger the dipole–dipole interaction between the atoms, larger is the degree of entanglement.

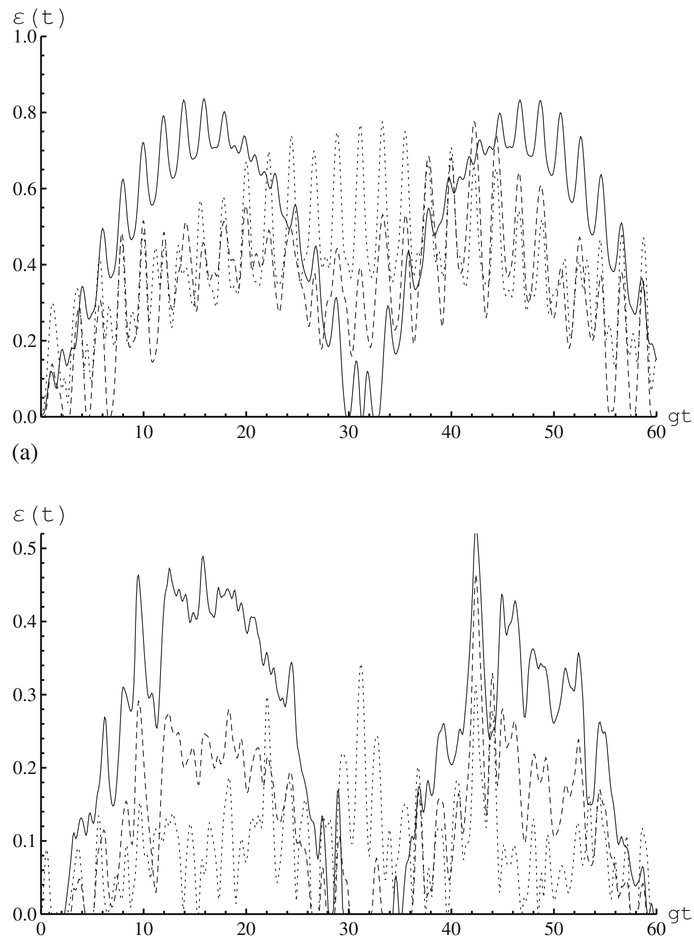

Figure 4. The negativity as a function of gt for the model in coherent atomic state with $\theta_1 = \pi/4$, $\theta_2 = \pi/4$ and $\Delta\varphi = 0$ (dotted line), $\Delta\varphi = 0.5\pi$ (dashed line), $\Delta\varphi = \pi$ (solid line). The mean photon numbers are (**a**) $\bar{n}_1 = \bar{n}_2 = 0.01$ and (**b**) $\bar{n}_1 = \bar{n}_2 = 1$. The dipole strength is $\alpha = 0.1$.

But such a behaviour takes place only for relatively small values of dipole strength α ($\alpha \leq 1$). In this case, the entanglement can reach values close to $\varepsilon = 1$. Figure 3 shows that dipole strength dependence of the entanglement has a nonmonotonic character for large α. Figure 4 displays the dependence of entanglement on the relative phase $\Delta\varphi = \varphi_2 - \varphi_1$ when $\theta_1 = \theta_2 = \pi/4$. The results indicate that the amount of entanglement dramatically depends on the relative phase of two initial atomic coherent states. Clearly, the entanglement may be stronger for appropriate values of relative phase $\Delta\varphi$.

In the case of flux superconducting qubits, the cavities (LC-superconducting circuits) are at tens of mK [12]. We have also investigated the entanglement behaviour for high cavity temperatures. Figure 5 shows the entanglement time dependence for intensive two-mode thermal field. If initial atomic state is incoherent, the value of atom–atom negativity dramatically decreases with increase in the mean photon numbers [5–13]. For the model with one-photon transitions, Hu *et al* [10] have shown that, for large intensities of the cavity fields the initial atomic coherence significantly enhances the atomic entanglement. The negativity calculations have confirmed this effect for the model with two-photon nondegenerate transitions. In figure 5 the evolution of negativity is plotted by the dashed line for two atoms initially in incoherent states and by the solid line for atoms initially prepared in the coherent states with $\theta_1 = \pi/4$, $\theta_2 = -\pi/4$ for large values of the mean photon numbers $\bar{n}_1 = \bar{n}_2 = 10$ when $\alpha = 0.1$. Negativity revealed by the solid line (coherent state) is much stronger than that revealed by the dashed line (incoherent state). Obviously, the dipole–dipole interaction can produce appreciable amount of entanglement for atoms in coherent states. Comparing curves for large mean photon numbers with those for one-photon interaction [10], we can clearly find that they are similar. The maximum degrees of entanglement are the same for both models when input intensities are equal. The numerical calculations on the basis of eqs (6) and (7) for large mean photon numbers demonstrate the lack of entanglement for atoms induced by two-mode thermal field for coherent states when the amplitudes of the polarized atoms differ noticeably from the values $\theta_1 = \pi/4$, $\theta_2 = -\pi/4$.

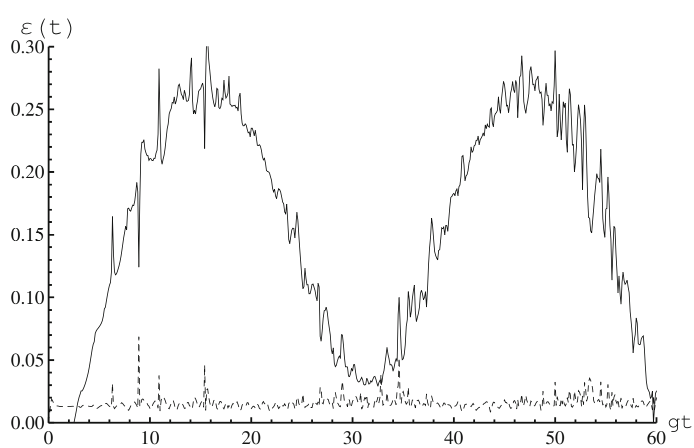

Figure 5. The negativity as a function of gt for the model with $\alpha = 0.1$ and $\theta_1 = \pi/4$, $\theta_2 = -\pi/4$ (solid line) and $\theta_1 = \pi/2$, $\theta_2 = 0$ (dashed line). The mean photon numbers are $\bar{n}_1 = \bar{n}_2 = 10$. The phases of the atomic states are $\varphi_1 = \varphi_2 = 0$.

Acknowledgements

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