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Half-width at half-maximum, full-width at half-maximum analysis for resolution of asymmetrically apodized optical systems with slit apertures

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Abstract. Resolution for the modified point spread function (PSF) of asymmetrically apodized optical systems has been analysed by a new parameter half-width at half-maximum (HWHM) in addition to the well-defined parameter full-width at half-maximum (FWHM). The distribution of half-maximum energy in the centroid of modified PSF has been investigated in terms of HWHM on good side and HWHM on bad side. We observed that as the asymmetry in PSF increases, FWHM of the main peak increases and then decreases and is being aided by the degree of amplitude apodization in the central region of slit functions. In the present study, HWHM (half-width at half-maximum) of the resultant PSF has been defined to characterize the resolution of the detection system. It is essentially a line of projection, which measures the width of the main lobe at its half-maximum position from the diffraction centre and has been computed for various amplitudes and antiphase apodizations of the slit aperture. We have noticed that HWHM on the good side decreases at the cost of the increased HWHM on the bad side in the presence of asymmetric apodization.

Keywords. Asymmetric apodization; resolution; half-width at half-maximum; full-width at half-maximum; point spread function.

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1. Introduction

Half-power diameter is an important parameter for measuring the width of an object in an image field which does not have sharp edges [1]. It is important when the image quality is limited by factors external to the optical system, like image movement or atmospheric turbulence. For large aberrations, it is very useful to evaluate PSF with non-zero minima, when well-established Rayleigh criterion is inapplicable [2]. Apodization is the process of deliberate modification in the feet of diffraction so as to improve the resolving power of the optical system [3]. Performing apodization in the focal region of an optical image forming system gives different effects, viz. improvement of encircled energy [4], resolving composite image of two object points in close proximity [5], suppress the optical side-lobes in the diffraction pattern resulting in steep central maxima [6], reduction of effects of aberrations on imaging of an optical system [7], etc. All these studies belong to symmetric apodization, in which the PSF is perfectly symmetric with respect to positions and intensities of the minima and maxima. The study of asymmetric apodization may yield new applications of apodization in image forming system, and similarly in microwave gratings and radar antenna arrays.

Asymmetric apodization is achieved by considering asymmetric aperture function in optical systems. An asymmetric PSF is obtained that divides into two sides: the ‘clean’ side, which has suppressed the side lobes and narrow central peak and the ‘messy’ side, which on the contrary has both fairly high side lobes and broad central peak. The idea of asymmetric apodization is introduced in 1991 by Cheng and Siu [8] and by continuing their work [9], they found that the asymmetric apodization is a relevant process in the field of diffraction. It results in the suppression of optical side lobes and narrowing the central peak which takes place simultaneously in the diffraction pattern. Apodization is ineffective in astronomical imaging, because the diffraction spots of stars or quasars due to the atmospheric turbulence are much larger than diffraction limits.

However, De and Hazra [10] presented methods of compensating the atmospheric turbulence. In such situations, the asymmetric apodization is useful. Marek Kowalczyk *et al* [11] demonstrated the effect of asymmetric apodization to achieve super-resolution in confocal scanning systems. Marek Kowalczyk *et al* [12] have designed an analytical model of phase-only pupil filters for asymmetric axial resolution of PSF. By employing asymmetric apodization, the performance of chirped fibre gratings improved significantly [13].

Based on our investigation on apodization, we have found that applying asymmetric apodization to aperture systems can be a solution, when we require high-performance apodizers or optical systems with high resolving power. In this context, we have designed complex one-dimensional pupil filter. Slit aperture images in optics has intrinsic significance and important practical imaging applications. Slit apertures are simple and widely useful in communication engineering and in astronomy. Employing asymmetric pupil functions in telescopic imaging systems may produce high resolution images of line objects. Employing asymmetric apodization technique across the proposed pupil function, yields significant improvement in the distribution of energy in the focal region to achieve instrumental line shape. The study of asymmetric apodization has found interesting applications in the fields of spectroscopy, astronomy and confocal microscopy.

In the present paper, following Cheng and Siu [8,9], we have made half-maximum analysis for improved resolution of asymmetrically apodized optical systems with slit apertures. Usually, resolution of an optical system greatly depends on aperture function and its transmittance. The proposed criterion HWHM is inferred from the configuration of the aperture function, which is useful to evaluate the resolution in the focal plane apodized optical systems, whereas FWHM is the most widely used resolution criterion to evaluate the resolving power of apodized systems and it is also applied to calculate the spectral width of sources used for optical communications and resolution of spectrometers. So, we can conclude that the well-defined Rayleigh criteria and FWHM are ineffective in evaluating the performance of asymmetrically apodized optical imaging systems. In such cases, HWHM can be considered as an image resolution criterion, used to measure an object in the image plane, where the diffraction field of the imaging system is asymmetrically distributed.

2. Theory and formulation

We have investigated HWHM as a newly introduced quality assessment parameter for PSF of asymmetrically apodized optical system, which carries non-zero minima for higher degree of apodization. HWHM is defined as the distance from the centre of diffraction to the point where the intensity of main peak becomes 50% of its peak value. We have discussed the widely studied image quantifying parameter FWHM, defined as twice the distance of the point from the centre of diffraction pattern where intensity of PSF becomes one-half of the maximum value. It is the direct PSF image criterion used to evaluate apodized optical system which is suffering from large aberrations, when the famous Rayleigh criterion is not applicable. In scalar diffraction optics, impulse response of one-dimensional optical imaging systems is the Fourier transform of the complex pupil function consisting of three zones, viz. two narrow strips at the edges with opposite phase transmittances of the form $\exp(-i\pi/2)$ and $\exp(i\pi/2)$ and a central zone with Straubel amplitude apodizer, where β is the amplitude of the apodization controlling parameter which determines transmittance function $(1-\beta r^2)$ over the specified region $(1-b)$ of the slit aperture. Here, b is the width of the edge strip. The range of values β takes are $0 \leq \beta \leq 1$. It is clear that for $\beta = 0$, the transmittance of this zone is uniform. The analytical model of the one-dimensional asymmetric slit pupil function is given in figure 1 in detail.

$A_0(z)$ is the diffraction amplitude contributed by the central cylindrical region. $A_1(z)$, $A_2(z)$ are diffraction amplitudes contributed by left- and right-edge strips, respectively. The diffraction field $A(z)$ on the image plane can be written as

$$A(z) = A_0(z) + A_1(z) + A_2(z), \tag{1}$$

$$A(z) = \int_{-1/2+b}^{1/2-b} (1-\beta r^2)\exp(i2zr)\mathrm{d}r + \int_{-1/2}^{1/2+b} -i\exp(i2zr)\mathrm{d}r + \int_{1/2-b}^{1/2} i\exp(i2zr)\mathrm{d}r, \tag{2}$$

$$I(z) = |A(z)|^2 . \tag{3}$$

The intensity $I(z)$ of the asymmetric PSF is a measurable quantity, used to evaluate image-quality criterions such as HWHM, FWHM for one-dimensional optical imaging system with asymmetric slit aperture of amplitude and phase type.

Studies done on spectral resolution revealed that FWHM studies are helpful to analyse image resolution of symmetrically apodized optical systems. But this criterion is not unique for evaluating the performance of asymmetrically apodized optical systems. So the current study on HWHM of PSF is a breakthrough in the field of diffraction optics.

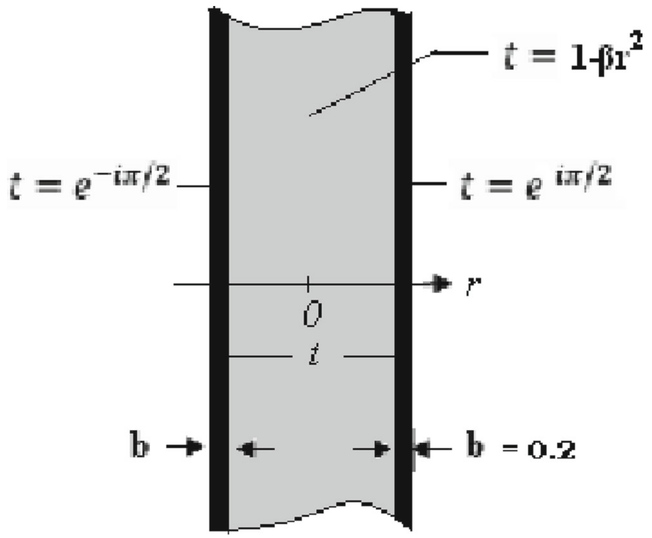

Figure 1. Design of one-dimensional asymmetric slit pupil function.

3. Results and discussion

The results from the investigations on the HWHM and its associated image quality parameters used to evaluate resolution of asymmetrically apodized optical imaging systems with slit aperture have been studied analytically from eq. (3) as functions of optical coordinate z varying from -12 to $+12$ by employing twelve-point Gauss quadrature method of numerical integration. It is an extremely useful and accurate method developed and applied to obtain the proposed image quality criterion of asymmetric PSF apodized by one-dimensional phase and amplitude filters.

Table 1 lists FWHM of asymmetric PSF as a function of edge strip width (b) for various values of amplitude apodization control parameter (β) depicted in figure 2. These values are obtained for unapodized case ($b = 0$) and asymmetrically apodized case with

Table 1. FWHM of asymmetric PSF for all values of β and b.

b	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1$
0	2.783	2.8032	2.8246	2.8472	2.8712	2.8967
0.02	2.8872	2.9058	2.9253	2.9458	2.9674	2.9902
0.04	2.9687	2.9839	2.9998	3.0163	3.0334	3.0512
0.06	3.0159	3.0264	3.0372	3.0482	3.0595	3.0711
0.08	3.0239	3.0293	3.0348	3.0402	3.0457	3.0512
0.1	2.9977	2.999	3.0001	3.0013	3.0023	3.0032
0.12	2.9508	2.9494	2.9479	2.9463	2.9447	2.9430
0.14	2.8972	2.8945	2.8918	2.889	2.8863	2.8835
0.16	2.8471	2.844	2.841	2.8381	2.835	2.8320
0.18	2.8056	2.8028	2.7999	2.7971	2.7942	2.7914
0.2	2.9579	2.966	2.9741	2.9823	2.9905	2.9987

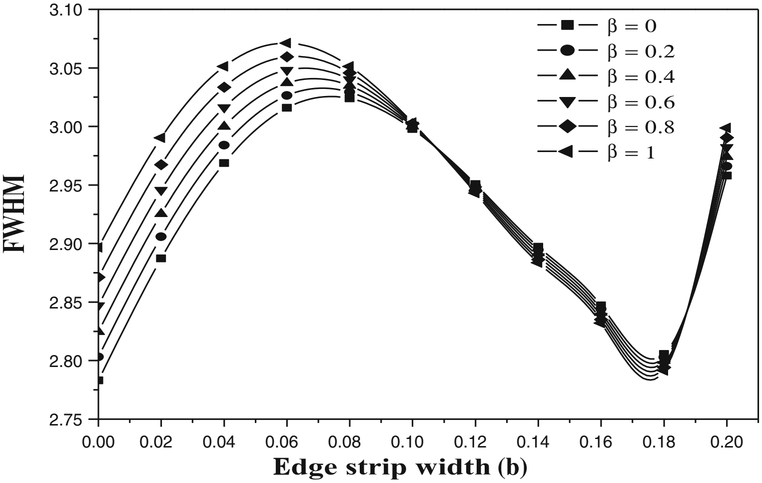

Figure 2. FWHM as a function of edge strip width b for different values of β.

transparent central region ($b \neq 0$). It shows that as edge strip width b increases from 0 to 0.08, the FWHM initially increases and then decreases with edge strip width. Further, the decrease in FWHM is achieved by increasing amplitude apodization parameter. For transparent central region ($\beta = 0$), as b increases from 0 to 0.08 in steps of 0.02, FWHM intially increases from 2.783 to 3.0239 and as b further increases from 0.1 to 0.18, FWHM decreases from 2.9977 to 2.8056. The PSFs are graphically presented in figure 3 for

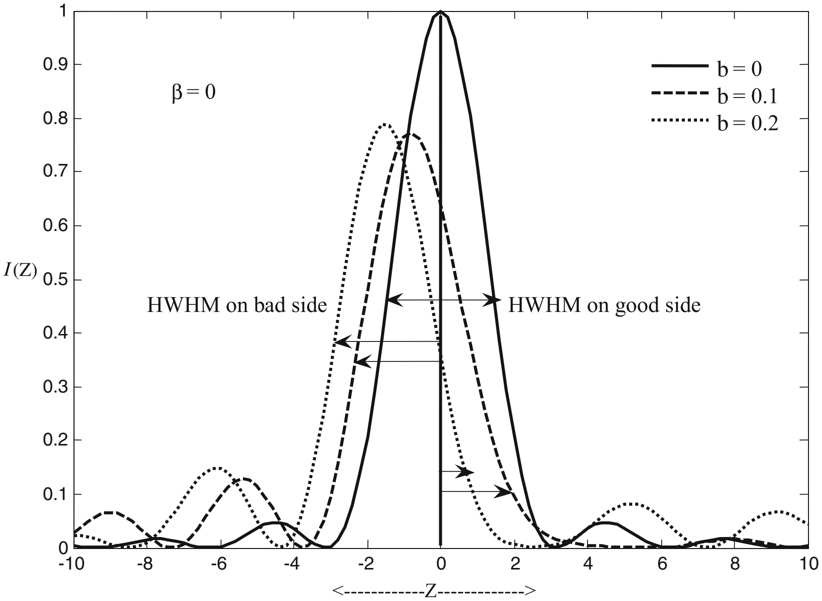

Figure 3. Shift in PSF curves given as a shift in half-width at half-maximum area for different values of edge strip width, b.

Table 2. HWHM on the good side of asymmetric PSF for all values of β and b.

b	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1$
0	1.3915	1.4016	1.4123	1.4236	1.4356	1.4483
0.02	1.3311	1.3376	1.3445	1.3517	1.3593	1.3673
0.04	1.2412	1.2435	1.2459	1.2484	1.2509	1.2535
0.06	1.117	1.1148	1.1125	1.11	1.107	1.1045
0.08	0.9591	0.953	0.9466	0.9399	0.9329	0.9257
0.1	0.7763	0.7676	0.7586	0.7494	0.7399	0.7301
0.12	0.583	0.5734	0.5636	0.5536	0.5434	0.5330
0.14	0.393	0.3838	0.3745	0.365	0.3555	0.3459
0.16	0.2152	0.2071	0.199	0.1909	0.1826	0.1743
0.18	0.0535	0.0468	0.0401	0.0334	0.0266	0.0198
0.2	−0.0916	−0.097	−0.1023	−0.1076	−0.113	−0.1184

different b values when the central region of the slit aperture is unapodized. Figure 3 clearly depicts that, on the left half axis the central peak is broadened and shifted, while on the right axis the central peak is narrower. However, the magnitude of this effect depends on the edge strip width b and for $b = 0.2$, this effect increases relative to unapodized case. As the figure reveals, the asymmetry in PSF does not depend on the degree of amplitude of apodization in the central region of pupil function, whereas for $b = 0.2$, the dark region occurs very close to the diffraction head and occupies certain distance. This is the usual effect of super-resolver, promoted by degree of amplitude apodization in the central region of the slit.

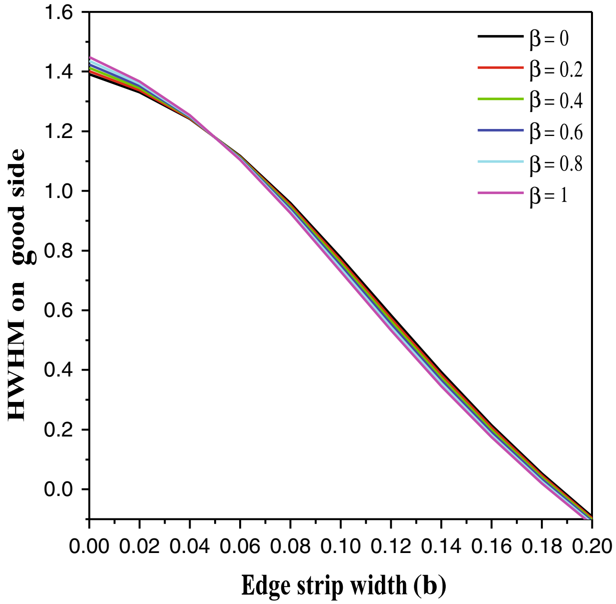

Figure 4. HWHM on the good side of the main peak.

Table 3. HWHM on the bad side of the asymmetric PSF for all values of β and b.

b	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1$
0	−1.3915	−1.4016	−1.4123	−1.4236	−1.4356	−1.4483
0.02	−1.5561	−1.5682	−1.5808	−1.5941	−1.6081	−1.6228
0.04	−1.7275	−1.7404	−1.7539	−1.7679	−1.7825	−1.7977
0.06	−1.8989	−1.9116	−1.9247	−1.9382	−1.9522	−1.9666
0.08	−2.0648	−2.0763	−2.0882	−2.1003	−2.1128	−2.1255
0.1	−2.2214	−2.2314	−2.2415	−2.2519	−2.2624	−2.2731
0.12	−2.3678	−2.376	−2.3843	−2.3927	−2.4013	−2.4100
0.14	−2.5042	−2.5107	−2.5173	−2.524	−2.5308	−2.5376
0.16	−2.6319	−2.6369	−2.642	−2.6472	−2.6524	−2.6576
0.18	−2.7521	−2.756	−2.7598	−2.7637	−2.7676	−2.7715
0.2	−2.8662	−2.869	−2.8718	−2.8746	−2.8774	−2.8803

The values listed in table 2 and figure 4 reveal that HWHM on the good side of the asymmetric PSF is a function of degree of amplitude apodization and edge strip width b. As edge strip width b increases from 0 to 0.2 in steps of 0.02, the distribution of half-maximum area on the good side from the diffraction centre decreases with degree of amplitude apodization in the central region of the slit aperture. For large β, HWHM on the good side are found to be narrow for all values of b. This is an important aspect of our study whereas HWHM on the good side is found narrower at the cost of broad HWHM on the bad side. The values listed in table 3 and figure 5 demonstrate that as b increases from 0 to 0.2, HWHM on the bad side gets broader for various values of β. This study

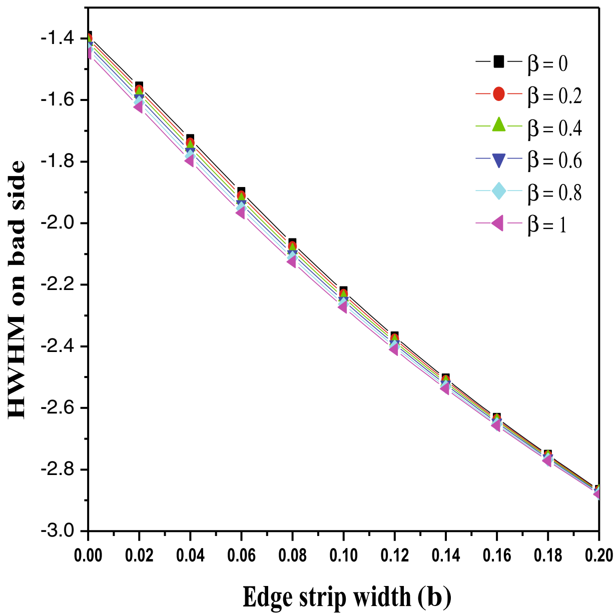

Figure 5. HWHM on the bad side of the main peak.

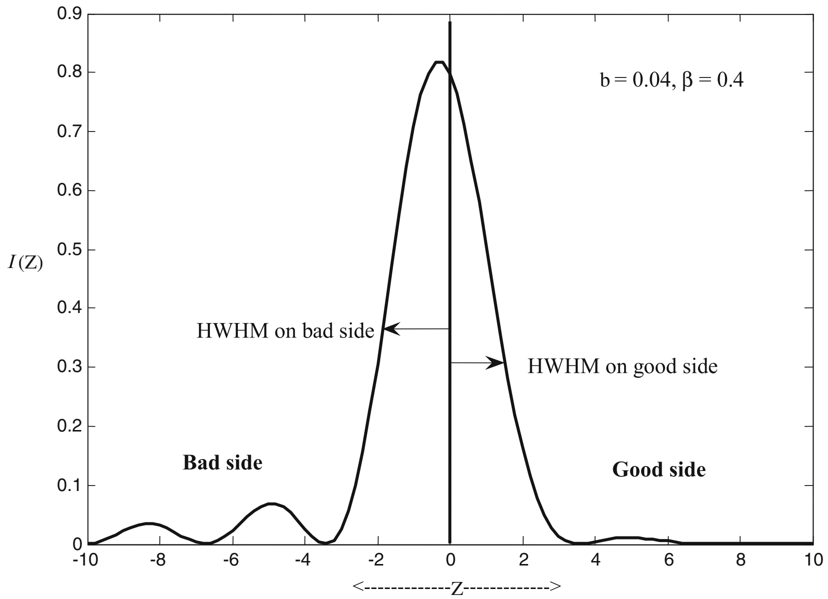

Figure 6. Half-maximum area distribution profile on either side of PSF.

is particularly important in the case of resolution of two point objects which are widely varying in their intensity and very close to each other. The intensity profile depicted in figure 6 clearly says that the main peak is narrowing on the good side with edge strip width b for any degree of amplitude apodization β. HWHM on the good side with respect to the central peak position of asymmetric PSF given in table 4 and presented in figure 7 demonstrates that as edge strip width b increases from 0 to 0.08, HWHM on the good side with respect to the central peak position initially increases and is seen to decrease with

Table 4. HWHM on the good side of the asymmetric PSF with respect to the central peak position.

b	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1$
0	1.3915	1.4016	1.4123	1.4236	1.4356	1.4483
0.02	1.4631	1.4733	1.4841	1.4954	1.5074	1.5202
0.04	1.5279	1.5372	1.5469	1.5571	1.5677	1.5788
0.06	1.5754	1.5824	1.5898	1.5973	1.6049	1.6130
0.08	1.5968	1.6009	1.6049	1.609	1.6131	1.6173
0.1	1.591	1.5922	1.5933	1.5943	1.5953	1.5963
0.12	1.5654	1.5646	1.5637	1.5628	1.5618	1.5607
0.14	1.5303	1.5285	1.5267	1.5247	1.5228	1.5209
0.16	1.4938	1.4917	1.4896	1.4876	1.4854	1.4832
0.18	1.4607	1.4587	1.4567	1.4548	1.4527	1.4507
0.2	1.616	1.625	1.6339	1.6427	1.6527	1.6608

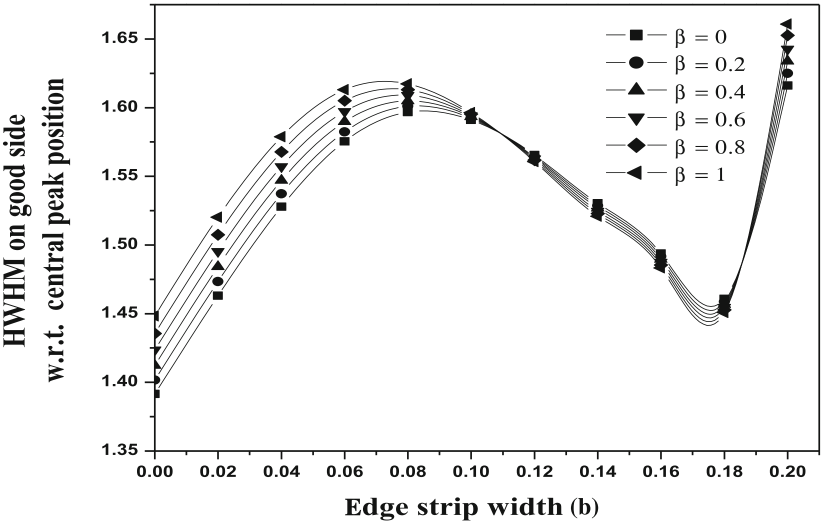

Figure 7. Effect of asymmetric apodization on HWHM (good side) with respect to main peak position.

increased apodization of the central region. It shows that for large edge strip width b, the resolution of point image increases with degree of amplitude apodization β in the central region, which is illustrated in figure 6.

4. Conclusions

Our study introduced a new image quality quantifying parameter called HWHM which can be used to analyse the resolution of asymmetrically apodized optical system along with well-defined FWHM. We computed HWHM to evaluate the improved resolution of asymmetrically apodized optical system, dealing with PSF with non-zero minima. We obtained FWHM with lower values at higher degree of amplitude and antiphase apodization. We have noticed similar trend for HWHM analysis on the good side with respect to central peak position. When $b = 0.18$ and $\beta = 1$, we obtained optimized values for this study. The present study emphasized that as the asymmetry in PSF increases, HWHM on the good side from the diffraction centre became narrow at the cost of broadened HWHM on the bad side. This implies that the spot size is reducing and subsequently narrowing the main peak on the good side of the pattern which improves the resolution in the sense of the classical criteria.

The term HWHM is a potent merit function in spectroscopic observation in the study of instrumental line shape (ILS).

In astronomical photography, as the diffraction spots of stars or quasars appear much larger than diffraction limits due to atmospheric turbulence. In such cases, this study can be a solution for quantifying the resolution of diffraction spots of distant objects.

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