PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 84, No. 1
January 2015
pp. 69–85

Form factors and charge radii in a quantum chromodynamics-inspired potential model using variationally improved perturbation theory

BHASKAR JYOTI HAZARIKA[1,*] and D K CHOUDHURY[1,2]

[1]Centre for Theoretical Studies, Pandu College, Guwahati 781 012, India
[2]Department of Physics, Gauhati University, Guwahati 781 014, India
*Corresponding author. E-mail: bh53033@gmail.com

MS received 2 August 2013; revised 8 April 2014; accepted 22 April 2014
DOI: 10.1007/s12043-014-0839-x; ***e*Publication:** 29 November 2014

Abstract. We use variationally improved perturbation theory (VIPT) for calculating the elastic form factors and charge radii of D, D_s, B, B_s and B_c mesons in a quantum chromodynamics (QCD)-inspired potential model. For that, we use linear-cum-Coulombic potential and opt the Coulombic part first as parent and then the linear part as parent. The results show that charge radii and form factors are quite small for the Coulombic parent compared to the linear parent. Also, the analysis leads to a lower as well as upper bounds on the four-momentum transfer Q^2, hinting at a workable range of Q^2 within this approach, which may be useful in future experimental analyses. Comparison of both the options shows that the linear parent is the better option.

Keywords. Variationally improved perturbation theory; form factor; charge radii.

PACS Nos 12.39.−x; 12.39.Jh; 12.39.Pn

1. Introduction

The potential model description in the nonperturbative regime of QCD is tremendously successful in providing both qualitative and quantitative descriptions of the hadron spectrum and the deacy modes. On this basis, we have pursued a nonrelativisic constituent quark model (NRQM) [1–4] for mesons containing a heavy (light) quark and a light (heavy) antiquark. Although nonrelativistic in nature, relativistic effect is to be introduced from outside [5,6] due to the light quarks involved. Basically, the model relies on the work of Rujula *et al* [7] who used a nonrelativistic treatment of potential model which was quite successful in describing different hadronic properties. We have solved the Schrödinger equation for the spin-independent Fermi–Breit Hamiltonian (used in Rujula *et al* [7]) consisting of linear-cum-Coulombic potential for the ground state [2]. The solution, i.e. wavefunction has been obtained using different approximation methods like Dalgarno method [8] and variationally improved perturbation theory (VIPT) [9–11] which is then used in predicting the Isgur–Wise (I–W) function [2–4,12,13], mass, decay constant, charge radii [1] etc. We note that with the linear-cum-Coulomb potential of QCD, we have two options in choosing the parent or the child (i.e. perturbation): (i) we can consider the linear one as the perturbation (i.e. the Coulombic one as the parent) and then (ii) linear one as the parent (i.e. Coulombic one as the perturbation). As we have already successfully used VIPT for both the options in the calculation of slope and curvature of I–W function for D, D_s, B, B_s and B_c mesons [12,13], extending it for predicting elastic form factors and charge radii (which provide important insight on the distribution of different charge constituents of a hadron) definitely makes sense.

It is well known that the form factor and charge radius are dependent on the momentum transform of the wavefunction. So, getting an appropriate wavefunction is very essential for a fruitful analysis. With the success of VIPT in the calculation of Isgur–Wise function as pointed out in [12,13], one can expect a similar success here too. It is worthwhile to note that while investigating the form factor, one must take into account the proper range of four-momentum transfer Q^2. The Q^2 range usually determines the applicability of perturbative QCD (pQCD) or nonperturbative QCD (npQCD). So, an accurate selection of Q^2 range within the nonperturbative approach, which will also fall within the experimental regime, is necessary. This fecilitates a direct comparison between theory and experiment. This has been done both theoretically and experimentally since long [14–19] for the light π, K etc. mesons. However, for the mesons which contain at least one heavy quark, very little has been investigated theoretically [20–22]. In the absence of any experimental data for them, our results may be helpful in future in the experimental set-up regarding the Q^2 range.

As far as our model is concerned, the perturbative or nonperturbative regime of QCD can be interpreted through the relativistic factor $\epsilon = 1 - \sqrt{1-(4\alpha_s/3)}$ [1]. The reality constraint on the form factor $F(Q^2)$ leads to the condition $0 < \epsilon < 1$, where $\epsilon \to 0$ ($\epsilon \to 1$) corresponds to the perturbative (nonperturbative) limit of QCD. The $\epsilon \to 1$ limit demands large α_s or low Q^2. So, discussing the nonperturbative effects of QCD with large confinement parameter b ($= 0.183\ \text{GeV}^2$), we must consider the low Q^2 limit of α_s in this model. However, we have observed in ref. [1] that large value of b prohibits the use of low Q^2 compelling one to involve with small α_s which corresponds to the perturbative regime and thus cannot be accounted for in this nonperturbative approach.

We reanalyse all these observations in this approach of VIPT for both the cases – linear or confinement part as perturbation and Coulombic part as perturbation. We shall explore the possibility of incorporating significant value of α_s even with large confinement. This work will also check the status of both confinement and Coulombic parts as perurbation and observe the consequences regarding the usable range of Q^2 to work in the absence of experimental data for the said mesons. The calculations are done with a fixed value of α_s from V-scheme [23–25] with large confinement effect instead of variation in both. Even with this single value of α_s and b, one can draw similar conclusion regarding the effective range of Q^2. The calculated form factors are plotted graphically to show their variation with Q^2 for both the cases.

Basically, this work explores the possibility of improving the results for form factors and charge radii over those of ref. [1,22] with the help of VIPT. In that sense, this work can be thought of as the improved version of that in ref. [1], as it uses the reasonably new technique, VIPT, which has certain advantages [12,13] over the Dalgarno method used in ref. [1]. Besides, the present work deals with the linear potential, both as perturbation and parent, unlike in ref. [1] where only perturbation option was considered. As a result, the present work explores the effectiveness of VIPT in a more elaborate way for both the options at the same time. In the process, we shall try to find the possible lower and/or upper limits of Q^2 where the linear part as perturbation or parent is applicable.

The rest of the paper is organized as follows: §2 contains the formalism, §3 the result and calculation, while §4 includes the discussion and conclusion.

2. Formalism

2.1 *VIPT with Coulombic potential as parent*

2.1.1 *Wavefunction.* We breifly reformulate the VIPT with the expression for the wavefunction corrected upto the first order of jth state given by [10,12]

$$\psi_j = \psi_j^{(0)} + \sum_{k\neq j} \frac{\int \psi_k^{(0)*} H'_{P'j} \psi_j^{(0)} \mathrm{d}v}{E_j^{(0)} - E_k^{(0)}}, \tag{1}$$

where P' is the variational parameter (which is later optimized with respect to energy) considered instead of physical parameter P. For the Coulombic part as parent, the physical parameter is $\alpha = 4\alpha_s/3$ and the optimized variational parameter is $\bar{\alpha}'$ [12]. As done in ref. [12], we consider the wavefunction for triple term consideration of the summation given by eq. (1) above and rewrite the equation (viz. eq. (45) of ref. [12]) with the subscript 10 ($n = 1, l = 0$) being replaced by T:

$$\begin{aligned}\psi_T = {} & \psi_{10}^{(0)} - A\left(1 - \frac{\mu\bar{\alpha}'_{10}r}{2}\right)\mathrm{e}^{-(\mu\bar{\alpha}'_{10}r/2)} \\ & + B\left(1 - \frac{2\mu\bar{\alpha}'_{10}r}{3} + \frac{2\mu^2\bar{\alpha}'^2_{10}r^2}{27}\right)\mathrm{e}^{-(\mu\bar{\alpha}'_{10}r/3)} \\ & + D'\left(\frac{1}{4} - \frac{3\mu\bar{\alpha}'_{10}r}{16} + \frac{\mu^2\bar{\alpha}'^2_{10}r^2}{32} - \frac{\mu^3\bar{\alpha}'^3_{10}r^3}{8\times 96}\right)\mathrm{e}^{-(\mu\bar{\alpha}'_{10}r/4)},\end{aligned} \tag{2}$$

where the different parameters are given by

$$c'_1 = \frac{\mu\bar{\alpha}'_{10}}{\pi^{1/3}} \tag{3}$$

$$A = \frac{4\sqrt{\mu}}{3\sqrt{\pi}(\bar{\alpha}'_{10})^{1/2}}\left[\frac{4\mu\bar{\alpha}'_{10}(\alpha - \bar{\alpha}'_{10})}{27} - \frac{32b}{81\mu\bar{\alpha}'_{10}}\right] \tag{4}$$

$$B = \frac{\sqrt{\mu}}{\sqrt{\pi}(\bar{\alpha}'_{10})^{1/2}}\left[\frac{3\mu\bar{\alpha}'_{10}(\alpha - \bar{\alpha}'_{10})}{64} - \frac{27b}{256\mu\bar{\alpha}'_{10}}\right] \tag{5}$$

and

$$D' = \frac{(\mu\bar{\alpha}'_{10})^{3/2}}{\sqrt{\pi}}\left[\frac{36(\alpha - \bar{\alpha}'_{10})}{15625\bar{\alpha}'_{10}} - \frac{384b}{78125\mu^2\bar{\alpha}'^3_{10}}\right]. \tag{6}$$

The wavefunctions for single and double term consideration can be obtained by putting $B = D' = 0$ and $D' = 0$ respectively in the same eq. (2).

However, we shall consider the relativistic version ($\epsilon \neq 0$) of the above wavefunction, viz. [12]

$$\psi_{\mathrm{T,Rel}}\,(r) = \psi_{\mathrm{T}}(r\mu\bar{\alpha}'_{10})^{-\epsilon}. \tag{7}$$

The relativistic factor ϵ is given by [1]

$$\epsilon = 1 - \sqrt{1 - \frac{4\alpha_{\mathrm{s}}}{3}}. \tag{8}$$

2.1.2 *The elastic charge form factor and charge radii.* The form factor can be expressed as [26]

$$eF(Q^2) = \sum \frac{e_i}{Q_i} \int_0^{+\infty} r \left|\psi_{\mathrm{T,Rel}}\,(r)\right|^2 \sin Q_i r \,\mathrm{d}r, \tag{9}$$

where

$$Q_i = \frac{\sum_{j\neq i} m_j Q}{\sum m_i}. \tag{10}$$

Putting (2) and (7) in (9) we get the form factor as

$$\begin{aligned} eF(Q^2) &= \sum e_i N_3'^2 \Gamma(3-2\epsilon) \\ &\quad \times \left[q_1+q_2+q_3+q_4+q_5+q_6+q_7+q_8+q_9+q_{10}\right], \end{aligned} \tag{11}$$

where $N_3'^2$ is the same normalization constant as appeared in eq. (54) of ref. [12] and the different $q_i(Q_i),\ s(i = 1, 2, \ldots, 10)$ are defined in Appendix.

The charge radius is derived as [1]

$$\langle r^2 \rangle = -\frac{\mathrm{d}(eF(Q^2))}{\mathrm{d}Q^2}\bigg|_{Q^2=0} \tag{12}$$

which in the present model is

$$\begin{aligned} \langle r^2 \rangle &= N_3'^2 \Gamma(3-2\epsilon)[r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7 + r_8 \\ &\quad + r_9 + r_{10}], \end{aligned} \tag{13}$$

the different r_i^3s ($i = 1, 2, \ldots, 10$) are defined in Appendix.

2.1.3 *Status of linear potential as perturbation.* The momentum transform of eq. (7) is [27,28]

$$\psi_{\mathrm{T,Rel}}(Q^2) = \sum \frac{e_i}{Q_i}\sqrt{\frac{2}{\pi}} \int_0^{+\infty} r\psi_{\mathrm{T,Rel}}\,(r) \sin\, Q_i r \,\mathrm{d}r \tag{14}$$

$$= \sum e_i N_3' \sqrt{\frac{2}{\pi}} \Gamma(3-2\epsilon)\left[p_1 - p_2 + p_3 + p_4\right]. \tag{15}$$

The p_is which depend on Q_i^2, ϵ etc. are given in Appendix.

Table 1. Values of lower limit of four-momentum transfer Q_0^2 with Coulombic parent taking single, double and triple terms in eq. (1). We have to use Q^2 values above these.

Meson	D^+	D^-	D_s^+	B^+	B^0	B_s^0	B_c^+
$Q^2_{0,S}$	0.0004	0.0004	0.001	0.053	0.053	0.075	0.211
$Q^2_{0,D}$	0.00036	0.00036	0.0009	0.052	0.052	0.072	0.209
$Q^2_{0,T}$	0.0003	0.0003	0.0007	0.05	0.05	0.07	0.205

Table 2. Values of charge radii (fermi) for different mesons with Coulombic parent for single, double and triple terms in eq. (1). The subscripts S, D, T correspond to single, double and triple terms respectively whereas F means finite mass consideration. The infinite mass limit (subscript ∞ is used) is shown for the triple term alone.

Meson	D^0	D^+	D_s^+	B^+	B^0	B_s^0	B_c^+
$\langle r^2_{S,F}\rangle^{1/2}$	−0.121	0.115	0.11	0.2545	−0.1822	−0.168	0.108
$\langle r^2_{D,F}\rangle^{1/2}$	−0.119	0.112	0.106	0.2512	−0.1788	−0.164	0.105
$\langle r^2_{T,F}\rangle^{1/2}$	−0.118	0.109	0.101	0.2464	−0.1736	−0.158	0.1034
$\langle r^2_{T,\infty}\rangle^{1/2}$	−0.131	0.12	0.113	0.263	−0.186	−0.1742	0.1325

If linear potential is treated as perturbation, then from eq. (15), the following inequality must be preserved:

$$p_1 > p_2 - p_3 - p_4. \tag{16}$$

This inequality leads to a lower limit of Q^2, namely Q_0^2 [1], above which one has to use the values of Q^2. Q_0^2 is determined from the condition

$$p_1 = p_2 - p_3 - p_4. \tag{17}$$

Due to the quark mass dependence, Q_0^2s have different values and they are shown table 1. In the Dalgarno method [1], the lower limits Q_0^2 were large and the formalism failed to account for large confinement effect in the nonperturbative QCD regime where α_s values were taken to be large. Only in the limit $b \to 0$, the Q_0^2 values were lowered and the formalism worked for low Q^2 range [1]. In this method of VIPT, the values of Q_0^2 are shown to be quite small even with large confinement effect enabling us to work in the nonperturbative QCD regime with large α_s.

We also note that for single-term consideration, only p_2 exists on the RHS of inequality and for double term consideration, both p_2 and p_3 exist. We have recorded the values of charge radii in table 2.

2.2 *VIPT with linear potential as parent*

2.2.1 *Wavefunction.* As pointed in refs [10,13], linear parent gives rise to Airy functions. The physical parameter is b and the optimized variational parameter is $\bar{b}'$. We reproduce the analogous wavefunction in this case also for three-term consideration of eq. (1) as was for the Coulombic parent:

$$\psi_{\mathrm{T}} = N''\left[\psi^{(0)} + \frac{(2\mu)^{1/3}}{(\rho_{02}-\rho_{01})\,\bar{b}'^{2/3}}\left((b-\bar{b}')\langle r\rangle_{2,1} - \alpha\left\langle\frac{1}{r}\right\rangle_{2,1}\right)\psi_{20}(r) + \frac{(2\mu)^{1/3}}{(\rho_{03}-\rho_{01})\,\bar{b}'^{2/3}}\left((b-\bar{b}')\langle r\rangle_{3,1} - \alpha\left\langle\frac{1}{r}\right\rangle_{3,1}\right)\psi_{30}(r) + \frac{(2\mu)^{1/3}}{(\rho_{04}-\rho_{01})\,\bar{b}'^{2/3}}\left((b-\bar{b}')\langle r\rangle_{4,1} - \alpha\left\langle\frac{1}{r}\right\rangle_{4,1}\right)\psi_{40}(r)\right], \quad (18)$$

where N'' is the normalization constant as appeared in ref. [13]. We note that for single (double) term consideration of eq. (1), the third and fourth terms (fourth term) are dropped from eq. (18) and normalization constants also change to different one (eqs (17) and (21) of ref. [13]).

For this case also, we take the relativistic version of the above wavefunction:

$$\psi_{\mathrm{T,Rel}} = \psi_{\mathrm{T}}(r\mu\bar{\alpha}'_{10})^{-\epsilon}. \quad (19)$$

The zeros of the Airy function ρ_{0n} is given by eq. (11) of ref. [13] as

$$\rho_{0n} = -\left[\frac{3\pi(4n-1)}{8}\right]^{2/3} \quad (20)$$

and

$$\langle r^k\rangle_{n,n'} = N_n N_{n'} \int_0^{+\infty} r^k \mathrm{Ai}\left((2\mu\bar{b}')^{1/3}r - \rho_{0n}\right)\mathrm{Ai}\left((2\mu\bar{b}')^{1/3}r - \rho_{0n'}\right)\mathrm{d}r, \quad (21)$$

where N_n, $N_{n'}$ are the normalization constants for n and n' states respectively.

Like the expressions, we have adopted the same values of b, $\bar{b}'$, ρ_{0n} from ref. [13] .

Table 3. Values of charge radii (fermi) for different mesons with linear parent for single, double and triple terms in eq. (1). The subscripts S, D, T correspond to single, double and triple terms respectively whereas F means finite mass consideration. The infinite mass limit (subscript ∞ is used) is shown for the single term alone.

Meson	D^0	D^+	D_s^+	B^+	B^0	B_s^0	B_c^+
C_S	8.24	8.24	5.916	14.22	14.22	10.91	4.50
C'_S	2.266	2.266	1.173	7.71	7.71	4.52	0.78
$\langle r^2_{S,F}\rangle^{1/2}$	−0.197	0.1494	0.104	0.425	−0.2996	−0.2227	0.1125
C_D	13.1	13.1	8.1	26.7	26.7	16.5	5.2
C'_D	2.69	2.69	1.67	9.89	9.89	5.7	1.1
$\langle r^2_{D,F}\rangle^{1/2}$	−0.21	0.161	0.121	0.473	−0.336	−0.2489	0.127
C_T	18.14	18.14	11.18	43.09	43.09	25.56	6.636
C'_T	3.365	3.365	2.199	13.14	13.14	7.655	1.363
$\langle r^2_{T,F}\rangle^{1/2}$	−0.24	0.182	0.143	0.555	−0.391	−0.289	0.148
$\langle r^2_{S,\infty}\rangle^{1/2}$	−0.246	0.174	0.125	0.453	−0.32	−0.246	0.144

2.2.2 *The elastic charge form factor and charge radii.* Putting (18) and (19) in the definition of form factor (9) given above, the form factor is found to be

$$eF(Q^2) = \sum e_i N''^2 \left[C - C' \frac{Q_i^2}{6} \right]. \tag{22}$$

The coefficients C, C's are given in table 3. They are of course different for single, double or more than two-term consideration. Numerical integrations are done in getting the above result. The corresponding charge radius is obtained by using eqs (18) and (19) in (12) which are recorded in table 3.

2.2.3 *Status of Coulombic potential as perturbation.* The momentum transform of (19) is

$$\psi_{\mathrm{T,Rel}}(Q^2) = \sum \frac{e_i}{Q_i} \sqrt{\frac{2}{\pi}} \int_0^{+\infty} r \psi_{T,\mathrm{Rel}}(r) \sin Q_i r \, \mathrm{d}r \tag{23}$$

$$= \sum e_i N_3' \sqrt{\frac{2}{\pi}} \Gamma(3-2\epsilon) \left[p_1' + p_2' + p_3' + p_4' - \frac{Q_i^2}{6}(p_5' + p_6' + p_7' + p_8') \right]. \tag{24}$$

The p_i's ($i = 1, 2, \ldots, 8$) are given in Appendix.

If Coulombic potential is treated as perturbation then from eq. (24) the following inequality must be preserved:

$$p_1' + p_2' + p_3' + p_4' > \frac{Q_i^2}{6}(p_5' + p_6' + p_7' + p_8'). \tag{25}$$

This inequality leads to an upper limit of Q^2, namely Q_0^2, below which one have to use the values of Q^2. Q_0^2 is determined from the condition:

$$p_1' + p_2' + p_3' + p_4' = \frac{Q_i^2}{6}(p_5' + p_6' + p_7' + p_8'). \tag{26}$$

Table 4. Values of upper limit of four-momentum transfer Q_0^2 with linear parent taking single, double and triple terms in eq. (1). We have to use Q^2 values lower than these.

Meson	D^+	D^-	D_s^+	B^+	B^0	B_s^0	B_c^+
$Q_{0,\mathrm{S}}^2$	2.297	2.297	2.92	1.43	1.43	1.676	3.11
$Q_{0,\mathrm{D}}^2$	2.1	2.1	2.67	1.31	1.31	1.56	2.89
$Q_{0,\mathrm{T}}^2$	1.88	1.88	2.387	1.177	1.177	1.386	2.55

Table 5. Values of $\bar{b}'$ for relativistic case only. The values are the same as recorded in table 2 of ref. [13].

Mesons	Reduced mass μ	$\alpha = 4\alpha_s/3$	$\bar{b}'$ with relativistic effect
D	0.2761	0.924	16.24
D_s	0.368248	0.924	19.8
B	0.31464	0.348	5.587
B_s	0.4401	0.348	5.954
B_c	1.1803	0.348	8.103

Table 6. Prediction of $\langle r^2 \rangle^{1/2}$ (fermi) for finite and infinite mass consideration in other models. The subscript F (∞) means finite (infinite) mass limit.

Meson	D^0	D^+	D_s^+	B^+/B^-	B^0	$B_s^0/\bar{B}_s^0$	B_c^+/B_c^-
$\langle r_F^2 \rangle^{1/2}$ [9]	–	0.506	0.491	0.258(B^-)	–	0.256($\bar{B}_s^0$)	0.236(B_c^-)
$\langle r_F^2 \rangle^{1/2}$ [10]	−0.551	0.43	0.352	0.612(B^+)	−0.432	−0.345(B_s^0)	0.207(B_c^+)
$\langle r_\infty^2 \rangle^{1/2}$ [10]	−0.704	0.498	0.425	0.704(B^+)	−0.498	−0.425(B_s^0)	–(B_c^+)
$\langle r_F^2 \rangle^{1/2}$ [11]	−0.484	0.366	0.355	1.72(B^+)	−1.21	−1.17(B_s^0)	1.43(B_c^+)
$\langle r_\infty^2 \rangle^{1/2}$ [11]	−0.6025	0.427	0.427	1.836(B^+)	−1.29	−1.29(B_s^0)	1.84(B_c^+)

Different values of the upper limit Q_0^2s for different terms, e.g., single, double etc. are shown in table 4.

3. Calculations and results

We have listed $\bar{b}'$ in table 5 while the lower and upper limits of Q_0^2 for single-, double- and triple-term considerations are given in tables 1 and 4. In table 3, we record the charge radii for single-, double- and triple-term consideration for Coulombic potential as parent; whereas the same is recorded for linear potential as parent in table 2. The infinite mass consideration shown by the subscript ∞ is also included for triple (single) term consideration for Coulombic (linear) parent. Table 6 shows charge radii of different mesons obtained from other models and data.

The α_s values are taken from the V-scheme [23–25] and the integrations are done numerically for all these calculations. Figures 1 and 2 show the variation $eF(Q^2)$ vs. Q^2 for D, D_s and B_c mesons for both the options.

4. Discussion and conclusion

We have analysed elastic form factors and charge radii in a QCD-inspired potential model with Cornell potential using the VIPT under two scenarios – linear potential (i)

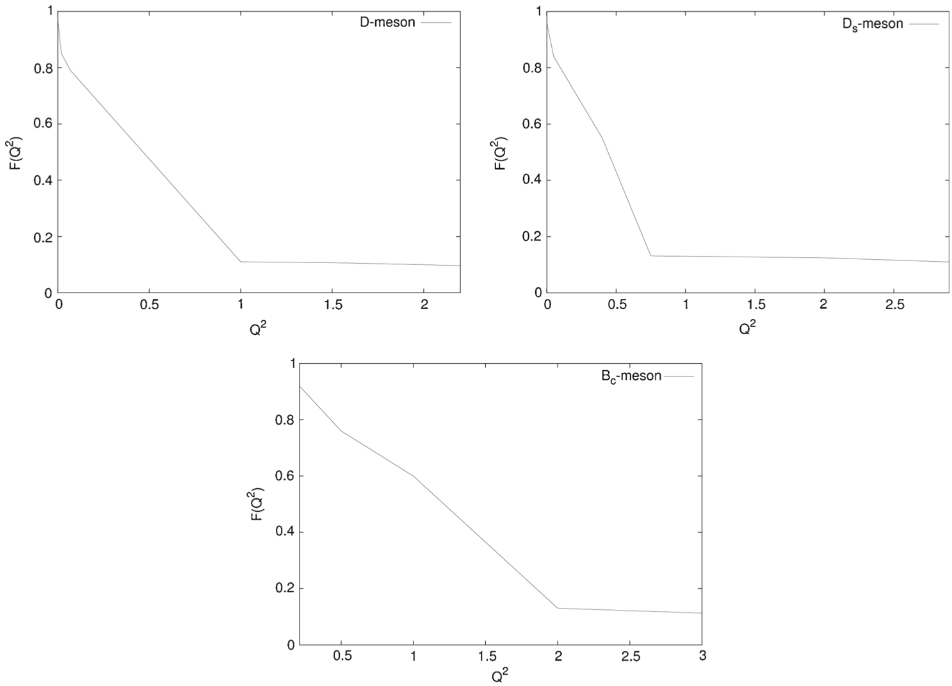

Figure 1. Variation of $eF(Q^2)$ vs. Q^2 for D, D_s and B_c mesons with Coulombic parent.

as perturbation (i.e. Coulombic part as parent) and (ii) as parent (i.e. Coulombic part as perturbation).

We summarize our findings below:

(1) The form factor $eF(Q^2)$ decreases with the increase of Q^2 (as it should) for both the scenarios.
(2) The form factor is either very small (for D-sector mesons) or small (for B-sector mesons) with linear perturbation as compared to those with linear parent. The charge radius is also observed to be smaller with linear perturbation than with linear parent.
(3) We used a fixed set of values for α_s under V-scheme [23–25] in the calculation. For example, it is 0.693 for the D, D_s mesons which is larger than the value 0.261 for the B, B_s, B_c mesons. This consideration directly results in the unexpectedly smaller values of charge radii for D, D_s mesons as compared to the B, B_s, B_c mesons. Larger α_s values are responsible for smaller charge radius.
(4) While checking the status of confinement as perturbation (or Coulombic part as perturbation, i.e. linear parent) we obtain a lower (or upper) limit on Q^2. It shows that the former option is valid for high Q^2 and the latter one for low Q^2 as it should.
(5) In the present analysis, even with large b, the lower limit of Q^2 (for linear perturbation) is really small as shown in table 1 for fixed α_s. We have seen that for $\alpha_s = 0.693$, the lower limit of Q^2 for D, D_s mesons are respectively 0.0003, 0.0007, whereas with $\alpha_s = 0.261$, the lower limit of Q^2 for B, B_s, B_c mesons are

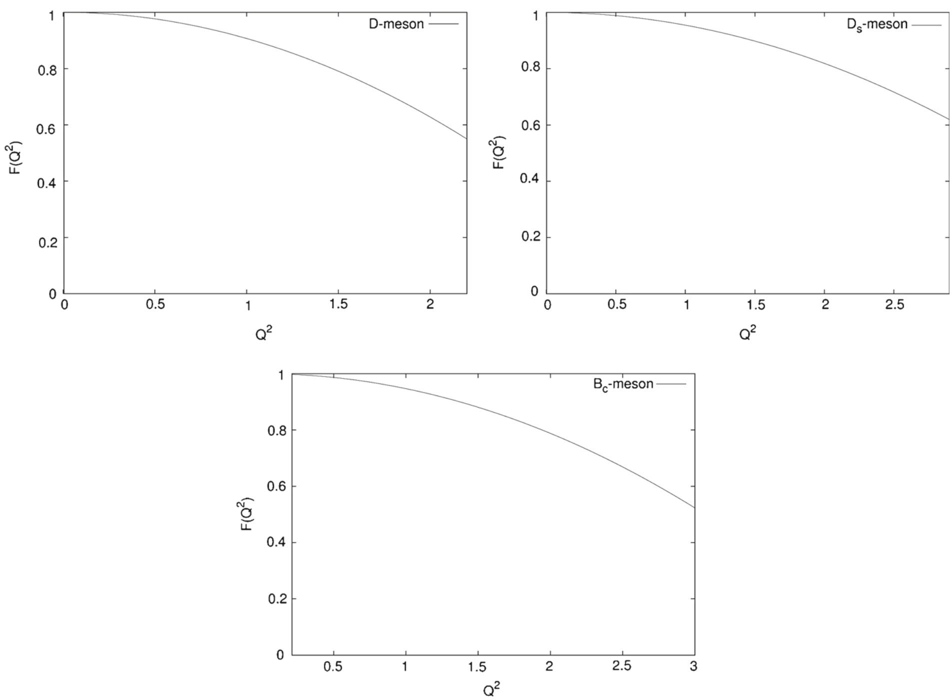

Figure 2. Variation of $eF(Q^2)$ vs. Q^2 for D, D_s and B_c mesons with linear parent.

respectively 0.05, 0.07, 0.205. These values for B, B_s, B_c mesons will be lowered if we put $\alpha_s > 0.261$. This is clearly advantageous over the Dalgarno method with linear perturbation as done in ref. [1] where the formalism broke down for large b. Thus, this approach allows a large value of $\alpha_s(Q^2)$ in the limit $Q^2 \to 0$ even with large confinement, an important feature absent in ref. [1].

(6) Further, if we look at eq. (1), consideration of different terms leads to different charge radii and the limiting values of Q^2 for both the cases. The charge radii and the lower limit of Q^2 decrease with more terms for the linear part as perturbation whereas the charge radii increase and upper limit of Q^2 decreases for the linear parent.

(7) The infinite mass consideration in this work shows that the charge radii are larger than those for finite-mass consideration to agree well with other models (table 6).

The above list as a whole suggests the success of VIPT over the Dalgarno method [1,22] as far as large confinement and limiting values of Q^2 are concerned. The difference in the values of form factors and charge radii for both the cases may be attributed to the use of same α_s (i.e. Q^2) under V-scheme for both the scenarios as the Coulombic potential is dominant for large Q^2 (i.e. low r) and the linear potential in the low Q^2 (i.e. large r) regime. It may be noted that we have used the low Q^2 assumption in the calculation of form factors and this clearly effects the upper limit of Q^2 corresponding to the validity of linear parent. The larger value of α_s for D-sector as compared to B-sector is also another point to be taken into account. Although, the linear parent has shown more flexibility and hence is the better option than the linear perturbation in VIPT, it has used terms up to a particular order in r in the integration involved with Airy function (which is an infinite series). This may lead to loss of certain information as far as physics is concerned. In the absence of any experimental results for these mesons, it is quite difficult to make a direct conclusion but there is clear indication that one must be careful in choosing the parameter $\alpha_s(Q^2)$ as well as the confinement parameter in the calculation of form factor and charge radius within the QCD framework.

The above discussion led to the conclusion that there is scope to use this approach in the study of meson decays. The lower and upper limits on Q^2 (i.e. range of Q^2) in this analysis may be useful in the experimental set-up to investigate cross-section and form factor in future for these mesons. Further, from the model-specific values of form factors and charge radii, the method allows to investigate the behaviour of α_s with respect to Q^2 in the nonperturbative regime of QCD.

Appendix

Expressions for q_is

$$q_1 = \frac{c_1'^2}{(4\mu^2\bar{\alpha}'^2 + Q_i^2)^{(1-\epsilon)}}, \tag{A1}$$

$$q_2 = A^2\left[\frac{1}{(\mu^2\bar{\alpha}'^2+Q_i^2)^{(1-\epsilon)}} - \frac{(3-2\epsilon)\mu\bar{\alpha}'}{(\mu^2\bar{\alpha}'^2+Q_i^2)^{(1.5-\epsilon)}} + \frac{(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{(\mu^2\bar{\alpha}'^2+Q_i^2)^{(2-\epsilon)}}\right], \tag{A2}$$

$$\begin{aligned} q_3 = B^2 \Bigg[& \frac{1}{(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2)^{(1-\epsilon)}} - \frac{4(3-2\epsilon)\mu\bar{\alpha}'}{3(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2)^{(1.5-\epsilon)}} \\ & + \frac{16(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{27\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(2-\epsilon)}} - \frac{8(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{81\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(2.5-\epsilon)}} \\ & + \frac{4(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4\bar{\alpha}'^4}{729\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(3-\epsilon)}}\Bigg], \end{aligned} \tag{A3}$$

$$\begin{aligned} q_4 = D'^2 \Bigg[& \frac{1}{16\left(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2\right)^{(1-\epsilon)}} - \frac{3(3-2\epsilon)\mu\bar{\alpha}'}{32\left(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2\right)^{(1.5-\epsilon)}} \\ & + \frac{17(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{256\left(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2\right)^{(2-\epsilon)}} - \frac{19(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{1536\left(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2\right)^{(2.5-\epsilon)}} \end{aligned}$$

$$+\frac{7(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4\bar{\alpha}'^4}{6144\left(\frac{\mu^2\bar{\alpha}'^2}{9}+Q_i^2\right)^{(3-\epsilon)}}$$

$$-\left.\frac{(7-2\epsilon)(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^5\bar{\alpha}'^5}{12288\left(\frac{\mu^2\bar{\alpha}'^2}{4}+Q_i^2\right)^{(3.5-\epsilon)}}\right], \tag{A4}$$

$$q_5 = 2c_1' A\left[\frac{1}{\left(\frac{9\mu^2\bar{\alpha}'^2}{4}+Q_i^2\right)^{(1-\epsilon)}} - \frac{(3-2\epsilon)\mu\bar{\alpha}'}{2\left(\frac{9\mu^2\bar{\alpha}'^2}{4}+Q_i^2\right)^{(1.5-\epsilon)}}\right], \tag{A5}$$

$$q_6 = 2c_1' B\left[\frac{1}{\left(\frac{16\mu^2\bar{\alpha}'^2}{9}+Q_i^2\right)^{(1-\epsilon)}} - \frac{2(3-2\epsilon)\mu\bar{\alpha}'}{3\left(\frac{16\mu^2\bar{\alpha}'^2}{9}+Q_i^2\right)^{(1.5-\epsilon)}}\right.$$

$$\left.+\frac{2(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{27\left(\frac{16\mu^2\bar{\alpha}'^2}{9}+Q_i^2\right)^{(2-\epsilon)}}\right], \tag{A6}$$

$$q_7 = 2c_1' D'\left[\frac{1}{4\left(\frac{25\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(1-\epsilon)}} - \frac{3(3-2\epsilon)\mu\bar{\alpha}'}{16\left(\frac{25\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(1.5-\epsilon)}}\right.$$

$$\left.+\frac{(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{32\left(\frac{25\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(2-\epsilon)}} - \frac{(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{768\left(\frac{25\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(2.5-\epsilon)}}\right], \tag{A7}$$

$$q_8 = -2AB\left[\frac{1}{\left(\frac{25\mu^2\bar{\alpha}'^2}{36}+Q_i^2\right)^{(1-\epsilon)}} - \frac{5(3-2\epsilon)\mu\bar{\alpha}'}{6\left(\frac{25\mu^2\bar{\alpha}'^2}{36}+Q_i^2\right)^{(1.5-\epsilon)}}\right.$$

$$\left.+\frac{20(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{\left(27\frac{25\mu^2\bar{\alpha}'^2}{36}+Q_i^2\right)^{(2-\epsilon)}} - \frac{(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{27\left(\frac{25\mu^2\bar{\alpha}'^2}{36}+Q_i^2\right)^{(2.5-\epsilon)}}\right], \tag{A8}$$

$$q_9 = -2AD'\left[\frac{1}{4\left(\frac{9\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(1-\epsilon)}} - \frac{5(3-2\epsilon)\mu\bar{\alpha}'}{16\left(\frac{9\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(1.5-\epsilon)}}\right.$$

$$+\frac{(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{8\left(\frac{9\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(2-\epsilon)}} - \frac{13(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{768\left(\frac{9\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(2.5-\epsilon)}}$$

$$\left.+\frac{(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4\bar{\alpha}'^4}{1536\left(\frac{9\mu^2\bar{\alpha}'^2}{16}+Q_i^2\right)^{(3-\epsilon)}}\right], \tag{A9}$$

$$q_{10} = 2BD'\left[\frac{1}{4\left(\frac{49\mu^2\bar{\alpha}'^2}{144}+Q_i^2\right)^{(1-\epsilon)}} - \frac{25(3-2\epsilon)\mu\bar{\alpha}'}{48\left(\frac{49\mu^2\bar{\alpha}'^2}{144}+Q_i^2\right)^{(1.5-\epsilon)}}\right.$$

$$+\frac{151(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{864\left(\frac{25\mu^2\bar{\alpha}'^2}{144}+Q_i^2\right)^{(2-\epsilon)}} - \frac{27.66(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{768\left(\frac{25\mu^2\bar{\alpha}'^2}{144}+Q_i^2\right)^{(2.5-\epsilon)}}$$

$$+\frac{3.66(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4\bar{\alpha}'^4}{1152\left(25\frac{\mu^2\bar{\alpha}'^2}{144}+Q_i^2\right)^{(3-\epsilon)}}$$

$$\left.-\frac{(7-2\epsilon)(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4\bar{\alpha}'^4}{10368\left(25\frac{\mu^2\bar{\alpha}'^2}{144}+Q_i^2\right)^{(3.5-\epsilon)}}\right]. \tag{A10}$$

Expressions for r_is

$$r_1 = 3c_1'^2\left(1+\frac{m_i}{m_j}\right)^{-2}(4\mu^2\bar{\alpha}'^2)^{\epsilon-2}(2-2\epsilon), \tag{A11}$$

$$r_2 = 3A^2\left(1+\frac{m_i}{m_j}\right)^{-2}[(2-2\epsilon)(4\mu^2\bar{\alpha}'^2)^{\epsilon-2} - 3\mu\bar{\alpha}'(3-2\epsilon)^2(\mu^2\bar{\alpha}'^2)^{\epsilon-2.5}$$

$$+0.75\mu^2\bar{\alpha}'^2(4-2\epsilon)^2(3-2\epsilon)(\mu^2\bar{\alpha}'^2)^{\epsilon-3}], \tag{A12}$$

$$r_3 = 3B^2\left(1+\frac{m_i}{m_j}\right)^{-2}\left[\left(\frac{4\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-2}(2-2\epsilon) - 4\mu\bar{\alpha}'(3-2\epsilon)^2\left(\frac{4\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-2.5}\right.$$

$$+\frac{16\mu^2\bar{\alpha}'^2}{9}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{4\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-3}$$

$$-\frac{8\mu^3\bar{\alpha}'^3}{81}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)\left(\frac{4\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-3.5}$$

$$\left.+\frac{4\mu^4\bar{\alpha}'^4}{729}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\left(\frac{4\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-4}\right], \tag{A13}$$

$$r_4 = 3D'^2\left(1+\frac{m_i}{m_j}\right)^{-2}\left[\left(\frac{\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-2}(2-2\epsilon) - \frac{9\mu\bar{\alpha}'}{32}(3-2\epsilon)^2\left(\frac{\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-2.5}\right.$$

$$+\frac{57\mu^2\bar{\alpha}'^2}{32}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-3}$$

$$-\frac{19\mu^3\bar{\alpha}'^3}{512}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)\left(\frac{\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-3.5}$$

$$+\frac{21\mu^4\bar{\alpha}'^4}{6144}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\left(\frac{\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-4}$$

$$\left.-\frac{3\mu^5\bar{\alpha}'^5}{12288}(7-2\epsilon)^2(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\left(\frac{4\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-4.5}\right], \tag{A14}$$

$$r_5 = 2c_1' A\left(1+\frac{m_i}{m_j}\right)^{-2}\left[3\left(\frac{9\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-2}(2-2\epsilon)\right.$$

$$\left.-1.5\mu\bar{\alpha}'(3-2\epsilon)^2\left(\frac{9\mu^2\bar{\alpha}'^2}{4}\right)^{\epsilon-2.5}\right], \tag{A15}$$

$$r_6 = 2c_1' B\left(1+\frac{m_i}{m_j}\right)^{-2}\left[3\left(\frac{16\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-2}(2-2\epsilon)\right.$$

$$-2\mu\bar{\alpha}'(3-2\epsilon)^2\left(\frac{16\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-2.5}$$

$$\left.+\frac{2\mu^2\bar{\alpha}'^2}{9}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{16\mu^2\bar{\alpha}'^2}{9}\right)^{\epsilon-3}\right], \tag{A16}$$

$$r_7 = 2c_1' D'\left(1+\frac{m_i}{m_j}\right)^{-2}\left[\left(\frac{25\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-2}\frac{3\,(2-2\epsilon)}{4}\right.$$

$$-\frac{9\mu\bar{\alpha}'(3-2\epsilon)^2}{16}\left(\frac{25\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-2.5}$$

$$+\frac{3\mu^2\bar{\alpha}'^2}{32}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{25\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-3}$$

$$\left.-\frac{3\mu^3\bar{\alpha}'^3}{768}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)\left(\frac{25\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-3.5}\right], \tag{A17}$$

$$r_8 = -2AB\left(1+\frac{m_i}{m_j}\right)^{-2}\left[\left(\frac{25\mu^2\bar{\alpha}'^2}{36}\right)^{\epsilon-2} 3\,(2-2\epsilon) - 2.5\mu\bar{\alpha}'(3-2\epsilon)^2\left(\frac{25\mu^2\bar{\alpha}'^2}{36}\right)^{\epsilon-2.5} + \frac{20\mu^2\bar{\alpha}'^2}{9}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{25\mu^2\bar{\alpha}'^2}{36}\right)^{\epsilon-3} - \frac{\mu^3\bar{\alpha}'^3}{9}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)\left(\frac{25\mu^2\bar{\alpha}'^2}{36}\right)^{\epsilon-3.5}\right], \quad \text{(A18)}$$

$$r_9 = -2AD'\left(1+\frac{m_i}{m_j}\right)^{-2}\left[0.75(\frac{9\mu^2\bar{\alpha}'^2}{16})^{\epsilon-2}\,(2-2\epsilon) - \frac{15\mu\bar{\alpha}'}{16}(3-2\epsilon)^2\left(\frac{9\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-2.5} + \frac{3\mu^2\bar{\alpha}'^2}{8}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{9\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-3} - \frac{13\mu^3\bar{\alpha}'^3}{256}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)\left(\frac{9\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-3.5} + \frac{\mu^4\bar{\alpha}'^4}{512}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\left(\frac{9\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-4}\right], \quad \text{(A19)}$$

$$r_{10} = 2BD'\left(1+\frac{m_i}{m_j}\right)^{-2}\left[0.75\left(\frac{49\mu^2\bar{\alpha}'^2}{144}\right)^{\epsilon-2}(2-2\epsilon) - \frac{17\mu\bar{\alpha}'}{16}(3-2\epsilon)^2\left(\frac{49\mu^2\bar{\alpha}'^2}{144}\right)^{\epsilon-2.5} + \frac{151\mu^2\bar{\alpha}'^2}{288}(4-2\epsilon)^2(3-2\epsilon)\left(\frac{49\mu^2\bar{\alpha}'^2}{144}\right)^{\epsilon-3} - \frac{83\mu^3\bar{\alpha}'^3}{768}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)\left(\frac{49\mu^2\bar{\alpha}'^2}{144}\right)^{\epsilon-3.5} + \frac{33\mu^4\bar{\alpha}'^4}{3456}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\left(\frac{9\mu^2\bar{\alpha}'^2}{16}\right)^{\epsilon-4} - \frac{\mu^5\bar{\alpha}'^5}{3456}(7-2\epsilon)^2(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\left(\frac{49\mu^2\bar{\alpha}'^2}{144}\right)^{\epsilon-4.5}\right]. \quad \text{(A20)}$$

Expressions for p_is

$$p_1 = \frac{c_1'}{(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2)^{\frac{(3-\epsilon)}{2}}}, \tag{A21}$$

$$p_2 = A\left[\frac{1}{\left(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2\right)^{\frac{(3-\epsilon)}{2}}} - \frac{(3-2\epsilon)\mu\bar{\alpha}'}{2\left(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2\right)^{\frac{(4-\epsilon)}{2}}}\right], \tag{A22}$$

$$p_3 = B\left[\frac{1}{\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{\frac{(3-\epsilon)}{2}}} - \frac{0.67(3-2\epsilon)\mu\bar{\alpha}'}{\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{\frac{(4-\epsilon)}{2}}} - \frac{2(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{27\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{\frac{(5-\epsilon)}{2}}}\right], \tag{A23}$$

$$p_4 = D'\left[\frac{0.25}{\left(\frac{\mu^2\bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(3-\epsilon)}{2}}} - \frac{3(3-2\epsilon)\mu\bar{\alpha}'}{16\left(\frac{\mu^2\bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(4-\epsilon)}{2}}} - \frac{(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{32\left(\frac{\mu^2\bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(5-\epsilon)}{2}}} - \frac{(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{768\left(\frac{\mu^2\bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(6-\epsilon)}{2}}}\right]. \tag{A24}$$

Expressions for p_i's

$$p_1' = n_1 \times (\bar{b}'\mu)^{2/3}, \tag{A25}$$

$$p_2' = n_2 \times (\bar{b}'\mu)^{2/3}, \tag{A26}$$

$$p_3' = n_3 \times (\bar{b}'\mu)^{2/3}, \tag{A27}$$

$$p_4' = n_4 \times (\bar{b}'\mu)^{2/3}, \tag{A28}$$

$$p_5' = n_5 \times (\bar{b}'\mu)^{2/3}, \tag{A29}$$

$$p_6' = n_6 \times (\bar{b}'\mu)^{2/3}, \tag{A30}$$

$$p'_7 = n_7 \times \left(\bar{b}'\mu\right)^{2/3}, \tag{A31}$$

$$p'_8 = n_8 \times \left(\bar{b}'\mu\right)^{2/3}. \tag{A32}$$

Each of the constants $n_1, n_2, \ldots, n_8$ are different for different mesons and they have been obtained by numerical integration.

References

[1] D K Choudhury, P Das, D D Goswami and J N Sharma, *Pramana – J. Phys.* **44**, 519 (1995)
[2] D K Choudhury and N S Bordoloi, *Int. J. Mod. Phys. A* **15**, 3667 (2000)
[3] D K Choudhury and N S Bordoloi, *Mod. Phys. Lett. A* **26**, 443 (2009)
[4] D K Choudhury and N S Bordoloi, *Mod. Phys. Lett. A* **17(29)**, 1909 (2002)
[5] J J Sakurai, *Advanced quantum mechanics* (Addison-Willey Publishing Company, Massachusetts, 1986) p. 128
[6] C Itzykson and J Zuber, *Quantum field theory*, International Student Edition (McGraw Hill, Singapore, 1986) p. 79
[7] A D Rujula, H Georgi and S L Glashow, *Phys. Rev. D* **12**, 147 (1975)
[8] A K Ghatak and S Lokanathan, *Quantum mechanics* (McGraw Hill, 1997) p. 291
[9] S K You, K J Jeon, C K Kim and K Nahm, *Eur. J. Phys.* **19**, 179 (1998)
[10] I J R Aitchison and J J Dudek, *Eur. J. Phys.* **23**, 605 (2002)
[11] F M Fernandez, *Eur. J. Phys.* **24**, 289 (2003)
[12] B J Hazarika and D K Choudhury, *Pramana – J. Phys.* **75**, 423 (2010)
[13] B J Hazarika and D K Choudhury, *Pramana – J. Phys.* **78**, 555 (2012)
[14] S J Brodsky, SLAC-PUB-5013 (1989)
[15] C J Bebek *et al*, *Phys. Rev. D* **17**, 1693 (1978)
[16] C R Ji and F Amiri, *Phys. Rev. D* **42**, 3764 (1990)
[17] T Applequist and E Poggio, *Phys. Rev. D* **10**, 3280 (1974)
[18] H Pagels and S Stoker, *Phys. Rev. D* **20**, 2947 (1979)
[19] N Isgur and C H Llewellyn Smith, *Phys. Lett. B* **217**, 535 (1989); CERN-TH 5013/88
[20] J N Pandya and P C Vinodkumar, *Pramana – J. Phys.* **57**, 821 (2001)
[21] C W Hwang, *Eur. Phys. J. C* **23**, 585 (2002)
[22] N S Bordoloi and D K Choudhury, *Ind. J. Phys.* **82(6)**, 779 (2008)
[23] M Peter, *Phys. Rev. Lett.* **78**, 603 (1997); *Nucl. Phys. B* **501**, 471 (1997)
[24] Y Schroeder, *Phys. Lett. B* **447**, 321 (1999)
[25] Y Schroeder, *Nucl. Phys. Proc. Suppl.* **86**, 525 (2000)
[26] D P Stanley and D Robson, *Phys. Rev. D* **21**, 3180 (1980); *Phys. Rev. D* **26**, 223 (1982)
[27] S Flugge, *Practical quantum mechanics* (Springer-Verlag, New York, Heidelberg, Berlin, 1974)
[28] H B Dwight, *Tables of integrals and other mathematical data* (McMillan Company, 1969)