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Spin polarizability of hyperons

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Abstract. We review the recent progress of the theoretical understanding of spin polarizabilities of the hyperon in the framework of $SU(3)$ heavy baryon chiral perturbation theory (HBChPT). We present the results of a systematic leading-order calculation of hyperon Compton scattering and extract the forward spin polarizability (γ_0) of hyperons. The results obtained for γ_0 in the case of nucleons agree with the known results of $SU(2)$ HBChPT when kaon loops are not considered.

Keywords. Compton scattering; polarizabilities; heavy baryon chiral perturbation theory.

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1. Introduction

Compton scattering is a source of valuable information of baryons because it offers access to some of the more subtle aspects of baryon structure such as polarizabilities [1–5], which parametrize the response of the target to an external quasistatic electromagnetic field. To probe the nucleon, the wavelength of the photon must be of the order of the size of the nucleon. At energies around 100 MeV and above, the effect of the nucleon structure become siginificant and can be detected in Compton scattering experiments. The spin-independent (SI) Compton amplitude is given by

$$\epsilon_1^{\mu}\,\mathcal{M}^{\rm SI}_{\mu\nu}\,\epsilon_2^{\nu}=\vec{\epsilon}\cdot\vec{\epsilon}^{\,*}\left(-\frac{Q_{\rm N}^2}{m_{\rm N}}+4\pi\alpha_{\rm N}\,\omega\omega'\right)+4\pi\beta_{\rm N}(\vec{\epsilon}\times\vec{q})\cdot(\vec{\epsilon}^{\,*}\times\vec{q}^{\,\prime})+\mathcal{O}(\omega^4), \quad (1)$$

where $N=$ p, n; $Q_{\rm N}, m_{\rm N}$ represent the charge and mass of the nucleon, while $\epsilon_\mu=(0,\vec{\epsilon})$, $\epsilon^*_\mu=(0,\vec{\epsilon}^{\,*})$ and $q_\mu=(\omega,\vec{q})$, $q'_\mu=(\omega',\vec{q}^{\,\prime})$ specify the polarization vectors and four-momenta of the initial and final photons, respectively. At this order, the Compton amplitude is defined in terms of two polarizabilities – electric ($\alpha_{\rm N}$) and magnetic ($\beta_{\rm N}$), which measure the response of the nucleon to the applied quasistatic electric and magnetic fields. By measuring differential cross-section one can extract $\alpha_{\rm N}$ and $\beta_{\rm N}$ provided the energy is large enough such that the second and third terms in eq. (1) contribute significantly with respect to the leading Thomson contribution, but not so large that higher order effects become significant. This extraction has been accomplished in the energy range 50 MeV $< \omega <$ 100 MeV [6–8]. According to the Particle Data Group [9] the current experimental numbers for $\alpha_{\rm N}$ and $\beta_{\rm N}$ are

$$\alpha_{\rm p} = (12.0 \pm 0.6) \times 10^{-4}\ {\rm fm}^3, \quad \beta_{\rm p} = (1.9 \pm 0.5) \times 10^{-4}\ {\rm fm}^3,$$
$$\alpha_{\rm n} = (11.6 \pm 1.5) \times 10^{-4}\ {\rm fm}^3, \quad \beta_{\rm n} = (3.7 \pm 2.0) \times 10^{-4}\ {\rm fm}^3. \tag{2}$$

However, the situation with regard to the scattering from polarized targets is less satisfactory, not least because until very recently no direct measurements of polarized Compton scattering had been attempted. The spin-dependent pieces of the scattering amplitude in the Breit frame, for incoming real photons of energy ω and momentum $\mathbf{q}$ to outgoing real photons of the same energy and momentum $\mathbf{q}^*$, is given by

$$\begin{aligned}\epsilon_1^{\mu}\,\mathcal{M}_{\mu\nu}\,\epsilon_2^{\nu} =\ & i\vec{\sigma}\cdot(\vec{\epsilon}^* \times \vec{\epsilon})A_3(\omega,\theta) + i\vec{\sigma}\cdot(\hat{q}' \times \hat{q})\vec{\epsilon}^* \cdot \vec{\epsilon} A_4(\omega,\theta)\\ & + (i\vec{\sigma}\cdot(\vec{\epsilon}^* \times \hat{q})\vec{\epsilon}\cdot\hat{q}^* - i\vec{\sigma}\cdot(\vec{\epsilon} \times \hat{q}^*)\vec{\epsilon}^*\cdot\hat{q})A_5(\omega,\theta)\\ & + (i\vec{\sigma}\cdot(\vec{\epsilon}^* \times \hat{q}^*)\vec{\epsilon}\cdot\hat{q}^* - i\vec{\sigma}\cdot(\vec{\epsilon} \times \hat{q})\vec{\epsilon}^*\cdot\hat{q})A_6(\omega,\theta)\\ & + i\vec{\sigma}\cdot(\hat{q}^* \times \hat{q})\vec{\epsilon}^*\cdot\hat{q}\vec{\epsilon}\cdot\hat{q}^* A_7(\omega,\theta),\end{aligned} \tag{3}$$

where $\hat{q}$ indicate unit vectors. By crossing symmetry the functions A_i are odd in ω. The leading pieces in an expansion in powers of ω are given by low-energy theorems, and the next terms contain the spin polarizabilities γ_i

$$\begin{aligned}A_3(\omega,\theta) =\ & \frac{e^2\omega}{2m_{\rm N}^2} Q(Q+2\kappa) - (Q+\kappa)^2\cos(\theta) + 4\pi\omega^3(\gamma_1 + \gamma_5\cos\theta)\\ & - \frac{e^2 Q(Q+2\kappa)\omega^3}{8m_{\rm N}^4} + O(\omega^5),\end{aligned} \tag{4}$$

$$A_4(\omega,\theta) = -\frac{e^2\omega}{2m_{\rm N}^2}(Q+\kappa)^2 + 4\pi\omega^3\gamma_2 + O(\omega^5), \tag{5}$$

$$A_5(\omega,\theta) = -\frac{e^2\omega}{2m_{\rm N}^2}(Q+\kappa)^2 + 4\pi\omega^3\gamma_4 + O(\omega^5), \tag{6}$$

$$A_6(\omega,\theta) = -\frac{e^2\omega}{2m_{\rm N}^2} Q(Q+\kappa) + 4\pi\omega^3\gamma_3 + O(\omega^5), \tag{7}$$

$$A_7(\omega,\theta) = O(\omega^5), \tag{8}$$

where the charge of the nucleon is $Q = (1+\tau_3)/2$ and its anomalous magnetic moment is $\kappa = (\kappa_{\rm s} + \kappa_v\tau_3)/2$. Only four of the polarizabilities are independent because three are related as $\gamma_1 + \gamma_2 + 2\gamma_4 = 0$. It is important to note that the polarizabilities are also isospin-dependent.

The nucleon polarizabilities have been studied in a number of theoretical approaches based on dispersion relations [3,10–15], phenomenological Lagrangians [16–20], constituent quark models [21–23], chiral-soliton type of models [24–28] and lattice QCD using the external electromagnetic field method in quenched [29,30] and unquenched approximation [31]. Additional insights into the polarizabilities have come from chiral perturbation theory (ChPT), an effective theory of the low-energy strong interaction [32,33], specifically from heavy baryon chiral perturbation theory (HBChPT) which is an extension of ChPT that includes the nucleon [34,35]. The ChPT is based on the assumption that (i) masses of the light quarks u, d and s can be treated as a perturbation compared to the typical hadronic scale of 1 GeV and (ii) in the limit of zero quark masses, the chiral symmetry is spontaneously broken to its vectorial subgroup and the resulting pseudoscalar mesons are the Goldstone bosons. ChPT is a systematic low-energy expansion around the chiral limit. Though it is a well defined quantum field theory (QFT), it has to be renormalized order by order. The ChPT gained importance after the work of Weinberg [32], which showed how loop diagrams can be incorported systematically to the tree-level diagrams. It was shown that the loop diagrams are suppressed by powers of $(E/\Lambda)^2$, where E is the energy/four momentum and Λ is the chiral symmetry breaking scale (the mass of the first non-Goldstone boson resonance $\Lambda \simeq M_\rho$). The method was systematized by Gasser and Leutwyler [33]. The low-energy expansion in ChPT involves two parameters: the external momentum q and the Goldstone masses or the quark mass $\mathcal{M}$. The expansion is carried out by keeping the ratio $\mathcal{M}/q^2$ fixed.

The straightforward extension of the ChPT to baryon sector leads to the non-vanishing mass of the baryon in the chiral limit. It should be noted that the baryon four-momenta are never small compared to the chiral symmetry breaking scale. Hence, HBChPT was developed. In HBChPT, baryons are treated as very heavy and only baryon three-momenta relative to the rest mass will count. Using the idea of heavy quark effective field theory, Jenkins and Manohar [34] gave a new formulation for baryon ChPT, by taking extreme non-relativistic limit of the fully relativistic theory by expanding in powers of the inverse baryon mass. Hence, the mass dependence lies in the vertices which can be expanded in powers of $1/m$. Therefore, HBChPT is a dual expansion in small momenta p vs. the chiral symmetry breaking scale and vs. inverse powers of the baryon mass.

In ChPT, the leading-order (LO) results for the nucleon polarizabilities is a prediction because no low-energy constants (LEC) appear in the LO calculations. The polarizabilities of the nucleon are given in terms of the mass of the nucleon m_{N} and pion m_π and the pion–nucleon coupling constant ($g_{\pi\mathrm{NN}}$). The first such calculations of nucleon polarizabilities within ChPT were carried out in [36,37]. However, HBChPT has an important deficiency in that the chiral perturbative series fails to converge in part of the low-energy region. The problem is generated by a set of higher-order graphs involving insertions in nucleon lines. It has been shown that infrared singularities of the various one-loop graphs occurring in the chiral perturbation series can be extracted in a relativistically invariant fashion. This procedure is known as infrared dimensional regularization (IDR) [38]. The IDR respects the constraints of chiral symmetry as expressed through the chiral ward identities. The manifestly Lorentz invariant form of baryon chiral perturbation theory (BChPT) with the IDR prescription has been successfully applied to calculate α_{N} and β_{N} and the results of these polarizabilities differ substantially from the corresponding HBChPT results [39,40]. In addition, HBChPT has been employed to analyse the virtual Compton scattering processes because, as an effective field theory, it satisfies the gauge invariance, Lorentz invariance and crossing symmetry [41]. New predictions for generalized polarizabilities have been made using HBChPT at $O(p^4)$ next to leading order

(NLO) [42–44] and, using ChPT, Compton scattering from deuteron has been computed to order $O(p^4)$ [45].

The spin-dependent (SD) pieces of the forward scattering amplitude for real photons of energy ω and momentum q is [4,46–49]

$$\epsilon_1^{\mu}\, \mathcal{M}_{\mu\nu}^{\rm SD}\, \epsilon_2^{\nu} = i e^2\, \omega\, W^{(1)}(\omega)\, \vec{\sigma} \cdot (\vec{\epsilon} \times \vec{\epsilon}^{*}) + \cdots . \tag{9}$$

From the theoretical perspective there is particular interest in the low-energy limit of the amplitude:

$$e^2 W^{(1)}(\omega) = 4\,\pi\, (f_2(0) + \omega^2\, \gamma_0^{\rm N}) + \cdots , \tag{10}$$

where γ_0 is the forward spin polarizability, which is related to the photoabsorption cross-sections for parallel (σ_+) and antiparallel (σ_-) photons and target helicities via

$$\gamma_0^{\rm N} = \frac{1}{4\pi^2} \int_W^{\infty} \frac{{\rm d}s}{s^3} \left[\sigma_-(s) - \sigma_+(s)\right], \tag{11}$$

where $W = M_\pi + M_\pi^2/(2m_{\rm N})$ is the threshold energy for an associated neutral pion in the intermediate state. The Low–Gell–Mann–Goldberger low-energy theorem states that

$$f_2(0) = -\frac{\alpha\, \kappa_{\rm N}^2}{2\, m_{\rm N}^2}, \tag{12}$$

where $\alpha = e^2/(4\pi) = 1/137.036$ is the fine-structure constant and $\kappa_{\rm N}$ is the anomalous magnetic moment of nucleon [50].

The forward spin polarizability $\gamma_0^{\rm N}$ has been calculated to $O(p^3)$ LO [51] in the framework of HBChPT yielding, at lowest order in the chiral expansion,

$$\gamma_0^{\rm N} = \frac{\alpha\, g_A^2}{24\,\pi^2\, F^2\, M_\pi^2} = 4.54 \times 10^{-4}\ {\rm fm}^4 \tag{13}$$

both for protons and neutrons, where the entire contribution comes from πN loops. (Hereafter we shall use units of 10^{-4} fm^4 for spin polarizability.) This LO calculation of spin polarizability is a prediction, because any low-energy constant associated with the polarizability enters only at next order. At LO the polarizability is given entirely by the loop contribution in terms of well-known parameters such as nucleon and pion masses and the pion–nucleon coupling constant ($g_{\pi\rm NN}$). The effect of including $\Delta(1236)$ enters in counterterms at fifth order in standard HBChPT, and has been estimated to be so large as to change the sign. The forward nucleon spin polarizability γ_0 has been computed in an extension of HBChPT with an explicit Δ in [47].

This calculation has also been carried out to NLO in the framework of HBChPT [52–55]. The contribution to $\gamma_0^{\rm N}$ up to and including NLO contributions is found to be $\gamma_0^{\rm p/n} = 4.5 - (6.9 \pm 1.5)$ – the NLO contributions are large. The corresponding relativistic chiral one-loop calculation of the forward spin polarizability was carried out by Bernard *et al* [51] and the computed value of $\gamma_0^{\rm N}$ was found to be smaller than the LO result of HBChPT. The generalized $\gamma_0^{\rm N}$ has been calculated in the Lorentz invariant formulation of BChPT to NLO which reproduces the results of LO and NLO HBChPT and demonstrates a large next-to-next leading-order (NNLO) contribution [56,57]. Electroproduction data have been used to extract $\gamma_0^{\rm N}$ using the sum rule given earlier. In particular, in [58] it was found that $\gamma_0^{\mathrm{p}} = -1.3$ and $\gamma_0^{\mathrm{n}} = -0.4$, while the more recent analysis of [59] gives a smaller value of $\gamma_0^{\mathrm{p}} = -0.6$. While rather large amount of work has been devoted, both theoretically and experimentally, to the study of the nucleon polarizabilities, very little is known about hyperon polarizabilities. However, with the advent of hyperon beams at FNAL and CERN, the experimental situation is likely to change, and this possibility has triggered a number of theoretical investigations. Already, predictions for electric and magnetic polarizabilities have been made for low-lying octet baryons in the framework of HBChPT to LO [60], and in the context of several other models, yielding a broad spectrum of predictions [61–66]. At present, no experimental data are available for the forward spin polarizabilty of the hyperons and no theoretical calculations have been published. Motivated by this situation, in this work, we extend the analysis of $SU(2)$ HBChPT to the $SU(3)$ version to compute γ_0 for hyperons. This could serve as a test of low-energy structure of QCD in the three-flavour sector. However, there is also a need to compute the spin polarizabilities in the framework of BChPT with the IDR prescription.

This paper is organized as follows. Section 2 contains an overview of the $SU(3)$ version of HBChPT relevant for calculating forward spin polarizability γ_0 of hyperon. The relevant Feynman rules for Σ^+ polarizability are listed in Appendix A, and the required loop integrals are listed in Appendix B. The explicit expressions for $\Sigma^+\pi^+(K^+)$ loops in terms of loop integrals are listed in Appendix C. In §3 explicit expressions for calculating electric polarizabilities are given. In §4 the explicit results for hyperon spin polarizability γ_0 and the corresponding numerical results are discussed. Brief conclusions are given in §5.

2. Effective Lagrangian

In this section, we discuss briefly the most general effective Lagrangian in the single baryon sector which is relevant for calculating hyperon polarizabilities. The effective Lagrangian consists of the sum of the purely mesonic ($\mathcal{L}_{\phi\phi}$) and the mesonic baryonic ($\mathcal{L}_{\phi\mathrm{N}}$) Lagrangians, respectively,

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\phi\phi} + \mathcal{L}_{\phi\mathrm{N}},$$

both of which are organized in a chiral derivative and quark mass expansion

$$\begin{aligned}\mathcal{L}_{\phi\phi} &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots,\\ \mathcal{L}_{\phi\mathrm{N}} &= \mathcal{L}_{\phi\mathrm{N}}^{(1)} + \mathcal{L}_{\phi\mathrm{N}}^{(2)} + \mathcal{L}_{\phi\mathrm{N}}^{(3)} + \mathcal{L}_{\phi\mathrm{N}}^{(4)} + \cdots,\end{aligned} \tag{14}$$

where the superscripts in $\mathcal{L}_{\phi\phi}(\mathcal{L}_{\phi\mathrm{N}})$ refer to the order in the expansion. The leading term in mesonic Lagrangian ($\mathcal{L}_{\phi\phi}^{(2)}$) contains two derivatives or one quark mass matrix. The quark masses are counted as $O(q^2)$. The mesonic Lagrangian contains only even powers whereas the baryonic Lagrangian involves both even and odd powers due to the additional spin degrees of freedom. The complete one-loop calculation in the meson–baryon system will have three terms of order p, the loop contribution of order p^2, p^3 and also counterterms of order p^2 and p^3. Hence, complete one-loop diagram to order $O(p^3)$ involves one insertion from $\mathcal{L}_{\phi\mathrm{N}}^{(1)}$ and tree-level diagrams. In HBChPT, the amplitude for any process has a chiral dimension D, and the chiral dimension for the meson–baryonic system is given by

$$D = 2L + 1 + \sum_d (d-2) N_d^M + \sum_d (d-1) N_d^{\mathrm{MB}},$$

where D keeps track of the powers of external momenta and meson masses, L is the number of loops, N_d^M is the number of vertices of dimension d from the meson Lagrangian with $d = 2, 4, \ldots,$ d corresponds to the chiral dimension of the Lagrangian and N_d^{MB} corresponds to the meson–baryon vertices with $d = 1, 2, 3 \ldots$. Therefore, tree diagrams start to contribute at order (p) and one-loop diagram at order p^3. Two loop diagrams are suppressed by two more powers of p so that within one-loop approximation one should consider tree diagrams from $\mathcal{L}^{(1)}_{\phi\mathrm{N}} + \mathcal{L}^{(2)}_{\phi\mathrm{N}} + \mathcal{L}^{(3)}_{\phi\mathrm{N}}$.

The lowest-order $SU(3)$ HBChPT Lagrangian involving the pseudoscalar meson octet, ϕ

$$\phi = \sqrt{2} \begin{pmatrix} (1/\sqrt{2})\pi^0 + (1/\sqrt{6})\eta & \pi^+ & K^+ \\ \pi^- & -(1/\sqrt{2})\pi^0 + (1/\sqrt{6})\eta & K^0 \\ K^- & \bar{K}^0 & -(2/\sqrt{6})\eta \end{pmatrix} \quad (15)$$

and the baryon octet B

$$B = \begin{pmatrix} (1/\sqrt{2})\Sigma^0 + (1/\sqrt{6})\Lambda & \Sigma^+ & p \\ \Sigma^- & -(1/\sqrt{2})\Sigma^0 + (1/\sqrt{6})\Lambda & n \\ \Xi^- & \Xi^0 & -(2/\sqrt{6})\Lambda \end{pmatrix} \quad (16)$$

consist of two basic pieces: the lowest-order chiral effective meson Lagrangian $\mathcal{L}^{(2)}_{\phi\phi}$ [32,33]

$$\mathcal{L}^{(2)}_{\phi\phi} = \frac{F^2}{4} \langle \nabla_\mu U \, \nabla^\mu U^\dagger + \chi_+ \rangle \quad (17)$$

and the lowest-order meson–baryon Lagrangian [4,34,35]:

$$\mathcal{L}^{(1)\ \mathrm{HBChPT}}_{\phi\, B} = \langle \bar{B}(i\, v \cdot D) B \rangle + \frac{D}{F_0} \langle \bar{B}\, S^\mu \{u_\mu, B\} \rangle + \frac{F}{F_0} \langle \bar{B}\, S^\mu [u_\mu, B] \rangle, \quad (18)$$

where the superscripts $(1, 2)$ attached to the above Lagrangians denote their low-energy dimension and the symbols $\langle\ \rangle$, $[\ ]$, $\{\ \}$ denote the trace over flavour matrices, commutator and anticommutator, respectively. We use the following notations: $U = u^2 = \exp(i\phi/F_0)$, where F_0 is the octet decay constant (in our calculations we use $F_0 = F_\pi = 92$ MeV), $u_\mu = i\{u^\dagger, \nabla_\mu u\}$; ∇_μ and D_μ are the covariant derivatives acting on the chiral and baryon fields, respectively, including external vector (v_μ) and axial (a_μ) fields:

$$\begin{aligned} \nabla_\mu U &= \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu), \\ D_\mu B &= \partial_\mu B + [\Gamma_\mu, B] \end{aligned} \quad (19)$$

with Γ_μ being the chiral connection given by

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger (v_\mu + a_\mu) u - \frac{i}{2} u (v_\mu - a_\mu) u^\dagger. \quad (20)$$

The covariant spin operator is $S_\mu = \frac{i}{2}\gamma_5\,\sigma_{\mu\nu}v^\nu$, obeying the following relations in d dimensions [4]:

$$S\cdot v = 0\,,\quad S^2 = \frac{d-1}{4},\quad \{S_\mu, S_\nu\} = \frac{1}{2}(v_\mu\,v_\nu - g_{\mu\nu}),$$
$$[S_\mu, S_\nu] = i\,\epsilon_{\mu\nu\alpha\beta}\;v^\alpha S^\beta. \tag{21}$$

Finally, $\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$ with $\chi = 2B\mathcal{M} + \cdots$, where $B = |\langle 0|\bar{q}q|0\rangle|/F^2$ is the quark vacuum condensate parameter and $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, \hat{m}_\mathrm{s}\}$ is the mass matrix of current quarks (we work in the isospin symmetry limit with $\hat{m}_u = \hat{m}_d = \hat{m} = 7$ MeV. The mass of the strange quark $\hat{m}_\mathrm{s}$ is related to the non-strange one via $\hat{m}_\mathrm{s} \simeq 25\,\hat{m}$). The parameters D and F are fixed from hyperon semileptonic decays to be $D = 0.8$ and $F = 0.46$ with $D + F = g_A = 1.26$ being the nucleon axial charge. In the above equations, m denotes the average baryon mass in the chiral limit.

3. Electromagnetic polarizabilities α, β

The electric (α) and magnetic (β) polarizabilities for octet baryons were calculated by Bernard *et al*. The relevant loop diagrams and the details can be found in [60]. For the calculation of α, the contributions of both the pion and kaon loops have been taken into account. The electric polarizabilities in terms of the D and F axial gauge couplings [67] are then given by [60]

$$\begin{aligned}
\alpha_\mathrm{n} &= (D+F)^2\Gamma_\pi + (D-F)^2\Gamma_K,\\
\alpha_\mathrm{p} &= (D+F)^2\Gamma_\pi + \left(\frac{2}{3}D^2 + 2F^2\right)\Gamma_K,\\
\alpha_{\Sigma^-} &= \left(\frac{2}{3}D^2 + 2F^2\right)\Gamma_\pi + (D-F)^2\Gamma_K,\\
\alpha_{\Sigma^0} &= 4F^2\Gamma_\pi + (D^2+F^2)\Gamma_K,\\
\alpha_{\Sigma^+} &= \left(\frac{2}{3}D^2 + 2F^2\right)\Gamma_\pi + (D+F)^2\Gamma_K,\\
\alpha_\Lambda &= \frac{4}{3}D^2\Gamma_\pi + \left(\frac{1}{3}D^2 + 3F^2\right)\Gamma_K,\\
\alpha_{\Xi^-} &= (D-F)^2\Gamma_\pi + \left(\frac{2}{3}D^2 + 2F^2\right)\Gamma_K,\\
\alpha_{\Xi^0} &= (D-F)^2\Gamma_\pi + (D+F)^2\Gamma_K,
\end{aligned}$$

where

$$\Gamma_\pi = \frac{5e^2}{384\pi^2F_\mathrm{P}^2}\frac{1}{M_\pi} \approx 6.7\times 10^{-4}\ \mathrm{fm}^3,$$

$$\Gamma_K = \frac{5e^2}{384\pi^2F_\mathrm{P}^2}\frac{1}{M_K} \approx 1.9\times 10^{-4}\ \mathrm{fm}^3.$$

Table 1. The electric polarizabilities for octet baryons (in units of 10^{-4} fm^3).

α	$D = 0.8$, $F = 0.5$	$D = 0.75$, $F = 0.5$
p	13.0	12.1
n	11.4	10.5
Σ^-	6.3	5.9
Σ^0	8.3	8.2
Σ^+	9.4	8.8
Λ	7.5	6.8
Ξ^-	2.3	2.1
Ξ^0	3.8	3.4

In the above equations, $F_{\rm P} \approx 100$ MeV is the pseudoscalar decay constant, $M_\pi = 139.57$ MeV, $M_K = 493.65$ MeV and $e^2/4\pi = 1/137.036$. Neglecting the mass splittings, the magnetic polarizability (β), to the leading order in the $1/m_{\rm B}$ expansion is obtained from the result,

$$\alpha_{\rm B} = 10\beta_{\rm B}.$$

The electric polarizabilities for the octet baryons are given in table 1.

4. Forward spin polarizability γ_0

In order to calculate the forward spin polarizability γ_0, we work in the Breit frame in which the sum of the incoming and outgoing baryon three-momenta vanishes. We work in the Weyl (temporal) gauge $A_0 = 0$, which in the language of HBChPT means $v \cdot \epsilon = 0$, where $v_\mu = (1, 0, 0, 0)$ is the baryon four-velocity. At $O(p^3)$ only the loop diagrams contribute to γ_0. To one loop, the hyperon polarizabilities are pure loop effects. At LO these loop diagrams have insertions only from $\mathcal{L}_{\phi \rm B}^{(1)\ \rm HBChPT}$. Figure 1 shows all the possible loop diagrams, which contribute to γ_0 for Σ^+. Similarly for the other octet baryons the diagrams in figure 1 are the only ones which contribute to γ_0 (except that the incoming and outgoing particles are different). There do exist contact term graphs stemming from two insertions from $\mathcal{L}_{\phi\,\rm B}^{(2)\ \rm HBChPT}$ and a single insertion from $\mathcal{L}_{\phi\,\rm B}^{(3)\ \rm HBChPT}$, but these do not contribute to γ_0 and hence these diagrams are not shown in this paper. Appendix A lists the relevant Feynman rules for computing loop diagrams, while Appendix B contains the relevant loop integrals required for their evaluation. Appendix C gives the analytic results for $\Sigma^+\pi^+(K^+)$ loops contributing to the forward Compton scattering amplitude $\gamma\Sigma^+ \to \gamma\Sigma^+$. Note that both pion and kaon loops give finite contributions to γ_0 for all octet baryons.

The values of γ_0 are found from the calculation of $W^{(1)}(\omega)$ via [47]

$$\gamma_0 = \alpha \, \frac{\partial^2}{\partial \omega^2} W^{(1)}(\omega)\bigg|_{\omega=0} \tag{22}$$

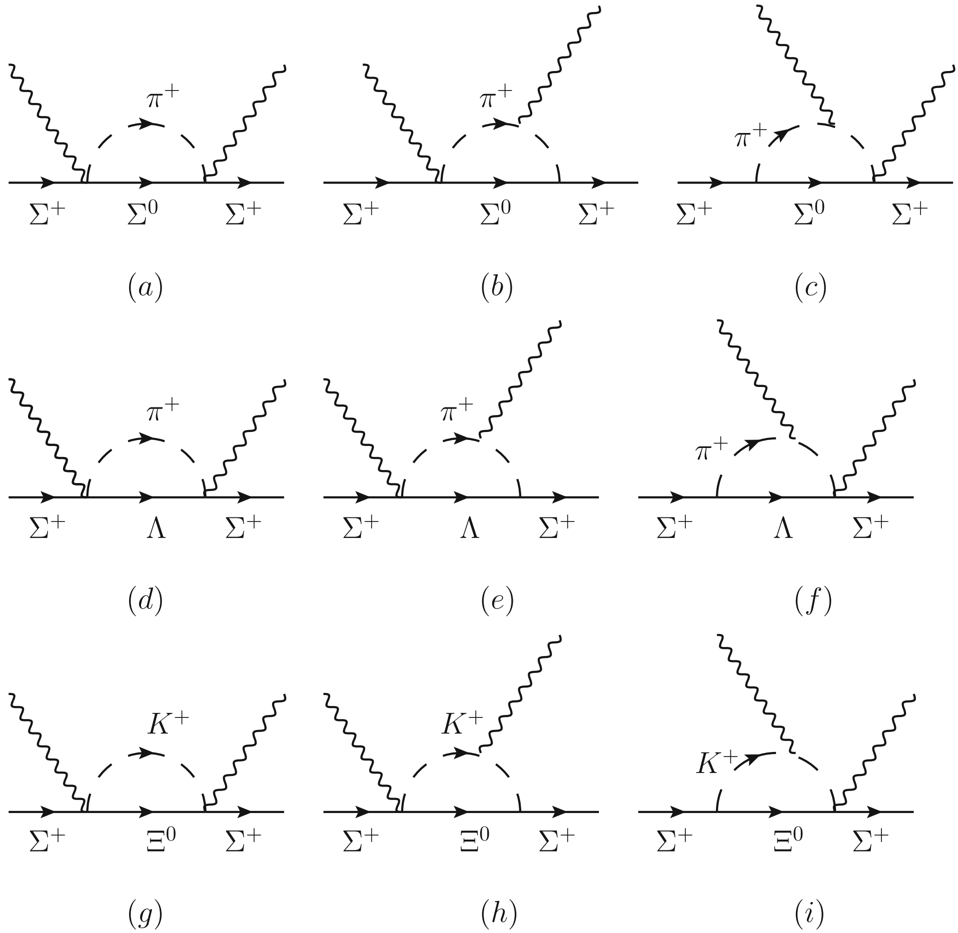

Figure 1. The one-loop diagrams contributing to forward Compton scattering of Σ^+ π^+ (K^+) at $O(p^3)$. Crossed diagrams are not shown.

and the expressions for γ_0 for all the low-lying octet baryons are listed below:

$$\gamma_0^{\mathrm{p}} = \gamma_0^{\mathrm{n}} = \frac{\alpha}{\pi^2 F_0^2}\left[\frac{(D+F)^2}{24}\left(\frac{1}{M_\pi^2}+\frac{1}{M_K^2}\right)+\frac{(D-F)^2}{96M_K^2}\right],$$

$$\gamma_0^{\Xi^0} = \frac{\alpha}{\pi^2 F_0^2}\left[\frac{(D-F)^2}{48M_\pi^2}+\frac{(D+F)^2}{32M_K^2}\right],$$

$$\gamma_0^{\Xi^-} = \frac{\alpha}{\pi^2 F_0^2}\left[-\frac{5D^2}{288M_K^2}+\frac{F^2}{32M_K^2}+\frac{(D+F)^2}{96M_K^2}+\frac{(D-F)^2}{48M_\pi^2}\right],$$

$$\gamma_0^{\Lambda} = \frac{\alpha}{\pi^2 F_0^2}\left[\frac{D^2}{144M_K^2}+\frac{F^2}{16M_K^2}+\frac{D^2}{72M_\pi^2}\right],$$

Table 2. The forward spin polarizability γ_0 of octet baryons (in units of 10^{-4} fm^4).

Baryon	The results at $O(p^3)$ with π loops	The results at $O(p^3)$ with π and K loops	$O(p^3)$ HBChPT [51]	$O(p^4)$ HBChPT and BChPT [52–54,56]	Electroproduction data
p	4.50	4.86	4.5	$4.5-(6.9+1.5)$	-1.3 [58], -0.6 [59]
n	4.50	4.86	4.5	$4.5-(6.9-1.5)$	-0.4 [58]
Σ^+	1.20	1.38			
Σ^0	0.60	0.70			
Σ^-	1.20	1.22			
Λ	0.60	0.70			
Ξ^-	0.16	0.26			
Ξ^0	0.16	0.43			

$$\gamma_0^{\Sigma^0} = \frac{\alpha}{\pi^2 F_0^2}\left[\frac{(D+F)^2}{96M_K^2}+\frac{(D-F)^2}{96M_K^2}+\frac{F^2}{24M_\pi^2}\right],$$

$$\gamma_0^{\Sigma^-} = \frac{\alpha}{\pi^2 F_0^2}\left[\frac{(D-F)^2}{48M_K^2}+\frac{D^2}{72M_\pi^2}+\frac{F^2}{24M_\pi^2}\right],$$

$$\gamma_0^{\Sigma^+} = \frac{\alpha}{\pi^2 F_0^2}\left[\frac{(D+F)^2}{48M_K^2}+\frac{D^2}{72M_\pi^2}+\frac{F^2}{24M_\pi^2}\right]. \qquad (23)$$

Note that, in the case of nucleon, the neglect of the kaon loops contributions, reproduces the well-known result of $SU(2)$ HBChPT [51]. The other results for spin polarizabilities are new predictions. In table 2, the second and third columns give the contributions to γ_0 from π and $\pi + K$ loops, respectively. In table 2 we also present the results for the nucleon γ_0 obtained in HBChPT at $\mathcal{O}(p^3)$ [51], in HBChPT and BChPT at order $\mathcal{O}(p^4)$ [52–54,56] and from the analysis of electroproduction data [58,59]. For computing the polarizabilities, we use $F_0 = 92$ MeV, $D = 0.8$, $F = 0.46$, $M_\pi = 139.57$ MeV and $M_K = 493.65$ MeV.

5. Conclusions

We have presented the LO contribution to spin-dependent and spin-independent Compton scattering in the framework of HBChPT. In LO HBChPT, all these contributions are meson loop effects, with no counterterm or resonance exchange contribution and hence are a test for the chiral sector of three-flavour QCD. There is a small but finite contribution from kaon loops for γ_0 for all the low-lying octet baryons. The γ_0 of the proton and neutron are remarkably well described and reproduces the results of the LO calculation of $SU(2)$ HBChPT and it remains to be seen how the predictions of the other baryons will compare with the future experiments. It should be noted that the baryon mass is not extremely large and hence there will be significant corrections from $1/m_{\rm N}$ terms. Therefore, on the theoretical side, one needs to perform $O(p^4)$ calculations to improve the predictions of the polarizabilities and to test the convergence of the chiral expansion. At order $O(p^4)$ within one-loop approximation, the loop diagrams will have one insertion from $\mathcal{L}^{(2)}_{\phi\mathrm{N}}$. Additional calculations are needed to compute γ_0 in the framework of BChPT with the IDR prescription to test the LO and NLO HBChPT results. Work in this direction is in progress.

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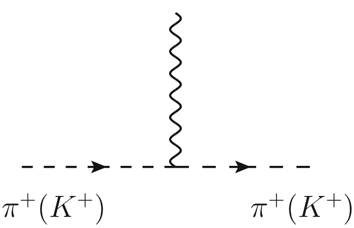
Appendix A. Feynman rules

Vertices from $\mathcal{L}^{(2)}_{\phi\phi}$

1. Photon–meson coupling: k_1 (in-momentum) and k_2 (out-momentum) stand either for π or for K mesons

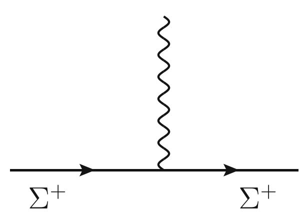
$$ie\epsilon \cdot (k_1 + k_2)$$



Vertices from $\mathcal{L}^{(1)\,\mathrm{HBChPT}}_{\phi\,\mathrm{B}}$

2. Photon-baryon coupling

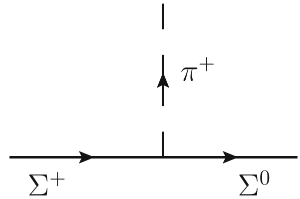
$$ie(\epsilon \cdot v)$$



Meson–baryon couplings

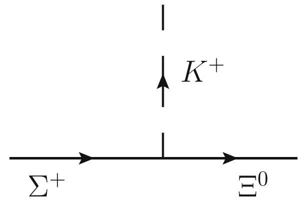
3. $\pi\Sigma\Sigma$ coupling

$$-2\,\frac{F}{F_0} s \cdot k$$



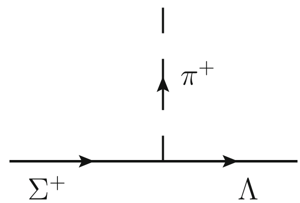
4. $K\,\Sigma\,\Xi$ coupling

$\frac{2(D+F)}{\sqrt{2}F_0}s \cdot k$



5. $\pi\,\Sigma\,\Lambda$ coupling

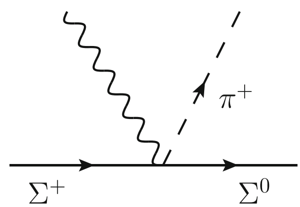
$\frac{2D}{\sqrt{3}F_0}s \cdot k$



Photon–meson–baryon couplings

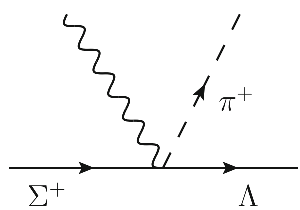
6. $\gamma\pi\,\Sigma\,\Sigma$ coupling

$-\frac{2eF}{F_0}s \cdot \epsilon$



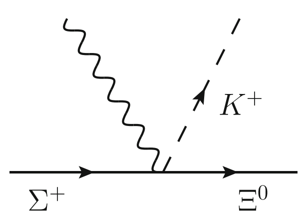
7. $\gamma\pi\,\Sigma\,\Lambda$ coupling

$\frac{2eD}{\sqrt{3}F_0}s \cdot \epsilon$



8. $\gamma\,K\,\Sigma\,\Xi$ coupling incoming photon

$\frac{2e(D+F)}{\sqrt{2}F_0}s \cdot \epsilon$



Appendix B. Loop integrals

Here, we have defined all the loop functions which occur in our calculations and we have given these functions in closed analytical form as far as possible. In the following all propagators are understood to have an infinitesimal imaginary part. The results of the integral are for real photons. The complete list of integrals can be found in [4]:

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d i} \frac{1}{M_\pi^2 - k^2} = \Delta_\pi, \tag{B1}$$

where

$$\Delta_\pi = 2M_\pi^2 \left[L + \frac{1}{16\pi^2} \log\left(\frac{4\pi}{\lambda}\right) + \mathcal{O}(d-4) \right],$$

$$L = \frac{\lambda^{d-4}}{16\pi^2} \left[\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \log(4\pi) \right] \tag{B2}$$

has a pole at $d = 4$. Here $\gamma_E = 0.557215$ and λ is the scale in dimensional regularization scheme used in the evaluation of integrals.

The relevant integrals are

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d i} \frac{(1, k^\mu, k^\mu k^\nu)}{(v \cdot k - \omega)[M_\pi^2 - k^2]} = (J_0^\pi(\omega), v_\mu J_1^\pi(\omega), g^{\mu\nu} J_2^\pi(\omega) + v^\mu v^\nu J_3^\pi(\omega)), \tag{B3}$$

where

$$J_0^\pi(\omega) = -4L\omega + \frac{\omega}{8\pi^2}\left(1 - 2\log\frac{M_\pi}{\lambda}\right) - \frac{1}{4\pi^2}\sqrt{M_\pi^2 - \omega^2} \arccos\left(-\frac{\omega}{M_\pi}\right) + \mathcal{O}(d-4), \tag{B4}$$

$$J_1^\pi(\omega) = \omega J_0^\pi(\omega) + \Delta_\pi, \tag{B5}$$

$$J_2^\pi(\omega) = \frac{1}{d-1}[(M_\pi^2 - \omega^2) J_0^\pi(\omega) - \omega\Delta_\pi], \tag{B6}$$

$$J_3^\pi(\omega) = \omega J_1^\pi(\omega) - J_2^\pi(\omega). \tag{B7}$$

Appendix C. $\Sigma^+\pi^+(K^+)$ loops in forward Compton scattering

Using the loop integrals defined in Appendix B, the $\Sigma^+ + \pi^+(K^+)$ loop diagrams of figure 1 can be written as

$$\mathrm{Amp}_{a+a'}^{\Sigma^+\pi^+} = C_1[S\cdot\epsilon^*, S\cdot\epsilon][J_0^\pi(\omega) - J_0^\pi(-\omega)], \tag{C1}$$

$$\mathrm{Amp}_{b+c+b'+c'}^{\Sigma^+\pi^+} = C_2[S\cdot\epsilon^*, S\cdot\epsilon]\frac{\partial}{\partial M_\pi^2}\int_0^1 [J_2^\pi(\omega z) - J_2^\pi(-\omega z)]\mathrm{d}z, \tag{C2}$$

$$\mathrm{Amp}_{d+d'}^{\Sigma^+\pi^+} = D_1[S\cdot\epsilon^*, S\cdot\epsilon][J_0^{\pi}(\omega) - J_0^{\pi}(-\omega)], \tag{C3}$$

$$\mathrm{Amp}_{e+f+e'+f'}^{\Sigma^+\pi^+} = D_2[S\cdot\epsilon^*, S\cdot\epsilon]\frac{\partial}{\partial M_\pi^2}\int_0^1 [J_2^{\pi}(\omega z) - J_2^{\pi}(-\omega z)]\mathrm{d}z, \tag{C4}$$

$$\mathrm{Amp}_{g+g'}^{\Sigma^+K^+} = E_1[S\cdot\epsilon^*, S\cdot\epsilon][J_0^{K}(\omega) - J_0^{K}(-\omega)], \tag{C5}$$

$$\mathrm{Amp}_{h+i+h'+i'}^{\Sigma^+K^+} = E_2[S\cdot\epsilon^*, S\cdot\epsilon]\frac{\partial}{\partial M_K^2}\int_0^1 [J_2^{K}(\omega z) - J_2^{K}(-\omega z)]\mathrm{d}z, \tag{C6}$$

where

$$C_1 = 2i\left(\frac{e\,F}{F_0}\right)^2, \qquad C_2 = -8i\left(\frac{e\,F}{F_0}\right)^2, \tag{C7}$$

$$D_1 = \frac{2i}{3}\left(\frac{e\,D}{F_0}\right)^2, \qquad D_2 = -\frac{8i}{3}\left(\frac{e\,D}{F_0}\right)^2, \tag{C8}$$

$$E_1 = i\left(\frac{e\,(D+F}{F_0}\right)^2, \quad E_2 = -4i\left(\frac{e\,(D+F}{F_0}\right)^2. \tag{C9}$$

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