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Heavy flavour hadron spectroscopy: An overview

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Abstract. A comprehensive overview and some of the theoretical attempts towards understanding heavy flavour hadron spectroscopy are presented. Apart from the conventional quark structure (quark, antiquarks structure for the mesons and three-quarks structure of baryons) of hadrons, multiquark hadrons the hadron molecular states etc., also will be reviewed. Various issues and challenges in understanding the physics and dynamics of the quarks at the hadronic dimensions are highlighted. Looking into the present and future experimental prospects at different heavy flavour laboratories like BES-III, CLEO-c, BaBar, Belle, LHC etc., the scope for theoretical extensions of the present knowledge of heavy flavour physics would be very demanding. In this context, many relevant contributions from the forthcoming PANDA Facility are expected. Scopes and outlook of the hadron physics at the heavy flavour sector in view of the future experimental facilities are highlighted.

Keywords. Heavy flavour; spectroscopy; potential models; exotics.

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1. Introduction

In recent years, the investigation of hadrons containing heavy quarks has dominated in the experimental efforts to improve our understanding of quantum chromodynamics (QCD) at different hadronic scale. Experimental groups at Belle, BaBar, DELPHI, CLEO, CLEO-c, CDF, LHC, BES-III, SELEX etc., are providing additional statistically significant data on various hadronic properties. Of late, a plethora of new hadronic states were discovered in the heavy flavour sector which do not really fit into our understanding of the conventional mesons ($q\bar{q}$ states) and baryons (qqq states). Such states are known as the exotic hadrons. Apart from the challenges posed by the exotics, there are also many states which are the radial and orbital excited states of the known hadrons. Many of the newly observed states are above the $D\bar{D}$ and $B\bar{B}$ thresholds [1]. Very recently, larger data samples of many hadronic states are observed at the BES-III upgraded facility, KEK at Japan and other Charm and Beauty factories elsewhere and it is expected that they will continue to supply valuable data for many more years. Later on, the LHC experiments at CERN and Panda at GSI etc., are also expected to offer future opportunities and challenges in the field of heavy flavour physics. At the same time, on the theoretical front, lattice quantum chromodynamics (QCD), heavy quark effective theory (HQET), non-relativistic quantum chromodynamics (NRQCD), non-relativistic quark models (NRQM), QCD sum rules, potential models, etc. are highly evolved to provide precise quantitative predictions of various hadronic properties [2–10].

2. An overview of recent experimental observations

After having played a major role in the foundation of QCD, heavy hadron spectroscopy has witnessed a renaissance in the last few years driven by recent experimental reports of various hadronic states by the high-energy experimental facilities the world over. So far, the greatest activity has occurred in the charm sector with energy range 3–5 GeV. In the bound state region of energy less than 4 GeV, major contributions in the charm sector came from BES and CLEO, while above 4 GeV, the charmonium like states have been reported from CLEO, Belle and BaBar. The recent discoveries include a number of narrow D_{sJ} states and a host of charmonium like X, Y, Z states by Belle, BaBar, CLEO, BES-III, LHCb, etc. Although the spectrum of charmonium-bound states is complete, we are far from understanding the true nature of the $c\bar{c}$ hyperfine interaction. We do not know if there is an intrinsic long-range hyperfine interaction, and if so, what is its origin? Is there a vector component in the confinement part of the potential? With the discovery of η_b by the BaBar B-factory at SLAC [11], better refinement on the theoretical estimation of the hyperfine interaction among $Q\bar{Q}$ became possible. Though the spectroscopy of quarkonia are now getting established experimentally, the excited states of open flavour mesons are not yet well established. About five years ago, all that was known above the $D\bar{D}$ threshold was the four-vector states Ψ (3770, 4040, 4160 and 4415). Since then there has been significant amount of data by CLEO, Belle and BaBar about the properties of D and D_s mesons. However, the great excitement, often referred to as the renaissance in hadron spectroscopy, has come from the discovery of a host of new states $X(3872)$, X, Y, Z (in the range 3940), $Y(4260)$ from Belle and BaBar and more recently $X'(4160)$, $X''(4324)$, $X'''(4660)$ etc. [12–14]. The challenges possed by these new states include the right identification with the proper J^{PC} values and their decay modes. E.g., the $X(3872)$ state does not easily fit in the charmonium spectrum. As its mass is very close to $M(D) + M(D^*)$, the most popular conjecture is that it is a weakly bound D–D^* molecular state [15–17]. Besides, there are other conjectures for the state such as the hybrid $c\bar{c}g$, multiquark states $(qq\bar{q}\bar{q})$, etc. One needs more high precision measurements for its branching ratio, particularly the $B(X \to DD\pi)$. Each of these charmonium-like states, X, Y, $Z(3940)$ is produced in a different production channel and each decays into a different decay channel. Some of these may be the orbital excited states of the charmonium, e.g., $X(3943)$ as $\eta_c''(3S)$, $Z(3929)$ as the $\chi_{c2}'(2^3P_2)$ and the $Y(3943)$ meson is speculated to be a hybrid. The $Y(4260)$ meson observed by BaBar, CLEO and Belle is a vector meson but not likely to be a charmonium vector state. So it is suggested to be a $c\bar{c}g$ hybrid. If so, its partner states 0^{-+} and 1^{-+} ought to be also observed near this energy. So far it has not convincingly seen from the experiments. It is thus a real experimental challenge to clarify this situation before going for any theoretical conjecture seriously. In the bottonium sector, the Upsilon (1S–6S), χ_b, χ_b' and now the long-awaited η_b [11] are known and identified. However, the higher orbital states are yet to be observed. In general, the bottonium is a much better place to get insight to the quarkonium spectroscopy as the running strong coupling constant is much smaller ($\alpha_s = 0.2$) and the relativistic effects are less important compared to the charmonium case. For the Upsilon (1^{--}) states, only their masses, total widths and branching fractions for leptonic, radiative and $\Upsilon(nS) \to \pi^+\pi^-\Upsilon(n'S)$ decays are known. A scarce $\Upsilon(3S) \to \omega\chi_b(2S)$ transition has also been observed, but huge gaps remain.

Another challenging area of spectroscopic interest lies in the open flavour sector. Only the lowest 0^- and 1^- states are listed in the particle data group (PDG) [12]. However, the L3 Collaboration [18] first reported measurement of masses of the 1^3P_1 and 1^3P_2 states of B_q mesons at $5670 \pm 10 \pm 13$ and $5768 \pm 5 \pm 6$ MeV, respectively. Two years ago, DØ and CDF Collaborations have reported results on the spectroscopy of orbitally excited beauty mesons [19]. CDF found two states, 1^1P_1 and 1^3P_2 with masses $M(1^1P_1) = 5734 \pm 3 \pm 2$ and $M(1^3P_2) = 5738 \pm 6 \pm 1$ MeV. DØ also found the same states but with slightly different masses, $M(1^1P_1) = 5720.8 \pm 2.5 \pm 5.3$ and $M(1^3P_2) - M(1^1P_1) = 25.2 \pm 3.0 \pm 1.1$ MeV. In the strange sector, CDF reported two narrow $B_s(1^3P_1)$ and $B_s(1^3P_2)$ states with masses $M(B_s(1^3P_1)) = 5829.4$ and $M(B_s(1^3P_2)) = 5839$ MeV while DØ measured only the $B_s(1^3P_2)$ with mass $5839.1 \pm 1.4 \pm 1.5$ MeV.

Similar progress has been observed in the open charm sector. The BaBar Collaboration reported the observation of a charm-strange state, the $D^*_{sJ}(2317)$ [20]. It was confirmed by the CLEO Collaboration at the Cornell electron storage ring [21] and also by the Belle Collaboration at KEK [22]. Besides, BaBar has also pointed out the existence of another charm-strange meson, the $D_{sJ}(2460)$ [20]. This resonance was measured by CLEO [21] and confirmed by Belle [22]. Belle results are consistent with the spin-parity assignments of $J^P = 0^+$ for the $D^*_{sJ}(2317)$ and $J^P = 1^+$ for the $D_{sJ}(2460)$ states. Thus, these two states are well established, confirmed independently by different experiments. They present unexpected properties, quite different from those predicted by the quark potential models. If they would correspond to the standard P-wave mesons made of a charm quark and a strange antiquark, their masses would be larger [23], around 2.48 GeV for the $D^*_{sJ}(2317)$ and 2.55 GeV for the $D_{sJ}(2460)$. Therefore, they would be above the DK and D^*K thresholds, respectively, being broad resonances. However, the states observed by BaBar and CLEO are very narrow, $\Gamma < 4.6$ MeV for $D^*_{sJ}(2317)$ and $\Gamma < 5.5$ MeV for $D_{sJ}(2460)$.

Many heavy flavour baryons are also being observed in recent times at CLEO, BaBar and Belle [1]. Large number of these states include the single charm baryons and only a single unconfirmed observation of the double charm baryons has been made by Selex group. Even the positive-parity excited states are being observed. However, more refined high luminosity measurements are required to confirm their J^{PC} values. The progress in this sector is more encouraging as more number of charmed and beauty baryons are predicted previously by the extension of the Gell–Mann's $SU(3)$ quark model. Accordingly, 18 baryons with single charm, 6 baryons with double charm and one baryon with triple charm are expected to be observed in future experiments. The B-factories have already reported a large number of these baryon states. E.g. BaBar reported $\Lambda_c(2940)$, $\Omega_{cc}(2770)$, Belle has observed $\Sigma_c(2800)$, $\Sigma_c(2980)$, etc. Before the year 2006, only one bottom baryon (Λ_b) was known, now we have the Σ_b and Ξ_b. These are extremely challenging measurements resolving states at about 6 GeV separated by just 20 MeV.

During the last year many experimental groups (Belle, BaBar, BES-III) reported the discovery of a number of exotic charged charmonium-like states. More details can be found in [24,25]. Many of these exotics are interpreted as candidates for tetraquark states or dimesonic molecular states containing hidden charm.

3. Theoretical attempts

The investigation of the properties of mesons composed of a heavy quark and antiquark ($c\bar{c}$, $b\bar{c}$, $b\bar{b}$) gives very important insight into heavy quark dynamics and to the understanding of the constituent quark masses. The theoretical predictions of the heavy quarkonia $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ mesons have rich spectroscopy with many narrow states of charmonium lying under the threshold of open charm production and of bottonium lying under the threshold of B–B production. At the hadronic scale, the non-perturbative effects connected with the complicated structure of QCD vacuum play important roles. All these lead to theoretical uncertainties in the $Q\bar{Q}$ potential at large and intermediate distances. It is in this region of large and intermediate distances that most of the basic hadron resonances are formed. So the success of theoretical model predictions of the hadronic properties with respect to the new experimental results provides important information about the quark–antiquark interactions. Such information is of great interest, as it is impossible to obtain the $Q\bar{Q}$ potential starting from the basic principle of the QCD at the hadronic scale. Though lattice QCD is the hope, it still cannot provide a full quantitative description of the hadron spectroscopy.

The remarkable progress at the experimental side, with various high-energy machines for studying hadrons has opened up new challenges in the theoretical understanding of the heavy flavour hadrons. The existing results on excited heavy flavour mesons are partially inconclusive, and even contradictory in several cases. The theoretical predictions of the masses of heavy–light system for the ground state as well as the excited state are very few [26–31]. Spectroscopies of heavy flavour mesons ($Q\bar{Q}$, $Q\bar{q}$ systems) have been studied using Coulomb plus power potential (CPP$_\nu$) in the non-relativistic formalism with different choices of the potential index ν ($0.1 \le \nu \le 2.0$). A comprehensive study based on the CPP$_\nu$ model of the heavy flavour hadrons containing one or more heavy flavour quarks with minimum number of free parameters are being studied by us [10,32–36] and highlights of the results will be presented in this paper. The properties studied consist of the masses ($1S - 6S$, low-lying P-waves, D-waves and F-waves of $c\bar{c}$, $b\bar{b}$ and $b\bar{c}$ systems), the relative mean square velocities of bound quarks, mean square radii of the meson states, the decay constant $f_{P/V}$, the dilepton, digamma and digluon widths of different states and the $E1$ and $M1$ transitions rates. The heavy–light flavour sector includes the masses of a few low-lying states of $Q\bar{q}$ systems (D, D_s, B, B_s), the decay constant $f_{P/V}$, the inclusive semileptonic and leptonic branching ratios and the neutral flavour oscillations of B^0–$\bar{B}^0$ and B_s^0–$\bar{B}_s^0$ mesons. The model potential has also been extended to study the heavy flavour baryonic properties such as ground-state masses, mean square radii, hyperfine mass splitting and the magnetic moments of single heavy flavour (Qqq), double heavy flavour (QQq) and triple heavy flavour (QQQ) systems within a hypercentral formalism [37,38].

It is apparent that the predictions for the S-wave states ($1S$–$4S$) of $c\bar{c}$ systems in the potential index ν close to 1.1 and those of $b\bar{b}$ ($1S$–$6S$) states for the potential index ν around 0.8 are in good agreement with the respective experimental values and also the predicted results on the P-wave masses of $c\bar{c}$ mesons 1^1P_1(3514–3542 MeV), 1^3P_1(3514–3542 MeV), 1^3P_2(3524–3552 MeV) for the potential index $\nu = 1.0 - 1.1$ are in good agreement with the experimental values of $h_c(3526)$, $\chi_{c1}(3511)$, $\chi_{c2}(3556)$ [12] while the $1^3P_0(3414)$ at $\nu = 0.8$ matches exactly with the experimental value of $\chi_{c0}(3415)$ [12]. Similar agreement for $b\bar{b}$ states 1^3P_0(9817–9909 MeV), 1^3P_1(9831–9929 MeV) and 1^3P_2(9838–9938 MeV) for the potential index $\nu = 0.5$–0.7 are in agreement with the experimental average values of $\chi_{b0}(9859)$, $\chi_{b1}(9893)$, $\chi_{b2}(9912)$ [12]. In the same range of ν, the model predicts the h_b state around 9834–9932 MeV.

The predictions for 2^3P_2(3887–3970 MeV), 2^3P_1(3875–3958 MeV), 2^3P_0(3835–3912 MeV) and 2^1P_1(3877–3960 MeV) within the potential index between 1.0 and 1.1 for the $2P$-states of $c\bar{c}$ systems lie close to the experimental states reported by Belle group (around 3940) [39]. In the same range of potential index, $1.0 \leq \nu \leq 1.1$ the results for 3^1S_0(3895–3991 MeV) are closer to the experimental charmonium state of $X(3938)$ reported recently by Belle [39] and for 4^1S_0(4180–4325 MeV) is close to the $Y(4260)$ state reported by BaBar [40]. The predicted 2^3D_1 states (4130–4245 MeV) of the $c\bar{c}$ system in the same range of $1.0 \leq \nu \leq 1.1$ is closer to the experimental $\psi(4160, J^P = 1^{--})$ state [41]. The lone known 1^3D_1 (3770) is found to be closer to 3796 MeV at $\nu = 0.9$. The D-wave masses obtained here for $\nu = 0.9$ are close to the lattice predictions [42] for $c\bar{c}$. The other spin triplet $1D$ states ($^3D_2$, $^3D_3$) and spin singlet $^1D_2$ are expected to be narrow and are above 3770 MeV. Experimental production rates for these states in the hadronic collisions of B-meson decays are expected to be closer to that of $\psi(3770)$. It seems that the production of $c\bar{c}$ states with large relative angular momentum ($\ell \geq 2$) is suppressed and they do not mix with S- or P-waves.

The predictions for the $2P$-spin triplet states of $b\bar{b}$ meson, 2^3P_0(10214–10329 MeV), 2^3P_1(10208–10322 MeV) and 2^3P_0(10193–102304 MeV) for ν in the 0.7–0.8 range are nearer to the corresponding experimental χ_b states. Its spin singlet state 2^1P_1(10210–10234) in the same range of ν is close to the lattice predictions. The D-wave masses of $b\bar{b}$ system are found to be off the mark by around 100 MeV compared to the lone experimental state as well as with other model predictions for $\nu = 0.7$. It also supports the arguments of large angular momentum suppression for quarkonia. This shift towards the lower index for the higher angular momentum states probably suggests the orbital energy (n, ℓ) dependence on the potential strength A. However, more experimental datasets with high confidence level are required for further understanding of the spectroscopy at high ℓ values. The computed spectroscopic results for $b\bar{c}$ states are also found to lie within the same range of the potential index $0.7 \leq \nu \leq 1.1$ as compared to other model predictions.

Even though the spectroscopy of quarkonium states is well recorded experimentally, the S-wave masses of charmonium states beyond $3S$ and the bottonium states beyond $4S$ are still not very well resolved. There seemed to be mixing of other resonances nearby. E.g., the 1^{--} states such as $\psi(3770)$, $Y(4008)$, $Y(4260)$, $Y(4360)$, $X(4630)$, $Y(4660)$, $\Upsilon(10865)$, $\Upsilon(11020)$, $Y_b(10880)$, etc., may be the quarkonia states with or without mixing with the nearby resonance states. For instance, $\Upsilon(11020)$ state has recently been reported to be a mixed bottonium $\Upsilon(6S)$ and $\Upsilon(5D)$ states with a mixing angle of $\theta = 40^\circ \pm 5^\circ$ [43].

Moreover, the decay properties of the higher quarkonia states are interesting with reference to the two well-known puzzles. One is the $\rho - \pi$ puzzle [44–46] related to the branching ratio of hadronic and leptonic decays of $\psi(2S)$ states in comparison with the decays of $J/\psi(1S)$ state. The second one is the Vogel ($\Upsilon(\Delta n = 2)$) puzzle [46,47], where $\Upsilon(2S) \rightarrow \Upsilon(1S) + 2\pi$ has large branching ratio without σ (scalar meson), while $\Upsilon(3S) \rightarrow \Upsilon(1S) + 2\pi$ has large branching ratio with σ. Both the puzzles were recently being resolved by invoking these higher quarkonia states as admixtures of the respective $Q\bar{Q}$ states with $Q\bar{Q}g$ hybrids [46].

In the case of heavy–light flavour combinations (D, D_s and B, B_s) the CPP$_\nu$ potential model shows considerable shift in the potential index ν particularly for the open charm sector (D, D_s) beyond $\nu > 1.0$, while matching with the known experimental results upto $2S$ levels and with other theoretical predictions of levels upto $3S$ states.

The predicted masses of P-wave of D-mesons provide 1^3P_2(2342–2472 MeV) for $1.0 \leq \nu \leq 1.7$, 1^3P_1(2361–2464 MeV) for $1.0 \leq \nu \leq 1.5$, 1^3P_0(2312–2398 MeV) for $0.9 \leq \nu \leq 1.3$ and 1^1P_1(2269–2337 MeV) for $1.0 \leq \nu \leq 1.5$. The experimental candidate for $P = 1^+$ state of D-meson (2420–2460 MeV) observed by CLEO [48] and Belle [49] and $J^P = 0^+$ state observed in the range 2300–2400 MeV by Belle and Focus [50] lie within the predicted range.

In the case of open strange-charm mesons (D_s), the predictions for P-states provide 1^3P_2(2416–2552 MeV) for $1.0 \leq \nu \leq 2.0$ as against the latest experimental average value (PDG2008) of 2573 MeV, 1^3P_1(2397–2484 MeV) for $1.0 \leq \nu \leq 1.5$ as against the PDG(2008) value of 2460 MeV, 1^3P_0(2317–2350 MeV) for $0.8 \leq \nu \leq 1.0$ as against the recently reported value of 2317 by BaBar, CLEO and Belle group [1], while 1^1P_1 mass predicted by the CPP$_\nu$ model within the potential range of $1.0 \leq \nu \leq 2.0$ is below the experimental value of 2536 MeV [12]. The radial excitation of $D_s^*(2112)$ observed by Belle Collaboration [51] at 2715 MeV is found to be close to the 3^3S_1 state predicted in this model and at $\nu = 1.0$ as the 2^3S_1 values predicted here lie in the range of 2474–2730 MeV for $1.0 \leq \nu \leq 1.7$, which are below the experimental value in the expected range of the potential index around $\nu = 1.0$. Even higher excited states of $c\bar{s}$ system has been observed by the BaBar Collaboration [52] with spin-parity 0^+, 1^+ and 2^+ and mass at 2856±1.5±5.0 which in this case corresponds to the $2P$ state with the predicted mass range of 2^3P_2(2668–2842 MeV) for $1.0 \leq \nu \leq 1.3$, 2^3P_1(2651–2820 MeV) for $1.0 \leq \nu \leq 1.3$, 2^3P_0(2612–2858 MeV) for $1.0 \leq \nu \leq 1.5$ and 2^1P_1(2656–2935 MeV) for $1.0 \leq \nu \leq 1.5$. Thus, the present study on open charm and open strange-charm mesons are being identified with the recently discovered D- and D_s-meson states. Other predicted high angular momentum states $\ell \geq 2$ of these mesons are expected to be seen in the future experiments at BES-III, BaBar, Belle and CLEO Collaborations.

While in the case of open beauty systems (B, B_s), the spectral predictions are in better agreement with the known experimental states and with other theoretical model predictions at a potential index between $0.7 \leq \nu \leq 1.1$, the predicted mass for 1^1P_1(5724 MeV) and 1^3P_2 (5431 MeV) states at $\nu = 0.7$ of B-meson are very close to the recently observed $1^1P_1(5721 \pm 2.5 \pm 5.3$ MeV) and $1^3P_2(5738 \pm 6 \pm 1$ MeV) states by CDF and DØ [19]. In the case of B_s meson the recent CDF observation of 1^3P_1 (5829 MeV) and 1^3P_2 (5839 MeV) states lie well within the range of values predicted for 1^3P_2 (5816–5850 MeV) and 1^3P_2 (5809–5842 MeV) states in the potential index between 0.8 to 0.9. Unfortunately very few experimental data for B–B_s systems [12] exist. In future, high-luminosity B-factories are expected to provide more clean and high precision data in the open heavy flavour mesons.

4. Conclusions and summary

At the end, we summarize that the non-relativistic Coulomb plus power potential (CPP$_\nu$) with varying power index using numerical approach to solve the Schrödinger equation is an attempt to understand the nature of the interquark potential and their parameters that provide us the spectroscopic properties as well as the decay properties of the $Q\bar{Q}$ system with the potential index between 0.7 and 1.1. It also provides us the importance of the quark mass parameters and the state dependence on the potential strength for studying the spectral properties. The radial wave functions obtained from the study are not only important for determining hyperfine and fine splitting of their mass spectra but also essential inputs for evaluating decay constants, decay rates, NRQCD parameters and production cross-sections of quarkonia states.

In the heavy–light open flavour sector (D, D_s, B, B_s), the spectroscopic parameters are successfully employed to obtain the semileptonic, leptonic and flavour oscillations of the neutral B^0, B_s^0 mesons. The binding energy effect considered through the effective mass for the constituent quarks ($m_{Q/q}^{\mathrm{eff}}$) are found to be important in many of their decay properties. The spectroscopic mass difference due to the hyperfine/fine splitting are found to be sensitive to the choice of quark mass parameters. A closer look at the different properties of the heavy flavour mesons studied using phenomenological models reveals strong correlation between the model quark mass ($m_{Q/q}$) parameter and the confinement strength (A). A better understanding of the constituent mass for the quarks that do depend on the constituents of a hadron has been obtained.

Inspired by recent experimental observations of charmed and bottom baryons [1,53,54], we have investigated the masses of heavy baryons (Qqq, QQq and QQQ) systematically using the hypercentral model. The chromomagnetic (spin-hyperfine) splittings of the charmed baryons and of the bottom baryons obtained from this study agree well with the recent experimental data. The predicted results are also consistent with the recently observed Ξ_b^* state by CDF Collaboration [53]. The predictions for the masses of baryons in the single heavy flavour (qqQ) sector, double heavy flavour (QQq) sector and triple heavy flavour (QQQ) sector would be tested by the future heavy flavour high luminosity experiments.

It is seen that the three-body description based on hypercentral coordinates and the nature of the confinement potential assumed in the hCPP$_\nu$ model for heavy flavour baryons, have played significant roles in bringing out a possible saturation property of the interquark interactions within the heavy baryons, as seen from the mass saturation of baryons for the potential power index $\nu > 1$. We also observed that the model quark mass parameters contribute significantly to the right splitting of $J = 3/2$ and $J = 1/2$ states. The predictions of magnetic moments were found to be less sensitive to the choice of the potential power index ν [38].

It is interesting to note that the parameter free predictions of the magnetic moments using the hypercentral Coulomb plus power potential (hCPP$_\nu$) model do not vary appreciably with different choices of ν ranging from 0.5 to 2.0 and the magnetic moments of the single heavy flavour baryons with spin 1/2 are in accordance with the predictions of the relativistic quark model as well as with the non-relativistic approximation reported by [55].

From the study reported in [10], we found that $\psi(4040)$ is an admixture of 3^3S_1 and 3^3D_1 with mixing angle $\theta = 11.07^\circ$ corresponding to 96.31% of $3S$ state and 3.69% of $3D$ state with its leptonic decay width of 0.896 keV which is in close agreement with the experimental value of 0.86 ± 0.07 keV compared to 0.959 keV obtained for the pure $3S$ description of the state. The leptonic decay widths of $\psi(4160)$ obtained with the mixing configuration of $(3^3S_1, 3^3D_1)$ and $(4^3S_1, 2^3D_1)$ are in agreement with the experimental value 0.48 ± 0.22 and lie within the error bar reported by Belle and BES Collaborations [56,57] but completely in disagreement with the value of 0.83 ± 0.07 reported by [12]. Though $Y(4260)$ can be obtained by $4S$–$2D$ admixture state with mixing angle $\theta = 14.44^\circ$ that predicts its leptonic decay width as 0.588 keV, the mixing may not be possible as the $4S$ and $2D$ masses differ by more than 200 MeV. So, we consider $Y(4260)$ close to the $c\bar{c}(4S)$ state with its predicted leptonic decay width of 0.65 keV. However, experimental determination of this width is awaited. Though experimentally, the J^{PC} for $Z(4443)$ is not known, our predicted $c\bar{c}$ $(5S)$ is very close to this state, while the state $\psi(4421 \pm 4)$ does not qualify to be the pure $5S$ state or S–D admixture. Among the other charmonia-like states, the present study strongly favour $Y(4360)$ as the admixture of $(4^3S_1, 4^3D_1)$ with mixing angle $\theta = 40.33^\circ$ whose leptonic decay width is then predicted as 0.326 keV and $Y(4660)$ as admixture of $(6^3S_1, 5^3D_1)$ with $\theta = 31.05^\circ$ that yields its leptonic decay width as 0.259 keV.

We look forward to see the experimental leptonic decay widths of $Y(4260)$, $X(4630)$, $Y(4360)$ and $Y(4660)$ states before making further conclusion about their status. However, the states $\psi(4160)$, $\psi(4415)$, $\Upsilon(10860)$ and $\Upsilon(10996)$ do not qualify to be either pure S-wave or admixtures and they may be treated as exotic states as listed in [58]. $Y_b(10888)$ is identified to be close to the pure $b\bar{b}$ $(6S)$ 1^{--} state with its leptonic decay width of 0.16 keV and $\Upsilon(11019)$ is identified to be the $b\bar{b}(7S)1^{--}$ state whose predicted leptonic width of 0.134 keV is in very good agreement with the experimental result of 0.13 ± 0.03 keV.

The most challenging problem at present is the description of the recent observation of states such as $X(1835)$ observed at BES, the observations of $X(3940)4$, $Y(3940)$ at Belle and the observation of $Y(4260)$ at BaBar. These recent observations of X, Y and Z have clarified that potential models suffer from large systematic uncertainties in this region and that the inclusion of at least heavy–light meson degrees of freedom is necessary. Although NRQCD holds in this region, extracting information from it on the lattice is not simple, as in addition to heavy quarkonium, heavy–light meson pairs and hybrid states populate in this region of energy. It would be important to develop theoretical tools in order to bring current phenomenological approaches into QCD-based ones [59,60]. Many extremely interesting questions in hadron spectroscopy remain unanswered at present. However, there is hope that the upcoming facilities, PANDA at GSI, JPARC at KEK and the 12 GeV upgrade at Jefferson Laboratory (JLab), will throw more light on these exotics and will bring new challenges to the theorists and phenomenologists to give serious attention on heavy flavour spectroscopy.

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