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Nuclear triaxiality in the $A \sim 160$–170 mass region: the story so far

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Abstract. Research in nuclear triaxial deformation has revealed many exciting facts and figures over the last one and a half-decades. Although wobbling motion of nuclei was experimentally discovered at the beginning of the last decade, after almost 25 years of its prediction by Bohr and Mottelson, efforts are still being put to understand this rare nuclear phenomenon in greater detail. The concept of transverse wobbling is one such recent attempt which successfully explains the evolution of experimentally observed wobbling frequency with spin. The population of triaxial strongly deformed (TSD) bands in the $A\sim160$–170 region is favoured for which neutron number ($N = 92$ or 94) is a topic of current debate. Experimental efforts are being put following Bengtsson's calculations which indicate that the elevated yrast lines for $N = 92$ isotones favour TSD population. In $A\sim170$ mass region, the ambiguity over the real character of certain strongly deformed bands has recently been removed by extensive experimental and theoretical efforts, and the bands have now been firmly established as either enhanced deformed (ED) or superdeformed (SD).

Keywords. Triaxiality; wobbling excitation; ULTIMATE CRANKER; transverse/longitudinal wobbling; enhanced deformed; superdeformed.

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1. Introduction

Nuclear triaxiality has been a subject of major discussion and extensive theoretical and experimental investigations ever since its conception as one of the aspects of nuclear deformation. Considerable progress has been made over the last one and a half-decades to affirm nuclear triaxiality in terms of phenomena like wobbling excitation and nuclear chirality, which are uniquely related to nuclear triaxial shape. Wobbling motion of nucleus, which is an unambiguous fingerprint of stable nuclear triaxiality, has been observed in even-N $^{163,165,167}Lu_{92,94,96}$ [1–4], $^{167}Ta_{94}$ [5], and possibly in $^{161}Lu_{90}$ [6]. While the wobbling mode was first predicted in even–even systems, it has only been observed in odd-A

Lu and Ta isotopes. The wobbling bands in these odd-A nuclei are built on configurations that contain an odd $i_{13/2}$ proton. This highly aligned odd $i_{13/2}$ proton plays a pivotal role in generating wobbling excitations in the sense that it drives the nuclear shape toward large deformation, thereby stabilizing a triaxial strongly deformed (TSD) shape [7]. Also, the odd $i_{13/2}$ proton contributes in lowering the wobbling frequency/energy in these nuclei. As a result of this lowering of wobbling frequency, the experimental observation of one- and two-phonon wobbling excitations as separate bands in these nuclei has become possible. Otherwise, these bands could evade proper identification being immersed in the sea of several particle–hole excitations. However, apart from these odd-A wobbling candidates, a number of TSD bands have also been identified in neighbouring odd–odd (even-A) Lu isotopes, centred around ^{163}Lu$_{92}$. All these observations validate the predicted existence of TSD island in this mass region.

Theoretical calculations based on different approaches predicted large shell gaps at large triaxiality around $Z = 71, 72$ and $N = 94, 97$ [8]. But, the occurrence of TSD bands in Lu ($Z = 71$) and Hf ($Z = 72$) isotopes is not consistent with the predicted proton and neutron shell gaps, because the high-j orbitals at large triaxiality are poorly known. According to the calculated shell gaps, among Lu isotopes, ^{165}Lu$_{94}$ and ^{168}Lu$_{97}$ are good candidates for TSD structures. But, the strongest TSD band in ^{163}Lu$_{92}$ is three times stronger than that in ^{165}Lu$_{94}$, and the TSD structure has not been even observed in ^{168}Lu$_{97}$. In addition, more TSD bands are observed in ^{163}Lu, and the wobbling bands in ^{163}Lu are by far the best example for wobbling excitation. Also, no TSD bands have been observed in ^{166}Hf$_{94}$ till date, which is located at the centre of the predicted TSD island. These facts seem to indicate that at large triaxiality $N = 92$ could be a better shell gap than $N = 94$. On the other hand, due to the recent observation of wobbling mode of excitation in the ^{167}Ta$_{94}$ nucleus [5], the $N = 94$ shell gap seemed more favourable for triaxiality than the $N = 92$ gap. However, further attempts to look for wobbling excitation in the $N = 94$ isotones in this mass region has not yet been successful.

2. The elevated yrast lines of $N = 92$ isotones

Recently, it has come to light that the excitation energy of the TSD bands relative to the normal deformed (ND) yrast line may also play an important role. It is possible that the relative position of TSD and ND structures has a greater role on the population of TSD bands than the predicted neutron shell gaps. In a comprehensive calculation of TSD structures in the $A\sim160$ region, Bengtsson [8] determined that the yrast lines of the normal deformed $N = 92$ isotones are systematically higher in excitation energy at high spin than those of their neighbours (figure 1). In the experimental data, it is observed that in the Lu chain of isotopes, the TSD band in ^{163}Lu has the lowest excitation energy relative to the yrast line, and it has maximum population. This might be because of the fact that the yrast line of the $N = 92$ nucleus ^{163}Lu sits higher than ^{165}Lu and ^{167}Lu. So, population of TSD bands may not be because of the TSD bands coming down in energy in this mass region, but a consequence of the fact that the band at normal deformation going up in energy for this neutron number. The evolving shell effects that contribute to (or, influence) the normal deformed structures at high angular momentum for this neutron number ($N = 92$) might be responsible for such elevated yrast lines. However, more study is required at the normal deformation for an in-depth understanding of the underlying physics.

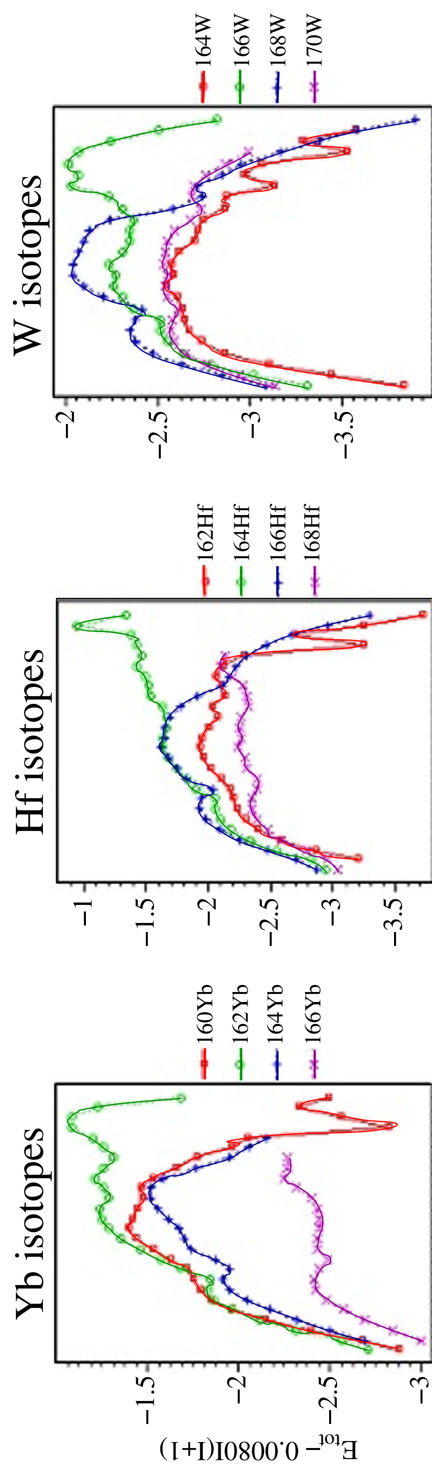

Figure 1. Excitation energies minus a common rigid-rotor reference for ND yrast lines for $^{160-166}$Yb, $^{162-168}$Hf, $^{164-170}$W. Colour codes are as follows: Red: $N = 90$; Green: $N = 92$; Blue: $N = 94$; Pink: $N = 96$.

Motivated by the above considerations, an experiment has recently been carried out at the ATLAS Facility at Argonne National Laboratory, USA, to look for TSD structures in ^{164}Hf, which has, indeed, 92 neutrons in the Hf ($Z = 72$) chain of isotopes. The reaction ^{94}Zr(^{74}Ge, 4n) with a beam energy of 330 MeV was used to populate the high-spin states in ^{164}Hf. Coincident γ-rays were measured using the Gammasphere array [9] which consisted of 99 Compton-suppressed Ge detectors at the time of this experiment. In the data analysis, two new bands of distinct character have been identified and linked to the known states. The crucial linking transitions allowed the determination of the level spins, energies and parities of the newly discovered bands. Based on their rotational properties and on comparisons with cranking calculations with a modified-oscillator potential, the bands are suggested to be the long-predicted TSD bands in ^{164}Hf [10]. Proposed configurations for the bands involve four quasiparticles, including the high-j intruder $(i_{13/2})^2$ proton orbitals. Furthermore, the new bands are substantially more intense and are observed at lower spins than the previously reported TSD bands in ^{168}Hf. These observations make ^{164}Hf the best even–even system so far for the study of TSD structures in the $A\sim160$ mass region [10]. Further investigations in this direction are in progress.

3. Assignment of true characters to some strongly deformed bands

In parallel to the above-mentioned investigations, efforts were being put to understand a number of high-spin strongly deformed bands in some heavier Hf isotopes, e.g., in ^{170}Hf, ^{173}Hf, ^{174}Hf and ^{175}Hf, which were reported earlier and were suggested to be of TSD nature [11–13]. But, such assignments raised several questions. In fact, a convincing description consistent with the experimental results was not available for any of the proposed TSD structures in these Hf isotopes. Based on a systematic investigation of the properties of all the strongly deformed bands in $^{170-175}$Hf, cranking calculations employing the ULTIMATE CRANKER (UC) code and cranked relativistic mean-field (CRMF) calculations [14], it was suggested that these structures might fall into two groups: the enhanced deformed (ED) and the superdeformed (SD) bands.

In order to clarify the situation, an experiment employing Gammasphere [9] was performed at the ATLAS Facility at Argonne National Laboratory to measure the transition quadrupole moments for the strongly deformed bands in ^{171}Hf and ^{172}Hf. For each transition of the speculated enhanced deformed band in ^{171}Hf, Doppler-broadened line shapes were clearly visible (figure 1 in [16]) at forward and backward angles of Gammasphere. The profile of the line shapes, which included the Doppler-broadened part as well as the stopped component for each transition, was indicative of the fact that the level lifetimes were comparable to the stopping time of the recoil. LINESHAPE analysis code of Wells and Johnson [15] was used to extract the mean lifetimes of the levels in the band. Using these lifetime values, the quadrupole moment of the band was obtained. The value of this quadrupole moment, Q_t ($\sim$9.5 eb), for band ED in ^{171}Hf, strongly supports the recent suggestion that this sequence and similar bands in the ^{168,170,175}Hf isotopes are associated with little triaxiality and deformations enhanced relative to that of the normal deformed structures [16]. Theoretical calculations indicate that these structures involve $i_{13/2}h_{9/2}$ proton configuration, which is largely responsible for such enhanced deformation.

In case of the band members (transitions) of the three strongly deformed bands (SD1, SD2 and SD3) in ^{172}Hf, total centroid shifts were observed in forward and backward angles (figures 2, 3 and figure 2 in [16]). This observation, which is in contrast to the Doppler-broadened line shapes in transitions in the ED band in ^{171}Hf (figure 1 in [16]), readily indicates that the bands in ^{172}Hf are more deformed than the ED band in ^{171}Hf. Centroid shift analysis using FITFTAU code [17] was carried out for these bands in ^{172}Hf to extract the lifetimes of the levels and, in turn, the quadrupole moments of the bands. The measured values of the quadrupole moments (Q_t = 13.6(9) eb for SD1 and Q_t = 11.6(10) eb for SD2) for the strongly deformed bands in ^{172}Hf are consistent with the measured values of similar strongly deformed bands in ^{173,174}Hf [18]. These values, when considered along with the other observables, such as excitation energies and the intensities of these bands, suggest that these sequences are associated with a prolate superdeformed shape [16]. Similar bands in $^{173-175}$Hf are also likely to be associated with superdeformed shapes.

UC calculations have predicted TSD bands for $^{172-174}$Hf at high spins. But, the predicted TSD bands lie much higher in excitation energies than the ND yrast line. In figure 4, it is seen that the calculated 4-quasiparticle TSD band in ^{172}Hf lies ∼4 MeV higher in excitation energy than the experimentally observed yrast state at spin ∼36ℏ. At such high excitation energy the feeding is expected to be very small, and as a consequence, such bands will be difficult to observe.

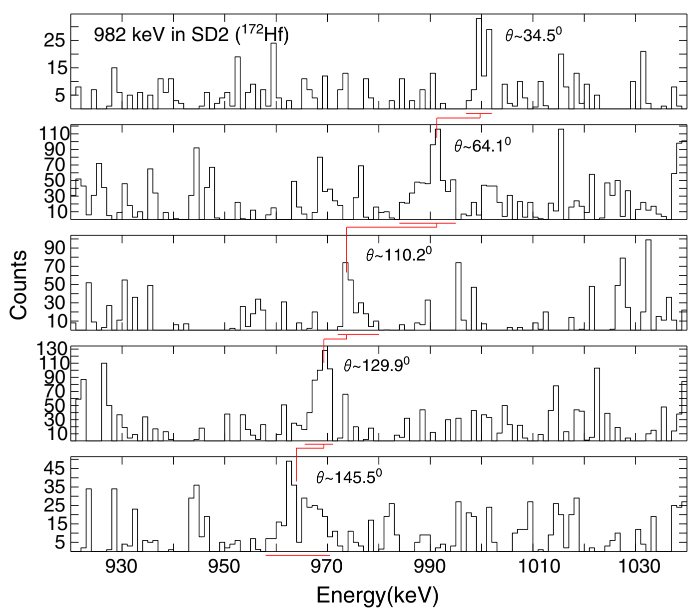

Figure 2. Representative, summed coincidence γ-ray spectra profile for the 982-keV transition in SD2 at different angles. An average detector angle is shown where two close lying rings of Gammasphere were summed for better enhancement of the peak shape.

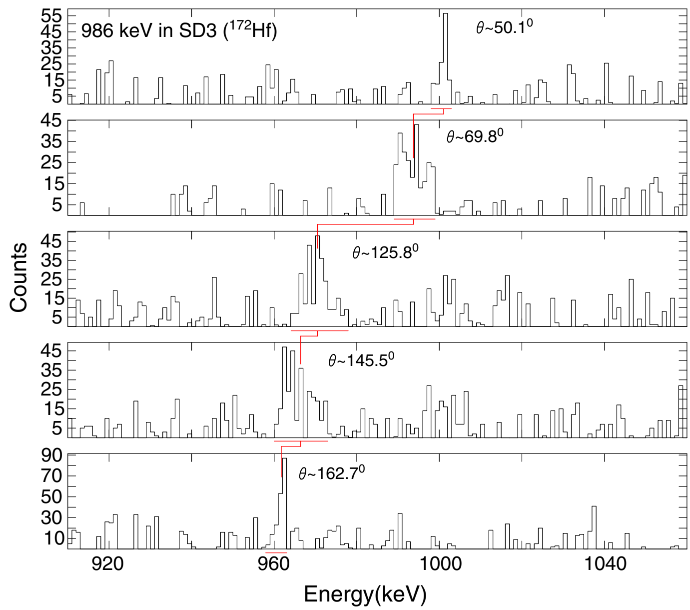

Figure 3. Representative, summed coincidence γ-ray spectra profile for the 986-keV transition in SD3 at different angles. An average detector angle is shown where two close lying rings of Gammasphere were summed for better enhancement of the peak shape.

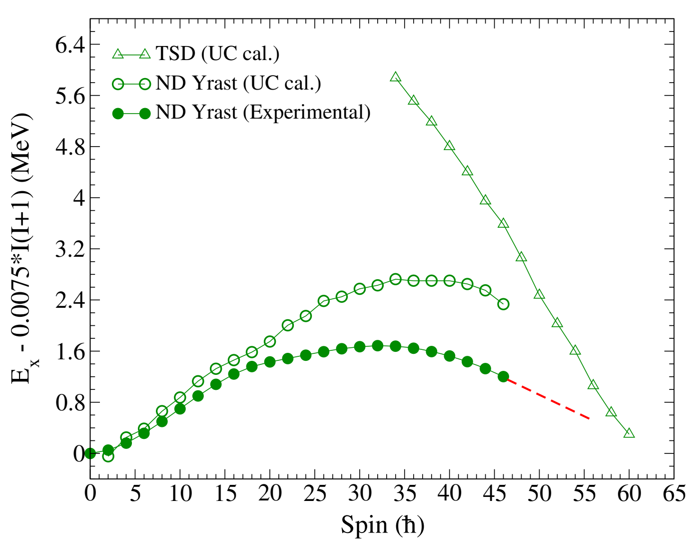

Figure 4. Experimental and UC calculated excitation energies minus a rigid-rotor reference for bands in ^{172}Hf. Upon extrapolation (red line), it appears that the UC calculated TSD comes closest to the experimental ND yrast line around spin $\sim 60\hbar$.

4. A new concept: Transverse wobbling

Although, a good understanding of the measured electromagnetic transition rates could be achieved in the work of Hamamoto and Hagemann [19], the observed decrease of the wobbling frequency or wobbling energy with increasing angular momentum had remained an open problem from the theoretical point of view. The wobbling energy is calculated as

$$\hbar\omega_\omega = E_1(I) - [E_0(I+1) + E_0(I-1)]/2. \quad (1)$$

Here, E_0 is the energy of the state in the $\pi i_{13/2}$ band and $E_1(I)$ is the energy of the state with spin I in the $n_\omega = 1$ wobbling band. The wobbling energy associated with the $n_\omega = 1$ wobbling phonon in the five known cases is depicted in figure 5. A clear trend of gradually decreasing wobbling energy with spin is observed. These experimental findings contradict the theoretical expectation because the associated Hamiltonian of a 'simple wobbler' suggests that the wobbling frequency should increase linearly with spin. The concept of transverse wobbling, proposed by Frauendorf and Dönau, provides a natural explanation for the decrease of the wobbling frequency with angular momentum and also for the enhanced $E2$ transitions between the wobbling bands [20].

The presence of a high-j quasiparticle, when rigidly aligns its angular momentum j with one of the principal axes, significantly changes the motion of the coupled system. As a result of this so-called 'frozen alignment' (FA) of the angular momentum, two types of wobbling motion appear: the longitudinal and the transverse. The longitudinal wobbler appears when the quasiparticle j is aligned (FA) with the largest moment of inertia (MoI). On the other hand, when the quasiparticle j is oriented (FA) perpendicular to the axis with the largest MoI, the transverse wobbling occurs. Using this assumption of

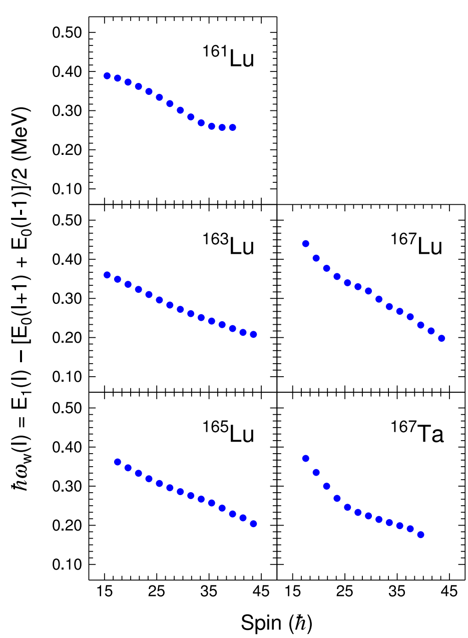

Figure 5. Wobbling frequency vs. spin for the five known cases of wobbling in Lu and Ta nuclei [7].

frozen alignment (FA) of the quasiparticle j with one of the principal axes, simple analytical expressions were derived for the wobbling frequency and $E2$, $M1$ transition rates by applying the harmonic approximation to the motion of a triaxial rotor (harmonic FA-HFA) [20]. The derived HFA expressions were helpful in understanding the fact that the monotonic increase of the wobbling frequency with the total angular momentum is caused by the longitudinal alignment, whereas, the gradual decrease of the wobbling energy with the total angular momentum is a result of the transverse alignment [20].

So, it is now evident that all the strongly deformed wobbling bands observed at high-spin in the Lu and Ta isotopes carry the signature of the transverse wobbling. In the $A{\sim}130$ region, interestingly, the two $h_{11/2}$ bands in the ^{135}Pr nucleus appear to exhibit the behaviour expected for a transverse wobbler. The wobbling bands in ^{135}Pr appear at a much lower spin. This can be attributed to the fact that the MoI of the weakly deformed ^{135}Pr are smaller by a factor of three when compared to the wobblers in the $A{\sim}160$ mass region. To perform detailed spectroscopic investigation of the $h_{11/2}$ bands in ^{135}Pr, two separate complementary experiments, one using the Gammasphere spectrometer at ANL and the other using the Indian National Gamma Array (INGA) spectrometer at TIFR, have recently been carried out. The results will be published in a forthcoming article [21]. In ^{135}Pr, the wobbling mode is transverse at low spin. But, the wobbling mode changes from transverse to longitudinal at the critical spin of $I = 29/2$. This may be caused by the realignment of the $h_{11/2}$ proton from the short to the medium axis (largest MoI).

5. Conclusion and future perspective

A brief review of recent discoveries and ongoing experimental and theoretical investigations in the context of nuclear triaxiality has been presented. The wobbling mode of nuclei looks all set to be termed as a general phenomenon in the nuclide chart after its discovery in nuclei (^{167}Ta and ^{135}Pr) other than the odd-A Lu isotopes. The recent concept of transverse wobbling has been intriguing. It has turned out to be an amazing fact that all the strongly deformed wobbling bands observed in Lu and Ta isotopes carry the signature of the transverse wobbling. However, the wobbling band in ^{135}Pr evolves from transverse at low spin values to longitudinal wobbling at higher spins. In future experimental efforts, it would be interesting to search for a 'simple wobbler' where wobbling frequency only increases with spin. The ^{164}Hf nucleus has turned out to be the best even–even nucleus to have a TSD structure. More experimental investigations are needed to explore the $N = 92$ TSD shell gap in this direction.

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