The structure of $^{12}\text{C}$

MARTIN FREER
School of Physics and Astronomy, University of Birmingham, Birmingham, B15 2TT, United Kingdom
E-mail: M.Freer@bham.ac.uk

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Abstract. The nucleus $^{12}\text{C}$ has a rather significant role in modern nuclear physics, but whose influence can be traced to the work of Hoyle in the 1950s, when it was concluded that there should be a state close to 7.68 MeV responsible for the synthesis of carbon in stellar nucleosynthesis. Although a state at 7.65 MeV was subsequently discovered, its properties have remained something of a mystery until rather recently. This paper explores our current understanding of the structure of $^{12}\text{C}$, in particular the nature of the Hoyle state.

Keywords. Nuclear spectroscopy; nuclear reactions; structure of $^{12}\text{C}$.

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1. Clusters and correlations

Many have wrestled with the uncomfortable thought that the Universe appears to be finely tuned. In his book “Just Six Numbers” [1] the Astronomer Royal, Martin Rees, explores how if one were to make rather small adjustments to the values of six numbers the consequences would be rather dramatic. These include what he terms ‘nuclear efficiency’, which is the ability of stars to transform the rest mass of the protons into energy through the synthesis of helium, a number that turns out to be 0.007. This is related to the strength of the strong interaction and indeed is sensitive to its finer details such as the spin dependence. Specifically, for the $n-p$ system the $S = 1$ system is bound by 2.2 MeV, whereas the $S = 0$ configuration is unbound by tens of keV. Small changes (0.001) to the nuclear efficiency have dramatic consequences. An increase in the strength of the strong interaction such that the spin singlet state ($S = 0$) of the nucleon–nucleon system becomes bound results in the $^2\text{He}$ nucleus becoming stable, consequently consuming protons and reducing or removing the raw material that stars such as the Sun consume to generate energy. A decrease in the strength would result in the triplet state being unbound and hence in $p-p$ burning in the Sun, it would not be possible to form a deuteron, the path to helium would be blocked, and there would be no net energy production.
This straddling of the singlet and triplet states across the zero binding threshold is a necessary condition for life.

The binding of the neutron and proton in the spin triplet state, compared to the spin singlet case owes its origins to the tensor component of the strong interaction, which in turn may be traced to both scalar (pion) and vector ($\rho$) meson exchange. Understanding how to embed the detail of the nucleon–nucleon interaction which arises from the understanding of the $n$–$p$ system and nucleon–nucleon scattering measurements into the computation of the properties of a nucleus is a significant challenge. Part of this arises from the fact that single meson exchange does not capture the full complexity of the nucleon–nucleon interaction, and higher-order exchanges can have a major contribution, but that three-body terms (and may be to a lesser extent higher-order terms) play a significant role. One way of capturing such contributions is through processes such as the Fujita–Miyazawa [2] type exchanges, in which a nucleon can get excited via a pion exchange, which then alters the nature of the interaction of the excited nucleon with further nucleons. The parametrization of the nuclear force based on such interactions has led to a rather successful description of the properties of the binding energies and spectroscopy of light nuclei up to $A = 12$, e.g., through the Greens function Monte Carlo approach [3–5]. However, in such approaches there are a number of challenges; including the mass range of nuclei that can be reached and the choice of parametrization of the two-body interaction leads to ambiguities in the nature of the 3-body force – there is no unique three-body interaction.

An alternative approach, which recognizes that ultimately the nuclear strong force emerges from the QCD degrees of freedom, and that the meson exchange is a manifestation of this, is to use chiral perturbation theory (ChPT) [6]. The QCD description is the simplest where the strong coupling constant, $\Lambda$, is the weakest. At low energies, that are appropriate for nuclear matter, the regime is such that, the coupling is so strong that it cannot be treated as a perturbation – the non-perturbative regime. One approach to circumvent this has been to calculate properties based on constituents being fixed on a lattice (lattice QCD). The second approach has been to use an effective field theory which draws on ideas developed for solid-state theory – chiral effective field theory (ChEFT) [6]. This theory draws on the fact that the up- and down-quarks display chiral-like symmetry which is broken only by their non-zero mass. In this description, the properties of both the hadrons (including protons and neutrons) and the mass of the pion emerges. In analogy with condensed matter chiral-symmetry breaking, the pion should be a spinless Goldstone boson, which couples only weakly. This last aspect is rather essential as it then permits a tractable, perturbative, approach [6].

This effective field theory then results in a series of exchange diagrams which relate to the perturbative expansion in orders of $(Q/\Lambda)^n$, where $Q$ is of the order of the pion mass ($\sim 140$ MeV) and $\Lambda$ is the chiral-symmetry breaking scale ($\sim 1$ GeV). In leading order ($n = 0$) there is a one-pion exchange component plus what is called a contact term, which in essence accounts for the heavier exchange mesons ($\rho$, $\sigma$ and $\omega$) whose range is significantly less than that of the pion. The next term ($n = 1$) vanishes due to parity and time invariance, leaving the $n = 2$ term as the next to leading order, NLO. This includes processes such as the two-pion exchange. The complexity increases as more orders are added which means that although the contribution from each term to the interaction is less by the factor $(Q/\Lambda)$ there are more processes to consider. The aim is to calculate
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the interaction to all orders, but this is clearly a challenge, and NNNLO is the current milestone (N$^3$LO).

The great advantage of this approach is that three-body forces emerge naturally, first appearing in the NNLO term, and the higher-order terms such as NNNLO introduce four-body interactions. Here, the two-body and $n$-body interactions emerge in a consistent fashion.

Such developments offer the opportunity to develop a consistent interaction which may be applied to the description of nuclei whose degrees of freedom may at least be traced to QCD, even if they are not explicitly those of QCD. The N$^3$LO interaction has been applied through models such as the no core shell model (NCSM) [7]. Here the complexity of the interaction is traded against the simplicity of the basis in which the interaction is applied. The states adopted in this approach are those of the harmonic oscillators (HO), which are attractive due to their analytic form. The NCSM has been extremely successful in terms of the reproduction of the spectroscopy of a range of light nuclei. However, it has been famously unsuccessful in the reproduction of one particular state in $^{12}\text{C}$ - the $7.65\text{ MeV}$, $0^+$, Hoyle state [8]. In order to capture the detail of this state, or more precisely its experimentally observed excitation energy, nucleons must be scattered into very high-lying levels within the HO potential. The feature indicates that for some reason the HO basis is inappropriate in its attempt to capture the structural properties of the state. It has been argued that this reflects the $3\alpha$-cluster structure of the state. However, it should be noted that the highly clustered nucleus $^{8}\text{Be}$ is well-described within the ab-initio NCSM [9]. This ability to reproduce the ground-state energy of $^{8}\text{Be}$, albeit with a model space of $6–8\hbar\omega$, and not the Hoyle state, is interesting. The harmonic oscillator basis is not ideally suited for the reproduction of the behaviour of weakly bound systems, as the asymptotics of the wave functions do not match well with the strong exponential character of the tails of wave functions for states close to the threshold. This may be partially responsible for the fact that in order to capture the ground-state energy of $^{8}\text{Be}$, whose properties are dominated by the $1p$ levels, $6–8\hbar\omega$ excitations are required. Similarly, if the Hoyle state is associated with a $(1s)^4(1p)^4(1d)^4$ configuration then it too would be expected to be reasonably reproduced by the NCSM. This configuration would most naturally be associated with a linear arrangement of $3\alpha$-particles [10]. This lack of reproduction of the experimental energy of $7.65\text{ MeV}$ by the NCSM may point to a rather different structure (i.e., not a linear arrangement).

Returning to the beginning, which rehearsed at least one of the tenants that lie behind the view of a finely-tuned Universe, the $^{12}\text{C}$ Hoyle state also fits into this doctrine. The synthesis of $^{12}\text{C}$ proceeds through what is known as the triple $\alpha$-process. This first relies on a state existing with appropriate properties in $^{8}\text{Be}$ and then similarly in $^{12}\text{C}$. The ground state of $^{8}\text{Be}$ lies just $92\text{ keV}$ above the $\alpha$-decay threshold, and has a width of $5.6\text{ eV}$ implying a lifetime of about $10^{-16}\text{ s}$. The energy of $92\text{ keV}$ is remarkably close to the peak of the Gamow window for $\alpha$-burning temperatures near to $10^8\text{ K}$ (peak of the Gamow window is approximately $85\text{ keV}$ with a width of $60\text{ keV}$) [11]. The energy width of the $^{8}\text{Be}$ ground state is constrained by the Coulomb barrier, through which the $\alpha$-decay (or capture) must proceed. If the $Q$-value had been somewhat higher then the width would have been significantly larger, the lifetime shorter and the state would be located outside the Gamow window. These factors would dramatically affect the equilibrium abundance of $^{8}\text{Be}$ in helium-seeded plasma.

Similarly, the energy of the Hoyle state lies 285 keV above the $^8\text{Be}+\alpha$ decay threshold. This corresponds to the peak of the Gamow window for temperatures close to $2.5 \times 10^8$ K. Also, the presence of the state close to the critically sensitive region for synthesis of $^{12}\text{C}$ is crucial, providing an enhancement by approximately eight orders of magnitude. A few hundred keV higher in energy and the influence of the state would be effectively lost. The serendipity that appears in these two stages of the triple $\alpha$-process appears to be more than remarkable and the degree of balancing is an order of magnitude greater than that which appears in the binding of proton and neutron into the ground state of deuteron.

In recent years, the magic of the production of carbon has focussed on the existence of the Hoyle state, but actually the properties of $^8\text{Be}$ are just as important. The focus on the latter is driven by the power of the prediction of Hoyle state [12]. These two states might have several things in common. The first is that they have $J\pi = 0^+$. In the case of $^8\text{Be}$ the origin of this might be trivial, it is an even–even nucleus and the state in question is the ground state, but in $^{12}\text{C}$ it is not. The second characteristic is that the two states are believed to be clustered; $^8\text{Be} = \alpha + \alpha$ and $^{12}\text{C}(\text{Hoyle}) = \alpha + \alpha + \alpha$. This observation may not be trivial and is rather crucial. The appearance of clusters, as described above, has a strong link to correlations and hence is fundamental to a deeper understanding of nuclear structure and the symmetries that underly the forces from which the clustering emerges.

The association with clustering is not a new observation, as the phenomenon dates back to the early days of the subject. For example, the work of Hafstad and Teller was formative [13]. It was recognised in this early work that $\alpha$-particle structures were likely to play significant roles in the structure of $N = Z$, $\alpha$-conjugate nuclei. In a rather simple piece of analysis it was recognized [13] that the binding energies of nuclei such as $^8\text{Be}$, $^{12}\text{C}$, $^{16}\text{O}$ scaled linearly with the possible number of $\alpha$-particle interactions, or bonds. The picture produced of $\alpha$-particles quasiformed in the ground states is not perfect, but is a good starting point for understanding the structure of such nuclei. However, in detail, the appearance of clustering is a rather deep subject, yet to be completely understood, which as identified above, links fundamentally to the understanding of the nucleon–nucleon interaction. It is known that for the $\alpha$-particles $S = 1$, $T = 0$, $(n-p)$ correlations play a significant role in the binding of the $^4\text{He}$ nucleus and that in the case of $^8\text{Be}$ there is a significant gain in the binding of the system, but these effects are localized within the $\alpha$-particles which form the two cluster subunits [14].

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Hafstad and Teller [13] identified that for such $\alpha$-particle systems, there should be a set of dynamical symmetries. In the case of $^{12}\text{C}$ the dynamical symmetries of $3\alpha$ system correspond to a spinning top with a triangular point symmetry. The rotational properties of these states are given by

$$E_{J,K} = \frac{\hbar^2 J(J+1)}{2I_{\text{Be}}} - \frac{\hbar^2 K^2}{4I_{\text{Be}}},$$

where $I_{\text{Be}}$ is the moment of inertia corresponding to two touching $\alpha$-particles, which can be determined from the $^8\text{Be}$ ground-state rotational band [13]. One would expect that based on this structure, there should be a number of rotational bands with different values.
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of $K$. For $K^\pi = 0^+$, the rotations will be around an axis which lies in the plane of the three $\alpha$-particles, generating a series of states $0^+, 2^+, 4^+ \ldots$. These corresponds to the rotation of a $^9\text{Be}$ nucleus – the rotation axis passing through the centre of the third $\alpha$-particle. The next set of rotations corresponds to the rotation around an axis perpendicular to the plane of the triangle, with each $\alpha$-particle having one unit on angular momentum – giving $L = 3 \times 1\hbar$; $K^\pi = 3^-$. Rotations around this axis and that parallel to the plane combine to give a series of states $3^-, 4^-, 5^- \ldots$. The next set of collective states then corresponds to each $\alpha$-particle having $L = 2$; $K^\pi = 6^+ \ldots$.

Such an arrangement possesses a $D_{3h}$ point group symmetry. The corresponding rotational and vibrational spectrum was also described by the form [15–17]

$$E = E_0 + A v_1 + B v_2 + C L(L+1) + D (K \pm 2l)^2,$$

(2)

where $v_{1,2}$ are vibrational quantum numbers and $v_2$ is doubly degenerate; $l = v_2, v_2-2, 1$ or 0, $L$ is the angular momentum, and its projection is $K$ on a body-fixed axis [17]. $A, B, C$ and $D$ are adjustable parameters. The spectrum of the states predicted by the choice $A = 7.0, B = 9.0, C = 0.8$ and $D = 0.0$ MeV is shown in figure 1.

The ground-state band $(v_1; v_2^l) = (0, 0^0)$, contains no vibrational modes and coincides well with the observed experimental spectrum. Here the states correspond to different values of $K$ ($K = 3n, n = 0, 1, 2 \ldots$) and $L$. For $K = 0, L = 0, 2, 4, \ldots$ which is a rotation of the plane of the triangle about a line of symmetry, whereas for $K > 0, L = K, K+1, K+2 \ldots$. In the present case, $K = 0$ or 3 is plotted with the parity being given by $(-1)^K$. The $K = 0$ states coincide well with the well-known $0^+$ (ground state), $2^+$ (4.4 MeV) and $4^+$ (14.1 MeV) states. The $K = 3$ states, as described above, correspond to a rotation about an axis which passes through the centre of the triangle. The first state has spin and parity $3^-$ and coincides with the 9.6 MeV, $3^-$, excited state. The next such state would be $K = 6, J^\pi = 6^+$. A prediction of this model is that there should be a $4^-$ state almost degenerate with the $4^+$ state. A recent measurement involving studies of the $\alpha$-decay correlations indicated that the 13.35 MeV unnatural-parity state possessed

![Figure 1. Spectrum of the energy levels of an equilateral triangle configuration. The bands are labelled by $(v_1; v_2^l)$ [17].](image-url)
$J^\pi = 4^-$ [18]. The close degeneracy with the 14.1 MeV $4^+$ state would appear to confirm the $D_{3h}$ symmetry.

Historically, one of the pre-eminent tests of our understanding of the structure of light nuclei lies in the nature of the second excited state in $^{12}$C. This system resides at the limits of many of the \textit{ab-initio} approaches, particularly the Greens function Monte Carlo approach. This state has character $J^\pi = 0^+$ and lies at $E_x = 7.65$ MeV – the Hoyle state. In the description illustrated in figure 1, the $0^+$ state at 7.65 MeV corresponds to a vibrational mode ($v_1 = 1$). If the structure of the excited state is close to that of the ground state, then the coupling of rotational modes would produce a corresponding $2^+$ state at 4.4 MeV above 7.65 MeV, i.e., 12.05 MeV. There is no known $2^+$ state at this energy, pointing to the more complex structure of this state. The closest state which has been reported with these characteristics is at 11.16 MeV [19]. This state was observed in the $^{11}$B$(^3$He, $d)^{12}$C reaction, but has not been observed in measurements subsequently. A re-measurement of this reaction using the K600 spectrometer at iThemba in South Africa demonstrates that the earlier observation of a state at 11.16 MeV was almost certainly an experimental artifact and no such state exists [20].

This introduces an interesting set of possibilities which lie at the heart of uncovering the structure of the Hoyle state. If the Hoyle state is more deformed than the ground state, and the system behaves in a rotational fashion, then the $2^+$ state would be lower in energy. Alternatively, the Hoyle state could possess no collective excitations. It has been suggested that due to the close proximity of the Hoyle state to the $3\alpha$-decay threshold, bound only by the presence of the Coulomb barrier, the system obtains a bosonic rather than fermionic identity and the $\alpha$-particle bosons behave like a weakly interacting bosonic gas or even a bosonic condensate [21]. The resolution of the structure may follow from the identification of the $2^+$ excitation or otherwise.

Recent studies of the $^{12}$C($\alpha, \alpha'$) and $^{12}$C($p, p'$) [22] reactions indicate the presence of a $2^+$ state close to 9.6–9.7 MeV with a width of 0.5–1 MeV. The state is only weakly populated in these reactions, presumably due to its underlying cluster structure, and is broad. Consequently, its distinction from other broad states and dominant collective excitations (e.g., the 9.6 MeV, $3^-$ state) made its unambiguous identification challenging. Further, and perhaps definitive, evidence for such an excitation comes from the measurements of the $^{12}$C($\gamma, 3\alpha$) reaction performed at the HIGS Facility, TUNL [23]. Here, a measurable cross-section for this process was observed in the same region of 9–10 MeV which cannot be attributed to known states in this region. Furthermore, the angular distributions of the $\alpha$-particles are consistent with an $L = 2$ pattern, indicating a dominant $2^+$ component. Based on a rather simple description of this state in terms of three $\alpha$-particles with radii given by the experimental charge radius (see figure 2 for possible arrangements), it is possible to use the 2 MeV separation between the Hoyle state and the proposed $2^+$ excitation to draw some conclusions as to the arrangements of the clusters [22]. A linear arrangement of the $3\alpha$-particles, in which the separation is close to that of the $^8$Be ground state, would give a separation between the Hoyle state and the $2^+$ state of close to 1 MeV as opposed to 2 MeV that is observed experimentally. The data then indicate that rather than a linear arrangement of the three clusters, a more appropriate description would be a loose arrangement of the $\alpha$-particles resembling a triangular structure.

However, in the BEC model [21], a $2^+$ excitation can also be produced at an energy close to 9.6 MeV which would correspond to one of the three $\alpha$-particles being excited.
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Figure 2. Different arrangements of $\alpha$-particles. The closest possibility fitting the experimental data is the triangular arrangement.

from the lowest level in the mutual potential to the next available level, which corresponds to a $D$-state. This would generate a $J^\pi = 2^+$ state. The nature of the Hoyle state in terms of the underlying structure then is ambiguous. A natural extension of the rotational model is that there should also be a collective $4^+$ state. Using the simple $J(J+1)$ scaling, a $4^+$ excitation close to $E_x(^{12}\text{C}) = 14$ MeV would be expected. Recent measurements of the two reactions $^{9}\text{Be}(\alpha, 3\alpha)n$ and $^{12}\text{C}(\alpha, 3\alpha)^4\text{He}$ have been performed [24]. These measurements indicate a candidate state close to $13.3$ MeV with a width estimated to be $1.7$ MeV. It is believed that this is not a contaminant and is observed with similar properties in all spectra. Angular correlation measurements made using the $^{12}$C target are not definite, but indicate a $4^+$ assignment.

2. Outlook

The current experimental understanding of the structure of $^{12}$C matches well with the dynamical symmetries predicted by Hafstad and Teller in 1938 [13] and those of the more recent model of Bijker and Iachello [17]. There remains, however, much to be done to cement this understanding. The $2^+$ excitation of the Hoyle state is confirmed, the $4^+$ excitation remains to be confirmed. The observation of the $5^-$ excitation of the $K = 3^-$ band would be a further confirmation of the $3\alpha$-structure. A key measurement would be the measurement of the $B(E2)$ corresponding to the decay from the $2^+$ excitation of the Hoyle state to the Hoyle state itself. This was the basis for much of the discussion in the present work and stands as one of the experimental challenges.

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Martin Freer

References