PRAMANA
— journal of
physics

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Vol. 83, No. 4
October 2014
pp. 619–630

Quark matter coupled to domain walls in Bianchi types II, VIII and IX Universes

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MS received 2 October 2013; revised 14 January 2014; accepted 11 February 2014
DOI: 10.1007/s12043-014-0804-8; ***e*Publication:** 16 September 2014

Abstract. In this study of Bianchi types II, VIII and IX Universes, quark matter coupled to domain walls in the context of general relativity are explored. To obtain deterministic solution of the Einstein's field equations, various techniques are adopted. The features of the obtained solution are discussed.

Keywords. Bianchi types II, VIII and IX Universes; domain walls; quark matter.

PACS Nos 98.80.Bp; 98.80.Es; 98.80.Jk

1. Introduction

It is still challenging to know the exact physical situation at the very early stages of the formation of the Universe. Certain grand unified field theory predicts topological defects at the very early stages of the evolution of the Universe. It is assumed that during phase transition, the symmetry of the Universe was broken spontaneously. It could give rise to topologically stable defects. Topological defects are called local or global as the symmetry is local or global. Topological defects in the field theories are stable field configuration with spontaneously broken discrete or continuous symmetries [1,2]. The symmetry is said to be spontaneously broken if the ground state is not invariant under the full symmetry of the Lagragian density. In quantum field theories at high temperature, the broken symmetries are restored [3]. After symmetry breaking, different regions of the Universe can settle into different parts of the vacuum with domain walls forming the boundaries between these regions [4,5].

Among all the cosmological structures, string and domain walls are the most interesting. Domain walls are important from cosmological stand point. Domain walls are also formed at low-temperature transition in various condensed matter systems [6], when the phase transition is induced by Higgs sector of the Standard Model. Due to domain walls, galaxies are formed during the phase transition after the time recombination of matter and radiation [7].

Another important phase transition of the Universe is the quark hadron phase transition which occurred at cosmic temperature, $T \approx 200\,\mathrm{MeV}$. In quark hadron phase transition of the Universe, quark gluon plasma (QGP) is converted into hadron gas. Strange quark matter can be found in two ways: (1) during the quark hadron phase transition in the early Universe and (2) during the conversion of ultrahigh density, neutron starts converting into strange stars [8–10]. According to quark bag models, breaking of physical vacuum takes place within hadrons and vacuum energy densities inside and outside a hadron are different. The quark bag models are based on strong interaction theories. The vacuum pressure on the bag wall equilibrates the pressure of quarks and the system becomes stable. According to this hypothesis, some of neutron stars got converted into strange quark matter [11,12]. The equation of state for quark matter is rudiment on the bag model of quark matter. The quark matter consists of massless u, d quark, massive s quark and electron. It is assumed that quarks are massless and non-interacting. We have quark pressure $p_{\mathrm{q}} = \rho_{\mathrm{q}}/3$, where ρ_{q} is the quark energy density. The total energy density is

$$\rho_m = \rho_{\mathrm{q}} + B_{\mathrm{c}}, \tag{1}$$

where B_{c} is the bag constant. The value of the bag constant in the MIT bag model is $B_{\mathrm{c}}^{1/4} = 146$ MeV, whereas in the Chiral bag model, it is $B_{\mathrm{c}}^{1/4} \cong 150$ MeV [40]. Jia *et al* [41] predicted the bag constant as $B_{\mathrm{c}} \cong 2 f_\pi^2 m_\pi^2$. Schaffner *et al* [42] found strangelets with 'magic' number of quark for bag constant value $145 \leq B_{\mathrm{c}}^{1/4} \leq 170$ MeV [43]. The total pressure is

$$p_m = p_{\mathrm{q}} - B_{\mathrm{c}}. \tag{2}$$

The equation of state for strange quark matter [13,14] is

$$p_m = \frac{1}{3}\left(\rho_m - 4B_{\mathrm{c}}\right). \tag{3}$$

Equation (3) deals basically with the equation of state (EoS) of gas of massless particles with correction due to the quantum chromodynamics (QCD) trace irregularity and perturbative interactions. It is clear that these corrections tend to be always negative, reducing the energy density at a given temperature by about a factor of two. The parameter B_{c} is just a linear parameter in the EoS for the MIT bag model. This is not true in the model, where it is assumed that the quark masses are density-dependent (QMDD model). The reason for the behaviour of the bag constant is that the pressure as a function of the energy density is more complicated in the QMDD model [44,45]. The self-bound state appears to be at $\rho = 4B_{\mathrm{c}}$. For this simplest model of strange matter, the maximum mass of the strange star cannot be larger than 2.6 times the mass of the Sun.

We consider another equation of state

$$p_m = (\gamma - 1)\rho_m. \tag{4}$$

This equation of state in the form of perfect fluid was proposed by Back *et al* [15], Adomas *et al* [16] and Adcox *et al* [17].

Yilmaz [18] investigated string cloud and domain walls with quark matter in 5D Kaluza–Klein cosmological model by considering eqs (3) and (4). Adhav *et al* [19] studied axially symmetric non-static domain walls in scalar tensor theories of gravitation. The domain walls are probes that enable one to distinguish large distance modified gravity from general relativity at short distances (shown by Dvali *et al* [20]). Rami [21] investigated domain walls, stiff matter and ultrarelativistic particle from a generalized double component dark energy model. Kandalkar *et al* [22] studied Kantowski Sachs domain walls in Saez–Ballester scalar tensor theories of gravitation. Ipser and Silivie [23], Windrow [24], Goetz [25], Mukherjee [26], Wang [27], Rahman and Bera [28], Rahman [29] and Reddy and Subba Rao [30] are some of the researchers who have investigated several aspects of domain walls. Burau *et al* [31] and Filha and Brage [32] have examined various aspects of quark gluon in different contexts.

Motivated by the aforesaid discussion in this paper, we consider quark matter coupled to domain wall in general relativity.

2. Metric and field equations

We consider a spatially homogeneous Bianchi types II, VIII and IX metrics of the form

$$\begin{aligned} \mathrm{d}s^2 = &-\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{d}y^2 \\ &+ \left(B^2 I^2(y) + A^2 h^2(y)\right)\mathrm{d}z^2 + 2A^2 h(y)\mathrm{d}x\mathrm{d}z, \end{aligned} \tag{5}$$

where A and B are functions of time t alone. It represents

Bianchi type II: if $I(y) = 1$ and $h(y) = y$.

Bianchi type VIII: if $I(y) = \cosh y$ and $h(y) = \sinh y$.

Bianchi type IX: if $I(y) = \sin y$ and $h(y) = \cos y$.

The energy–momentum tensor of the domain wall in conventional form is given by [18,33]

$$T^i_j = \rho u^i u_j + p(g^i_j + u^i u_j). \tag{6}$$

This form of energy–momentum tensor consists of domain wall with quark matter [34], where $\rho = p_m + \sigma_w$ and $p = p_m - \sigma_w$ in which the quantities ρ_m and p_m are given by eqs (3) and (4) as well as domain wall tension, σ_w. u^i is the time-like four-velocity vector satisfying $u^i u_j = -1$. In co-moving coordinate system, u^i (0, 0, 0, 1), we have

$$T^1_1 = T^2_2 = T^3_3 = p, \quad T^4_4 = -\rho \quad \text{and} \quad T^i_j = 0 \quad \text{for } i \neq j. \tag{7}$$

The Einstein field equations (in gravitational unit $8\pi G = C = 1$) for the line element (5) using eq. (7) take the form

$$2\frac{B_{44}}{B} + \frac{B_4^2 + \delta}{B^2} - \frac{3}{4}\frac{A^2}{B^4} = -p, \tag{8}$$

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{1}{4}\frac{A^2}{B^4} = -p, \tag{9}$$

$$2\frac{A_4 B_4}{AB} + \frac{B_4^2 + \delta}{B^2} - \frac{1}{4}\frac{A^2}{B^4} = \rho. \tag{10}$$

Equations (8)–(10) are three independent equations in four unknown quantities A, B, p, ρ. Therefore, to get determinate solution of the field equation, we need one extra condition. The observations suggest that the Hubble expansion of the Universe is isotropic today to within 30 percent [46,47]. More precisely, the red-shift studies place the limit $(\sigma/H) \leq 0.30$, where H is the Hubble parameter and σ is the shear scalar [48]. Thorne [49] showed that $\sigma \propto \theta$, where θ is the expansion. Taking motivation from this, we assume in the present paper, shear scalar is proportional to the expansion scalar leading to the equation

$$A = B^n, \tag{11}$$

where n is a constant.

Using eqs (11), (8) and (9), we obtain

$$2B_{44} + 2\frac{(1+n)}{B}B_4^2 = \frac{2}{1-n}B^{2n-3} - \frac{2\delta}{(1-n)\,B}, \quad n \neq 1. \tag{12}$$

Let $B_4 = f(B)$, so that $B_{44} = ff'$, where the prime denote differentiation with respect to B. Equation (12) further reduces to

$$f^2 = \frac{1}{2n\,(1-n)}B^{2n-2} - \frac{\delta}{\left(1-n^2\right)} + c^2 B^{-2(n+1)}, \tag{13}$$

where c is the integration constant.

3. Bianchi type II ($\delta = 0$)

Equation (13) for $n = 2$ and $\delta = 0$ leads to

$$f = \frac{1}{2B^3}(4c^2 - B^8)^{1/2}. \tag{14}$$

Equation (14) gives

$$A = (2c\sin(2t))^{1/2}, \tag{15}$$

$$B = (2c\sin(2t))^{1/4}. \tag{16}$$

We get the following expression for pressure and density:

$$p = p_m - \sigma_w = \frac{5}{4}\cot^2(2t) + \frac{11}{4}, \tag{17}$$

$$\rho = \rho_m + \sigma_w = \frac{5}{4}\cot^2(2t) - \frac{1}{4}. \tag{18}$$

Making use of eq. (3) in eqs (17) and (18), we get

$$\text{Quark density, } \rho_{\mathrm{q}} = \frac{30}{16}(\cot^2(2t) + 1). \tag{19}$$

$$\text{Quark pressure, } p_{\mathrm{q}} = \frac{10}{16}\left(\cot^2(2t) + 1\right). \tag{20}$$

$$\text{Tension of the domain wall, } \sigma_w = \frac{5}{8}\cot^2(2t) - B_{\mathrm{c}} - \frac{3}{8}. \tag{21}$$

From eqs (19) and (20), we can observe that $p_{\mathrm{q}} = (1/3)\rho_{\mathrm{q}}$, that is, we get inflationary domain wall solutions as proposed by [9,10].

If we use eq. (4) for $\gamma = 2$ in eqs (17) and (18), we get

$$\text{Quark density, } \rho_{\mathrm{q}} = \frac{5}{4}(\cot^2(2t) + 1) - B_{\mathrm{c}}. \tag{22}$$

$$\text{Quark pressure, } p_{\mathrm{q}} = \frac{5}{4}(\cot^2(2t) + 1t) + B_{\mathrm{c}}. \tag{23}$$

$$\text{Tension of the domain wall, } \sigma_w = -\frac{3}{2}. \tag{24}$$

From eqs (22) and (23), we see that for stiff domain wall, we have stiff quark matter solution in the absence of the term, B_{c}. The negative tension of the domain wall in eq. (24) means that domain walls behave like invisible matter due to their negative tension. This result is similar to the result obtained by Yilmaz [18] and Katore *et al* [37] in different context.

3.1 *Physical properties*

In this model, the physical parameters are given as

$$\text{Volume, } V = AB^2 = 2c\sin(2t). \tag{25}$$

$$\text{Hubble parameter, } H = \frac{V_4}{3V} = \frac{2}{3}\cot(2t). \tag{26}$$

$$\text{Mean anisotropic parameter, } \Delta = \frac{1}{3}\sum_{i=1}^{3}(H_i - H)^2 = \frac{1}{8}. \tag{27}$$

$$\text{Expansion scalar, } \theta = 3H = 2\ \cot(2t). \tag{28}$$

$$\text{Shear scalar, } \sigma^2 = \frac{3}{2}\Delta H^2 = \frac{1}{4}\cot^2(2t). \tag{29}$$

$$\text{Deceleration parameter, } q = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) - 1 = 3\sec^2(2t) - 1. \tag{30}$$

We observe that the Universe is anisotropic. The value of Δ is similar to the value obtained by Chaubey [39] in the investigation of Bianchi type III and Kantowski Sachs with wet dark fluid. The volume of the Universe is oscillatory and when time is zero we have big bang.

4. Bianchi type VIII ($\delta = -1$)

To solve the field eqs (8)–(10) for Bianchi type VIII metric, we follow Hajj-Boutros [35] and Shri Ram and Prem Sing [36].

From eqs (8) and (9), we obtain

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} - \frac{A_4 B_4}{AB} + \frac{B_4^2}{B} - \frac{1}{B^2} - \frac{A^2}{B^4} = 0. \tag{31}$$

Making the transformation, $\mathrm{d}T = A\mathrm{d}t$, eq. (31) leads to

$$\frac{B''}{B} + \frac{B'^2}{B} - \frac{A'^2}{A} - \frac{A''}{A} - \frac{1}{A^2 B^2} - \frac{1}{B^4} = 0. \tag{32}$$

We put $a = A^2$ and $b = B^2$ in eq. (32) and obtain

$$\frac{b''}{b} - \frac{a''}{a} = \frac{2}{ab} + \frac{2}{b^2}. \tag{33}$$

Assuming the ad hoc relation

$$\frac{a''}{a} = -\frac{2}{ab} - \frac{2}{b^2}, \tag{34}$$

we get

$$\frac{b''}{b} = 0. \tag{35}$$

Equation (35) leads to

$$B^2 = b = cT + c_1, \tag{36}$$

where c and c_1 are integration constants.

Using eq. (36) in eq. (34), we get

$$\frac{a''}{a} = -\frac{2}{a(cT + c_1)} - \frac{2}{(cT + c_1)^2}. \tag{37}$$

Making the scale transformation, $L = cT + c_1$ in eq. (37), we obtain

$$L^2 \frac{\mathrm{d}^2 a}{\mathrm{d}L^2} + \frac{2a}{c^2} = -\frac{2L}{c^2}. \tag{38}$$

The general solution of eq. (38) is

$$a = A^2 = c_2(cT + c_1)^{\alpha_1} + c_3(cT + c_1)^{\alpha_2} - (cT + c_1), \tag{39}$$

where c_2 and c_3 are two constants of integration; and α_1 and α_2 are the roots of following eq. (40):

$$m^2 - m + 2/c^2 = 0. \tag{40}$$

When $c^2 \geq 8, \alpha_1, \alpha_2$ are real.

For these values of scale factor (36) and (39), we obtain the pressure and density as

$$p = \frac{c_2\left(-c^2 (2\alpha_1 - 1) + 3\right)}{4} (cT + c_1)^{\alpha_1 - 2} + \frac{c_3\left(-c^2 (2\alpha_2 - 1) + 3\right)}{4} (cT + c_1)^{\alpha_2 - 2} + \frac{\left(c^2 + 1\right)}{4(cT + c_1)}. \tag{41}$$

$$\rho = \frac{c_2\big(c^2\,(2\alpha_1+1)-1\big)}{4}\,(cT+c_1)^{\alpha_1-2} + \frac{c_3\big(c^2(2\alpha_2+1)-1\big)}{4}\,(cT+c_1)^{\alpha_2-2} - \frac{\big(c^2+3\big)}{4(cT+c_1)}. \tag{42}$$

If we use eq. (3) in (41) and (42), we get

$$\text{Quark density, } \rho_{\rm q} = \frac{3c_2}{8}\,(cT+c_1)^{\alpha_1-2} + \frac{3c_3}{8}\,(cT+c_1)^{\alpha_2-2}\,. \tag{43}$$

$$\text{Quark pressure, } p_{\rm q} = \frac{c_2}{4}\,(cT+c_1)^{\alpha_1-2} + \frac{c_3}{4}\,(cT+c_1)^{\alpha_2-2}\,. \tag{44}$$

$$\text{Tension of the domain wall, } \sigma_w = \frac{c_2\big(2c^2\,(1-\alpha_1)-5\big)}{4}\,(cT+c_1)^{\alpha_1-2} + \frac{c_3\big(2c^2\,(1-\alpha_2)-5\big)}{4}\,(cT+c_1)^{\alpha_2-2} - \frac{\big(c^2+1\big)}{4(cT+c_1)} - B_{\rm c}. \tag{45}$$

In this case, we see that from eqs (43) and (44), the equation $p_{\rm q} = (1/3)\rho_{\rm q}$ is satisfied. By substituting $\gamma = 2$ in eq. (4) and using eqs (41) and (42) we get

$$\text{Quark density, } \rho_{\rm q} = \frac{c_2}{4}\,(cT+c_1)^{\alpha_1-2} + \frac{c_3}{4}\,(cT+c_1)^{\alpha_2-2} - B_{\rm c}. \tag{46}$$

$$\text{Quark pressure, } p_{\rm q} = \frac{c_2}{4}\,(cT+c_1)^{\alpha_1-2} + \frac{c_3}{4}\,(cT+c_1)^{\alpha_2-2} + B_{\rm c}. \tag{47}$$

$$\text{Tension of the domain wall, } \sigma_w = \frac{c_2\big(c^2\,(2\alpha_1+1)-2\big)}{4}\,(cT+c_1)^{\alpha_1-2} + \frac{c_3\big(c^2\,(2\alpha_2+1)-2\big)}{4}\,(cT+c_1)^{\alpha_2-2} - \frac{\big(c^2+1\big)}{4\,(cT+c_1)}. \tag{48}$$

For this case, we observe that eqs (46) and (47) satisfy the case of stiff quark matter.

4.1 *Physical properties*

The values of physical parameters are found to be

$$\text{Volume, } V = AB^2 = \big[c_2L^{\alpha_1} + c_3L^{\alpha_2} - L\big]^{1/2}\,L. \tag{49}$$

$$\text{Hubble parameter, } H = \frac{V_4}{3V} = \frac{c}{3}\left\{\frac{c_2\alpha_1L^{\alpha_1-1} + c_3\alpha_2L^{\alpha_2-1} - 1}{2\,[c_2L^{\alpha_1} + c_3L^{\alpha_2} - L]} - \frac{1}{L}\right\}. \tag{50}$$

$$\text{Mean anisotropic parameter, } \Delta = 2\frac{\left\{\frac{c}{2}\frac{\left[c_2\alpha_1 L^{\alpha_1-1}+c_3\alpha_2 L^{\alpha_2-1}-1\right]}{\left[c_2 L^{\alpha_1}+c_3 L^{\alpha_2}-L\right]}-\frac{c}{2L}\right\}^2}{\left\{\frac{c}{2}\frac{\left[c_2\alpha_1 L^{\alpha_1-1}+c_3\alpha_2 L^{\alpha_2-1}-1\right]}{\left[c_2 L^{\alpha_1}+c_3 L^{\alpha_2}-L\right]}+\frac{c}{L}\right\}^2}. \tag{51}$$

$$\text{Expansion scalar, } \theta = c\left\{\frac{c_2(\alpha_1-c)L^{\alpha_1}+c_3(\alpha_2-c)L^{\alpha_2}-(1-c)L}{\left[c_2 L^{\alpha_{1+1}}+c_3 L^{\alpha_2+1}-L^2\right]}\right\}. \tag{52}$$

$$\text{Shear scalar, } \sigma^2 = \frac{1}{3}\left\{\frac{c_2\alpha_1 L^{\alpha_1-1}+c_3\alpha_2 L^{\alpha_2-1}-1}{\left[c_2 L^{\alpha_1}+c_3 L^{\alpha_2}-L\right]}-\frac{c}{2L}\right\}. \tag{53}$$

Deceleration parameter,

$$q = 3\left\{\frac{\begin{array}{l} cc_2^2(\alpha_1-c)L^{2\alpha_1}+cc_3^2(\alpha_2-c)L^{2\alpha_2}+cc_2c_3\left(2\alpha_1\alpha_2+\alpha_1+\alpha_2-\alpha_1^2\right.\\ \left.-\alpha_2^2-2c\right)L^{\alpha_1+\alpha_2}+cc_2L^{\alpha_1+2}+cc_3L^{\alpha_2+2}+cc_2\left(\alpha_1^2-3\alpha_1-1+3c\right)\\ \times L^{\alpha_1+1}+cc_3\left(\alpha_2^2-3\alpha_2-1+3c\right)L^{\alpha_2+1}+2c(1-c)L^2-cL^3 \end{array}}{c\left[c_2(\alpha_1-c)L^{\alpha_1}+c_3(\alpha_2-c)L^{\alpha_2}-(1-c)L\right]^2}\right\}. \tag{54}$$

where $L = cT + c_1$. In this model, the volume of the Universe increases indefinitely with time. The value of the mean anisotropic parameter is non-zero, that is, the Universe is anisotropic in nature.

5. Bianchi type IX ($\delta = 1$)

In eq. (13), taking $c = 0$ and integrating, we obtain

$$A = \frac{4}{3}\sin^2\left(\frac{t}{2}\right). \tag{55}$$

$$B = \frac{2}{\sqrt{3}}\sin\left(\frac{t}{2}\right). \tag{56}$$

For these values of scale factor, pressure and density of the model is obtained as

$$p = \cot^2\left(\frac{t}{2}\right) - 1/2. \tag{57}$$

$$\rho = 2\cot^2\left(\frac{t}{2}\right) + 1/2. \tag{58}$$

Using eq. (3) in eqs (57) and (58), we get

$$\text{Quark pressure, } p_q = \frac{1}{4}\cot^2\left(\frac{t}{2}\right) + \frac{1}{4}. \tag{59}$$

$$\text{Quark density, } \rho_{\mathrm{q}} = \frac{3}{4}\cot^2\left(\frac{t}{2}\right) + \frac{3}{4}. \tag{60}$$

$$\text{Tension of the domain wall, } \sigma_w = \frac{5}{4}\cot^2\left(\frac{t}{2}\right) - 1/4 - B_c. \tag{61}$$

For this model, from eqs (59) and (60), we have $p_{\mathrm{q}} = \frac{1}{3}\rho_{\mathrm{q}}$.
Substituting $\gamma = 2$ in eq. (4) and using eqs (59) and (60), we can obtain

$$\text{Quark pressure, } p_{\mathrm{q}} = \frac{1}{2}\cot^2\left(\frac{t}{2}\right) + B_{\mathrm{c}} + \frac{1}{2}. \tag{62}$$

$$\text{Quark density, } \rho_{\mathrm{q}} = \frac{1}{2}\cot^2\left(\frac{t}{2}\right) - B_{\mathrm{c}} + \frac{1}{2}. \tag{63}$$

$$\text{Tension of the domain wall, } \sigma_w = 3\cot^2(1/2t) + 1. \tag{64}$$

In the absence of term B_{c} for this case, we observe that eqs (62) and (63) satisfy the case of stiff quark matter.

5.1 *Physical properties*

For this model, the physical parameter takes the form

$$\text{Volume, } V = \frac{16}{9}\sin^4\left(\frac{t}{2}\right). \tag{65}$$

$$\text{Hubble parameter, } H = \frac{2}{3}\cot\left(\frac{t}{2}\right). \tag{66}$$

$$\text{Mean anisotropic parameter, } \Delta = \frac{1}{8}. \tag{67}$$

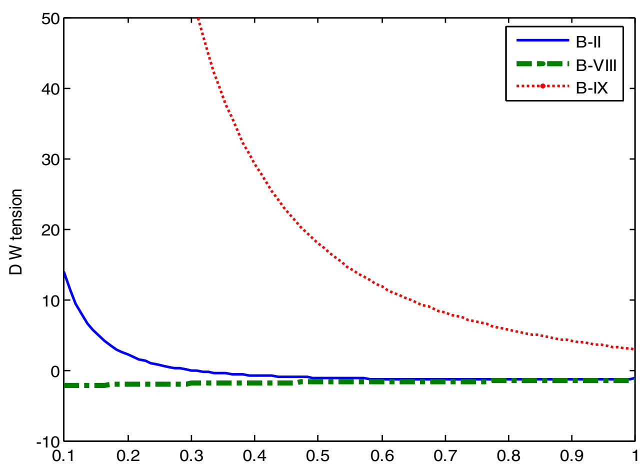

Figure 1. Plot of tension of the domain wall of quark matter vs. time.

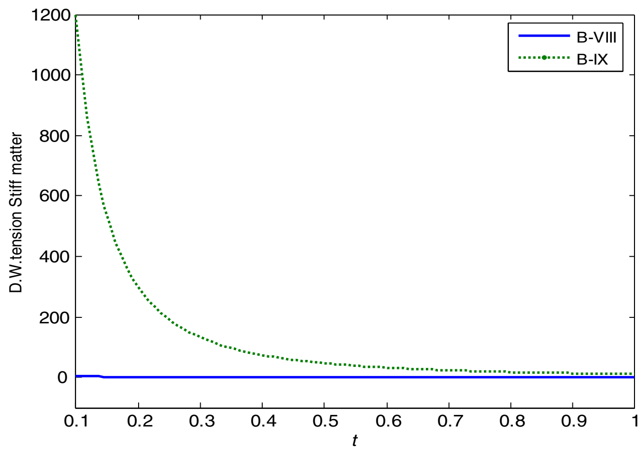

Figure 2. Plot of the tension of the domain wall of stiff matter vs. time.

$$\text{Expansion scalar, } \theta = 2 \cot\left(\frac{t}{2}\right). \tag{68}$$

$$\text{Shear scalar, } \sigma^2 = \frac{1}{12} \cot^2\left(\frac{t}{2}\right). \tag{69}$$

$$\text{Deceleration parameter, } q = \frac{3}{4} \sec^2\left(\frac{t}{2}\right) - 1. \tag{70}$$

We see that the value of the mean anisotropic parameter is constant, that is, the Universe is anisotropic. The volume of the Universe is a trigonometric function of time, that is, the volume of the Universe is oscillatory in nature.

In figures 1 and 2, we have taken the values of constant as $\alpha_1 = \alpha_2 = 0.5$, $B_c = c_2 = c_3 = c_1 = 1, c = 2\sqrt{2}$. In figure 1, we can see that the tension tends to zero for large time, that is the domain wall vanishes at large time.

In figure 2 we can see that as time increases the tension tends to zero, that is, domain walls vanish when the age of the Universe increases.

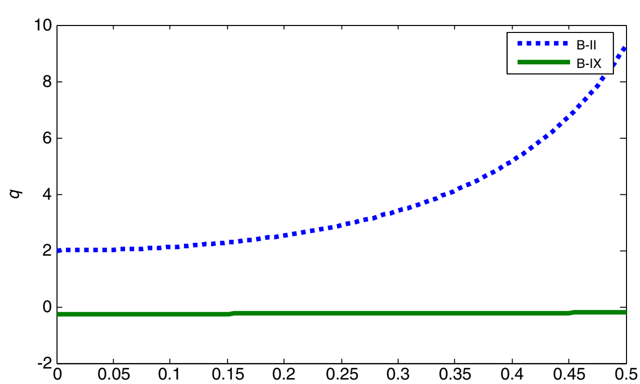

Figure 3. Plot of the deceleration parameter vs. time.

In figure 3, we observe that for Bianchi type II cosmological model, the deceleration parameter is greater than two. Therefore Universe is decelerating. For Bianchi type IX cosmological model, the Universe is accelerating as the deceleration parameter remains negative.

6. Conclusion

In this paper, we have presented domain walls of Bianchi types II, VIII and IX Universes with quark matter. To obtain solutions, it is expected that the equation of the domain wall will be stiffer than that of radiation, which is a realistic case in which the domain walls interact with the primodal plasma. We considered two cases of each solution of Bianchi types II, VIII and IX metrics.

In the case of strange quark matter coupled to domain walls, we get $p_{\mathrm{q}} = (1/3)\rho_{\mathrm{q}}$ as proposed by Bodmer [9] and Written [10] for Bianchi types II, VIII and IX. The domain walls are important in the structure formation of the Universe. The tension of the domain wall tends to be zero at large time which means that the domain wall vanishes at large time. The pressure is found to be positive, that is, the reality conditions are satisfied.

In the case of quark matter coupled to domain walls, when we have stiff quark matter solution, the tension vanishes at large time, that is, domain walls are invisible at large time.

It is interesting to note that Yilmaz [18], Katore *et al* [37] and Reddy *et al* [38] obtained similar types of conclusions in different contexts.

The value of the mean anisotropic parameter is non-zero, that is, in all cases the nature of Universe is anisotropic. The value of the volume is oscillatory for the Bianchi types II and IX and it is increasing with time for the Bianchi type VIII model. The Universe is accelerating in Bianchi type IX, whereas, it decelerate in Bianchi type II cosmological model.

Acknowledgements

We would like to thank the anonymous referees, whose comments and suggestions helped us to significantly improve the manuscript.

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