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Three-body calculation of Be double-Λ hypernuclei

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Abstract. Energy levels and $\Lambda\Lambda$ bond energy of the double-Λ hypernucleus are calculated by considering two- and three-cluster interactions. Interactions between constituent particles are contact interactions for reproducing the low binding energy of nuclei. The effective action is constructed to involve three-body forces. In this paper, we also compare the obtained binding energy result with experimental and other cluster and shell models. The results of all schemes agree very well showing the high accuracy of our method to calculate the other many-body hyperonic nuclei using three-cluster interactions. The experimental values of $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}) = (11.90 \pm 0.13)$ MeV, $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\mathrm{Be}) = (20.49 \pm 1.15)$ MeV and $B_{\Lambda\Lambda}({}^{12}_{\Lambda\Lambda}\mathrm{Be}) = (22.23 \pm 1.15)$ MeV seem to be more compatible with our calculated value of $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}) = 14.04$ MeV, $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\mathrm{Be}) = 19.31$ MeV and $B_{\Lambda\Lambda}({}^{12}_{\Lambda\Lambda}\mathrm{Be}) = 21.45$ MeV in comparison with the other calculated results by Hiyama *et al*, Gal *et al* and Guleria *et al*.

Keywords. Hypernuclei; cluster models; effective interactions in hadronic systems.

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1. Introduction

In nuclear physics, the fundamental problem is to describe the different facets of interactions among the nucleons in two- and many-body hyperonic nuclei. Recent advances in computational facilities, together with the development of new methods and refinements of older ones, allow very precise calculations for few-body systems. Better hyperon–nucleon forces would help us to understand the role of hyperonic degrees of freedom in high-density matter, e.g., compact stars. Theoretically, hyperon–nucleon interactions have many similarities but also many differences with nucleon–nucleon interactions: no one-pion-exchange in ΛN interaction, but one-kaon-exchange; important two-pion-exchange with the excitation of an intermediate sigma; and hyperon–nucleon–nucleon forces that may be relatively more important than three-nucleon forces, depending on whether sigmas are kept explicitly.

On the other hand, for baryon–baryon interactions with $S = -2$ sectors, studied presently, experimental information is very limited due to the extreme difficulties of two-body scattering experiments. Therefore, the observed $\Lambda\Lambda$ bond energies of double-Λ hypernuclei should be the most reliable source for the $S = -2$ interaction, and such data play decisive roles in determining the strength of the underlying $\Lambda\Lambda$ interactions. Theoretically, hyperon–nucleon interactions have many similarities but also many differences with nucleon–nucleon interactions: no one-pion-exchange in ΛN, but one-kaon-exchange; important two-pion-exchange with the excitation of an intermediate sigma; and hyperon–nucleon–nucleon forces that may be relatively more important than three-nucleon forces, depending on whether sigmas are kept explicitly.

So far, several cluster models have been proposed to estimate the ground-state binding energies of double-Λ species [1–6]. Recently, ΛN interaction models have been constructed by utilizing various Λ hypernuclear data to complement the limited ΛN scattering data. A recent finding of the double-Λ hypernucleus in the KEK-E373 experiment has a great impact not only on the study of baryon–baryon interactions in the strangeness $S = -2$ sector but also on the study of dynamics of many-body systems with multi-strangeness [5]. Hiyama *et al* reported a pioneering five-body $\alpha\alpha N\Lambda\Lambda$ cluster-model calculation of ${}^{11}_{\Lambda\Lambda}$Be in order to confront a possible interpretation of the KEK-E373 HIDA event [7]. Gal *et al* reported a six-body $\alpha\alpha NN\Lambda\Lambda$ calculation of ${}^{12}_{\Lambda\Lambda}$Be to confront another possible interpretation which is beyond reach at present [8]. They obtained binding-energy shell-model estimates for both ${}^{11,12}_{\Lambda\Lambda}$Be, using experimental B_{Λ} values with small corrections based on the recently determined ΛN spin-dependent interaction parameters. The results of their calculation conclude that neither ${}^{11}_{\Lambda\Lambda}$Be nor ${}^{12}_{\Lambda\Lambda}$Be provides satisfactory interpretation of the HIDA event.

Guleria *et al* also extended the Skyrme–Hartree–Fock (SHF) approach to study the double-Λ hypernuclei [9]. They applied the nucleon–nucleon (*NN*) and Λ–nucleon (ΛN) interactions to successfully describe the single-Λ hypernuclear systems. They discussed the sensitivity of the calculated $\Lambda\Lambda$ binding energy and the $\Lambda\Lambda$ bond energy to the $\Lambda\Lambda$ and ΛN force parameters.

Recently, we have calculated the energy levels and $\Lambda\Lambda$ bond energy of the double-Λ hypernucleus ^{11}Be by considering two- and three-nucleon forces. The interactions between the constituent particles are contact interactions that reproduce low-energy binding energy of the nuclei. The results of all the schemes agree very well, showing the high accuracy of our present study to calculate the many-nucleon bound state with three-body forces [10].

In this paper, we present the results of three-body Faddeev-type calculations for systems of three-clusters interacting through short-range nuclear as well as the long-range Coulomb interactions. We apply the three-cluster Faddeev formalism to $\alpha\alpha\Lambda\Lambda$ the four-body, $\alpha\alpha N\Lambda\Lambda$ the five-body and $\alpha\alpha NN\Lambda\Lambda$ the six-body clusters for calculating the bond energy of ${}^{10}_{\Lambda\Lambda}$Be, ${}^{11}_{\Lambda\Lambda}$Be and ${}^{12}_{\Lambda\Lambda}$Be hypernuclei, respectively. From the earlier hypernuclear study, $\Lambda\Lambda$Be hypernucleus is considered to be a prototype of cluster structures. We have succeeded in performing a many-body calculation of $\Lambda\Lambda$Be using the cluster model in comparison with the recent observation of the events for a new double-Λ hypernucleus. The calculated $\Lambda\Lambda$ binding energy also shows good agreement with the other theoretical methods.

This paper is organized as follows. In §2, we describe the interactions between particles and derive the Faddeev equations for scattering amplitude. The Faddeev equations will be solved by iteration yielding a multiple scattering series. We tabulate the calculated bond energy, discuss the theoretical errors, and compare our results with the corresponding experimental and theoretical values in §3. Summary and conclusions follow in §4.

2. Brief review of the theoretical framework

2.1 αN, ΛN, $\Lambda\Lambda$ *and the Coulomb interactions*

Similar to the formalism presented in ref. [10], we employ the potential $V_{\alpha N}$ between the clusters α and nucleon which have been often used in the OCM-based cluster-model study of light nuclei. The potential, in the following parity-dependent form with the central and spin-orbit terms, are given by [10]

$$V_{\alpha x}(r) = \sum_{i=1}^{i_{\max}} V_i\, \mathrm{e}^{-\beta_i r^2} + \sum_{i=1}^{i'_{\max}} (-)^l V_i^{\mathrm{p}}\, \mathrm{e}^{-\beta_i^{\mathrm{p}} r^2} + \left[\sum_{i=1}^{i''_{\max}} V_i^{ls}\, \mathrm{e}^{-\gamma_i r^2} + \sum_{i=1}^{i'''_{\max}} (-)^l V_i^{ls,\mathrm{p}}\, \mathrm{e}^{-\gamma_i^{\mathrm{p}} r^2} \right] \mathbf{l}\cdot\mathbf{s}_x\ , \tag{1}$$

where p and $\mathbf{l}$ are the parity and the relative angular momentum between α and the nucleon and $\mathbf{s}_x$ is the spin of x. The interaction between the Λ particle and the nucleon is described by folding the G-matrix type YN interaction into the density of the x cluster, and it is given in [11]

$$v_{\Lambda N}(r;k_{\mathrm{F}}) = \sum_{i=1}^{3} \left[\left(v_{0,\mathrm{even}}^{(i)} + v_{\sigma\sigma,\mathrm{even}}^{(i)} \boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N\right)\frac{1+P_r}{2} + \left(v_{0,\mathrm{odd}}^{(i)} + v_{\sigma\sigma,\mathrm{odd}}^{(i)} \boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N\right)\frac{1-P_r}{2} \right] \mathrm{e}^{-\mu_i r^2}, \tag{2}$$

where P_r is the space exchange (Majorana) operator (see table 1 for various parameters of YN interactions).

$V_{\Lambda\Lambda}(\mathbf{r})$, which is the field generated by the $\Lambda\Lambda$ interaction, is given by [10]

$$V_{\Lambda\Lambda} = \frac{1}{2}\lambda_0\rho_\Lambda + \frac{1}{8}(\lambda_1+3\lambda_2)\tau_\Lambda + \frac{3}{16}(\lambda_2-\lambda_1)\nabla^2\rho_\Lambda + \frac{1}{2}\lambda_3\rho_\Lambda\rho_N^\alpha\ , \tag{3}$$

where $\rho_\Lambda(\rho_N)$ is the hyperon(nucleon) density. The value of the parameters (λ_0 and λ_1) are -312.6 MeV fm^3 and 57.5 MeV fm^5, respectively. The parameter λ_2 is set to zero under the argument that the p wave contributions do not take part in the lowest single-particle levels. The last term in eq. (3) also corresponds to the three-body $\Lambda\Lambda N$ interaction. The Coulomb potential W_{C}, due to its long range, does not satisfy the mathematical properties required for the formulation of standard scattering theory as given in the previous section for short-range interactions V_α. We choose the screened Coulomb potential in configuration-space representation as

$$W_R(r) = W_{\mathrm{C}}(r)\mathrm{e}^{-(r/R)^n}, \tag{4}$$

Table 1. Parameters applied of potentials defined in eqs (1–3) (see [5,10] for more details).

Parameter	$\alpha\alpha$	αt	αd	αN
i	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
β_i	0.11, 0.27, 0.33	0.09, 0.16, 0.20	1, –, –	0.36, 0.9, –
V_i	−1.7, −396, 299	7, −43, −52	−64, –, –	−96, 77, –
V_i^{p}	0.0, 0.0, 0.0	6.9, 43.3, −51.7	−10.2, –, –	34.0, −85.0, 51.0
β_i^{p}	–, –, –	0.1, 0.1, 0.2	0.2, –, –	0.2, 0.5, 2.5
γ_i	–, –, –	0.2, –, –	0.3, –, –	0.4, 0.5, 2.2
V_i^{ls}	–, –, –	−1.2, –, –	−4.0, –, –	−20.0, −16.8, 20.0
γ_i^{p}	–, –, –	0.3, –, –	0.3, –, –	0.4, 2.2, –
$V_i^{ls,\mathrm{p}}$	–, –, –	1.2, –, –	−4.0, –, –	6.0, −6.0, –
	$\Lambda\alpha$	Λt	Λd	
β_i	0.27, 0.45, 0.61	0.28, 0.49, 0.67	0.31, 0.57, 0.85	
V_i	−17.4, −127.0, 497.8	−14.1, −108.0, 425.9	−10.8, −88.3, 167.2	
V_i^{s}	0.0, 0.0, 0.0	2.3, 10.9, −126.9	2.7, 14.3, −179.9	
γ_i	0.18, 0.18, 0.18	0.20, 0.20, 0.20	0.27, 0.27, 0.27	
δ_i	0.40, 0.96, 2.9	0.33, 0.82, 2.52	0.24, 0.48, 1.924	
U_i	−0.37, −12.94, −331.20	−0.27, −9.55, −231.60	−0.18, −5.84, −3.06	
U_i^{s}	0.0, 0.0, 0.0	−0.26, 1.43, 97.05	−0.27, 1.56, 100.40	
	ΛN	$\Lambda\Lambda$		
μ_i	0.5, 1.3, 6.2	0.5, 1.6, 8.1		
v_i^{even}	−10, −87, 1031	–, –, –		
$v_i^{\mathrm{even},\sigma}$	0.2, 17, −256.30	–, –, –		
v_i^{odd}	−6, −18, 4029	–, –, –		
$v_i^{\mathrm{odd},\sigma}$	−1, −9, −573	–, –, –		
v_i	–, –, –	−10, −93, 4884		
v_i^{σ}	–, –, –	0.1, 16.0, 915.8		

where R is the screening radius and n controls the smoothness of the screening. The Lippmann–Schwinger equation yields the two-particle transition matrix

$$T_R = W_R + W_R g_0 t_R, \tag{5}$$

where g_0 is the two-particle free resolvent.

2.2 *Three-body nuclear reactions*

Recently, the three-body description of nuclear reactions involving a number of approximate methods has been developed. The methods are the distorted-wave Born approximation, various adiabatic approaches [12] and continuum-discretized coupled-channels method [13]. The present method based on Faddeev is more technically and numerically evolved but has some disadvantages [10]. We take the one from [14,15] defined in the configuration space as

$$v_\gamma(\vec{r}', \vec{r}) = H_c(x)[V_c(y) + i W_c(y)] + 2\vec{S_\gamma} \cdot \vec{L_\gamma} H_s(x) V_s(y), \tag{6}$$

with $x = |\vec{r}' - \vec{r}|$ and $y = |\vec{r}' + \vec{r}|/2$. The description of the (α, Λ, N) three-particle system with real potentials is quite successful at low energies but becomes less reliable at higher energies where the inner structure of the α-particle cannot be neglected anymore [17]. The central part has real volume and imaginary surface parts, whereas the spin-orbit part is real; all of them are expressed in the standard way by Woods–Saxon functions. Some of their strength parameters were readjusted in [16] to improve the description of the experimental nucleon–nucleus scattering data. The methods based on Faddeev equations can also be applied in this case. However, the potentials within the pairs that are bound in the initial or final channel must remain real.

2.3 *The Faddeev integral equation*

In the framework of nonrelativistic quantum mechanics the centre-of-mass (c.m.) and the internal motion can be separated by introducing Jacobi momenta

$$\begin{aligned} \vec{p}_\alpha &= \frac{m_\gamma \vec{k}_\beta - m_\beta \vec{k}_\gamma}{m_\beta + m_\gamma}, \\ \vec{q}_\alpha &= \frac{m_\alpha (\vec{k}_\beta + \vec{k}_\gamma) - (m_\beta + m_\gamma)\vec{k}_\alpha}{m_\alpha + m_\beta + m_\gamma}, \end{aligned} \tag{7}$$

where $(\alpha\beta\gamma)$, $\vec{k}_\alpha$ and m_α are the cyclic permutations of (123), the individual cluster momentum and the mass, respectively. The Lippmann–Schwinger equations are given by

$$\Psi_0^{(c)} = \Phi_\beta^{(c)} + G_\beta U^\beta \Psi_0^{(c)}, \tag{8}$$

where $|\mathbf{p}_\beta\rangle^{(c)}$ is a two-cluster state, and the index $\beta = 1, 2, 3$ indicates the three choices of pairs characterized by the third particle or cluster, the spectator. $U^\beta = \sum_{\gamma \neq \beta} U_\gamma$, where U_γ $(\gamma = 1, 2, 3)$ are the pair forces. $G_\beta^{-1} = (E + i\varepsilon - H_0 - U_\beta)^{-1}$ is the Green's function. The corresponding two-particle transition matrices are calculated with the full channel interaction [10,16]

$$T_\beta^{(R)} = (V_\beta + W_{\beta R}) + (V_\beta + W_{\beta R}) G_0 T_\beta^{(R)}. \tag{9}$$

$$\begin{aligned} V_0 &= \sum_\theta t_\theta \Phi_0 + \sum_\theta t_\theta G_0 \sum_{\gamma \neq \theta} t_\gamma \Phi_0 + \sum_\theta t_\theta G_0 \sum_{\gamma \neq \theta} t_\gamma G_0 \sum_{\beta \neq \gamma} t_\beta \Phi_0 + \cdots , \\ &\equiv \sum_\theta V_\theta, \end{aligned} \tag{10}$$

where V_θ obeys the coupled set of Faddeev equations

$$V_\theta = T_\theta + T_\theta G_0 \sum_{\beta \neq \theta} V_\beta. \tag{11}$$

Fixing the neutron as the spectator and labelling it as '1' and the two others ($\alpha\alpha$ and $\Lambda\Lambda$ two-body clusters) as particles '2' and '3', the scattering wave function $\Psi_0^{(c)}$ must be antisymmetric under the exchange of particles '2' and '3'. Thus, defining the exchange operator P_{23}, the scattering wave function must fulfill $P_{23}\Psi_0^{(+)} = -\Psi_0^{(+)}$. By applying the driving terms to the free state, we obtain

$$V_1\Phi_{0,a} = T_1\Phi_{0,a} + T_1 G_0 (1 - P_{23}) V_2\Phi_{0,a}$$

$$V_2\Phi_{0,a} = T_2\Phi_{0,a} + T_2 G_0 \left(-P_{23} V_2\Phi_{0,a} + V_1\Phi_{0,a}\right), \tag{12}$$

where

$$\Phi_{0,a} \equiv (1 - P_{23})|\mathbf{p}_1\mathbf{q}_1\rangle|0m_2m_3\rangle\left|0\frac{1}{2}\frac{1}{2}\right\rangle,$$

which is antisymmetric under the exchange of the two clusters.

The matrix elements for $\alpha\alpha + \Lambda\Lambda \rightarrow ^{10}_{\Lambda\Lambda}$Be, $N + \alpha\alpha + \Lambda\Lambda \rightarrow ^{11}_{\Lambda\Lambda}$Be and $NN + \alpha\alpha + \Lambda\Lambda \rightarrow ^{12}_{\Lambda\Lambda}$Be processes are simply related to the time-reversed photodisintegration process of $^{10}_{\Lambda\Lambda}$Be, $^{11}_{\Lambda\Lambda}$Be and $^{12}_{\Lambda\Lambda}$Be into free clusters, respectively. It is necessary to formulate photodisintegration of Be-doubleΛ based on a three-body picture. Let O be the photon absorption operator and $|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle$, the $_{\Lambda\Lambda}$Be ground state, where $x = 10$, 11 and 12, for $^{10}_{\Lambda\Lambda}$Be, $^{11}_{\Lambda\Lambda}$Be and $^{12}_{\Lambda\Lambda}$Be hypernuclei. The break-up amplitude of $\alpha\alpha + \Lambda\Lambda$ the four-body, $N + \alpha\alpha + \Lambda\Lambda$ the five-body and $NN + \alpha\alpha + \Lambda\Lambda$ the six-body clusters can then be written as an infinite series of processes

$$\langle\Phi_{0,a}|V_0|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle = \langle\Phi_{0,a}|O|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + \sum_i \langle\Phi_{0,a}|U_i G_0 O|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + \sum_{ij}\langle\Phi_{0,a}|U_i G_0 U_j G_0 O|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + \cdots, \tag{13}$$

where U_i are the pair forces among the $\Lambda\Lambda$, ΛN and $\Lambda\alpha$ particles. This infinite series in terms of pair forces represents final-state interactions. The first term is the direct break-up process generated by O. By defining

$$\langle\Phi_{0,a}|V_0|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle = \langle\Phi_{0,a}|O|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + \sum_i \langle\Phi_{0,a}|V_{0i}|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle, \tag{14}$$

one can obtain the T-matrices T_i, leading to three coupled Faddeev equations ($i = 1, 2, 3$) [19],

$$V_{0i}|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle = T_i G_0 O|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + T_i G_0 \sum_{j\neq i} V_{0j}|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle \tag{15}$$

and the complete break-up amplitude

$$\langle\Phi_{0,a}|V_0|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle = \langle\Phi_{0,a}|O|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + \langle\Phi_{0,a}|V_1|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle + \langle\Phi_{0,a}|(1 - P_{23})V_2|\Psi_{^x_{\Lambda\Lambda}\mathrm{Be}}\rangle. \tag{16}$$

In order to take into account the full many-body degrees of freedom of our systems and the full correlations among all the constituent particles, we describe the total wave function, as a function of the sets of Jacobi coordinates for each hypernucleus (for more details, see [5,18,20]).

3. Results and discussion

We derived the two coupled Faddeev equations for the cluster scattering amplitudes. We obtained the results of three-body Faddeev-type calculations for systems of three-particles interacting through short-range forces plus the long-range Coulomb interaction. Realistic applications of three-body theory to three-cluster nuclear reactions only became possible in recent years, when a reliable and practical momentum-space treatment of interaction has been developed [16]. In the present many-body system for $\Lambda\Lambda$Be hypernucleus, it is absolutely necessary that any subcluster system composed of two, three, four or five constituent particles are reasonably described by taking the interactions among these systems as discussed in §2. We provided Faddeev equations for the $\alpha\alpha\Lambda\Lambda$, $N\alpha\alpha\Lambda\Lambda$ and $NN\alpha\alpha\Lambda\Lambda$ capture processes to the ${}^{10}_{\Lambda\Lambda}$Be, ${}^{11}_{\Lambda\Lambda}$Be and ${}^{12}_{\Lambda\Lambda}$Be ground states.

In the present many-body calculations, we employ the interactions, so that constraints are successfully met in our two-, three-, four- and five-body subsystems (see §2). By using other works, double-Λ hypernuclei cluster model interactions were determined so as to reproduce well the following observed quantities: (a) energies of the low-lying states and scattering phase shifts in the αN and $\alpha\alpha$ systems, (b) Λ-binding energies B_Λ in ${}^{5}_{\Lambda}$He (=$\alpha\Lambda$), ${}^{6}_{\Lambda}$He (=$\alpha N\Lambda$), ${}^{9}_{\Lambda}$Be (=$\alpha\alpha\Lambda$), ${}^{10}_{\Lambda}$Be (=$\alpha\alpha\Lambda N$) and ${}^{11}_{\Lambda}$Be (=$\alpha\alpha\Lambda NN$), (c) double-Λ binding energies $B_{\Lambda\Lambda}$ in ${}^{6}_{\Lambda\Lambda}$He (=$\alpha\Lambda\Lambda$), the NAGARA event [5] and ${}^{11}_{\Lambda\Lambda}$He (=$\alpha\Lambda\Lambda$) HIDA event [7].

In tables 2 and 3, we show the results for double-Λ binding energies for the double-Λ Be hypernuclei with three different atomic masses. In all the results, one can see that $B_{\Lambda\Lambda}$ increases with mass number of the hypernucleus corresponding to the observations made of the double-Λ hypernuclei [22]. It is noted that the binding energies calculated using the Skyrme–Hartree–Fock approach depend on the choice of the parameters. In calculations of three-body systems including two- or three-clusters, we used an $\alpha\alpha$ potential with a strong repulsive core so as to describe the Pauli exclusion role which prevents the double-α cluster from overlapping. The obtained $B_{\Lambda\Lambda}$ for ${}^{10}_{\Lambda\Lambda}$Be, ${}^{11}_{\Lambda\Lambda}$Be and ${}^{12}_{\Lambda\Lambda}$Be hypernuclei with the first two sets of parameters reproduce the experimental data better than those obtained with the last set. It is well known that the presence of a third particle makes the double-α clusters come closer to each other. The last two rows correspond to our

Table 2. Comparison of different $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be hypernuclei, calculated using the Skyrme–Hartree–Fock method [9], the shell model calculations [8] and our obtained results by cluster model using only the two-body interaction or both the two- and three-body interactions. The last row shows the experimental data [21].

Type of calculation	MeV	Error
$B_{\Lambda\Lambda}^{S^{\Lambda\Lambda 1}}$ [9]	19.78	66%
$B_{\Lambda\Lambda}^{S^{\Lambda\Lambda 2}}$ [9]	18.34	54%
$B_{\Lambda\Lambda}^{S^{\Lambda\Lambda 3}}$ [9]	15.19	28%
Present work (two-body)	13.45	13%
Present work (two-body and three-body)	14.04	18%
Experiment	11.90 ± 0.13	(Prowse *et al*, 1966)

Table 3. Comparison of different $B_{\Lambda\Lambda}$ of ${}^{11}_{\Lambda\Lambda}$Be hypernuclei, calculated using the Skyrme–Hartree–Fock method [9], the shell model calculations [8] and our obtained results by cluster model using only the two-body interaction or both the two- and three-body interactions. The last row shows the experimental data [7].

Type of calculation	MeV	Error
$B^{S^{\Lambda\Lambda 1}}_{\Lambda\Lambda}$ [9]	20.55	$<$ 1%
$B^{S^{\Lambda\Lambda 2}}_{\Lambda\Lambda}$ [9]	19.26	6%
$B^{S^{\Lambda\Lambda 3}}_{\Lambda\Lambda}$ [9]	16.27	20%
SM [8]	18.39	10%
CM [5]	18.23	11%
Present work (two-body)	18.93	8%
Present work (two-body and three-body)	19.31	6%
Experiment	20.49 ± 1.15	HIDA [7]

calculations using the two-body interaction only or both the two- and three-body interactions. The results for ${}^{11}_{\Lambda\Lambda}$Be, ${}^{11}_{\Lambda\Lambda}$Be and ${}^{12}_{\Lambda\Lambda}$Be hypernuclei by considering the two-body interaction (both the two- and three-body interactions are found to be 13.45(14.04),

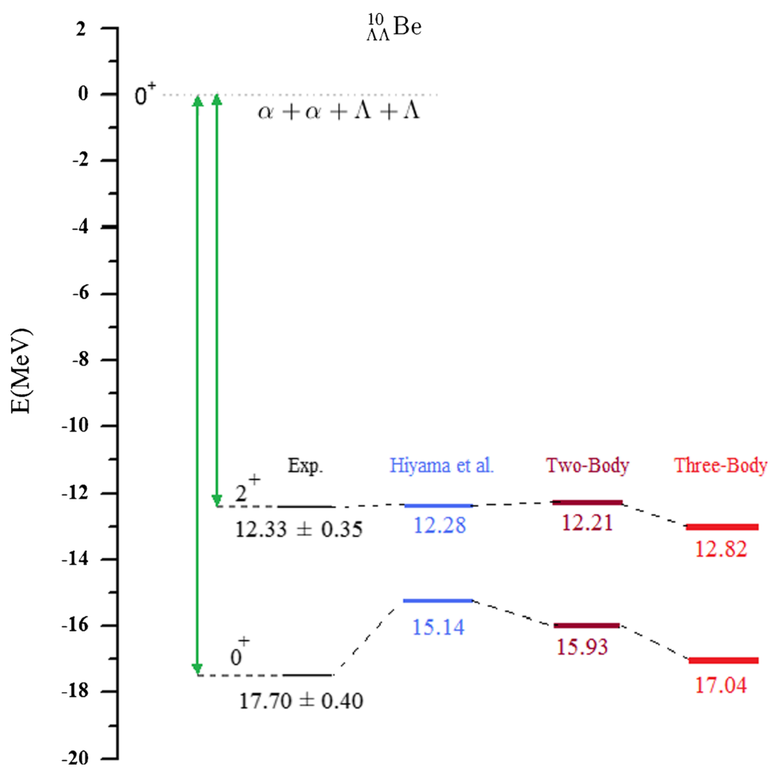

Figure 1. The calculated energy spectra of the low-lying states of ${}^{10}_{\Lambda\Lambda}$Be together with the results by Hiyama *et al* [5]. Experimental result is taken from [21].

18.93(19.31) and 20.96(21.45) MeV, respectively. The error for each method of calculations is also shown in the last column of each table.

For bond energies of $^{11}_{\Lambda\Lambda}$Be and $^{12}_{\Lambda\Lambda}$Be hypernuclei, our results with two- and three-body interactions show more agreement with the other theoretical and available experimental data. The binding energy of $^{10}_{\Lambda\Lambda}$Be hypernucleus shows an inconsistency when the three-body interactions are added to our calculations. This may be due to its cluster structure ($\alpha\alpha\Lambda\Lambda$) compared to the heavier system ($\alpha\alpha N\Lambda\Lambda$) or ($\alpha\alpha NN\Lambda\Lambda$), with the additional nucleons.

The results of $B_{\Lambda\Lambda}$ for the Skyrme–Hartree–Fock method [9] and the shell model calculations [8] are given in tables 2 and 3. For heavier systems, the obtained results from the two different methods are similar. We calculated energies for the $5/2_1^-$, $1/2_1^+$ and $1/2_1^-$ states of $^{10}_{\Lambda\Lambda}$Be and $^{11}_{\Lambda\Lambda}$Be as shown in figures 1 and 2. Figure 1 shows the calculated values of energy levels for the $5/2_1^-$, $1/2_1^+$ and $1/2_1^-$ states of $^{10}_{\Lambda\Lambda}$Be which are found to be 18.06, 17.35 and 16.95 MeV compared to the theoretical results 17.56, 16.45 and 16.04 MeV respectively by Hiyama *et al* [5]. In the emulsion analysis, there is no direct evidence for the production of $^{10}_{\Lambda\Lambda}$Be in an excited state. However, if the produced $^{10}_{\Lambda\Lambda}$Be is interpreted to be in the ground state, the resultant $\Lambda\Lambda$ bond energy becomes repulsive, contradictory to the NAGARA event. The calculated value of Hiyama studies [5] of the

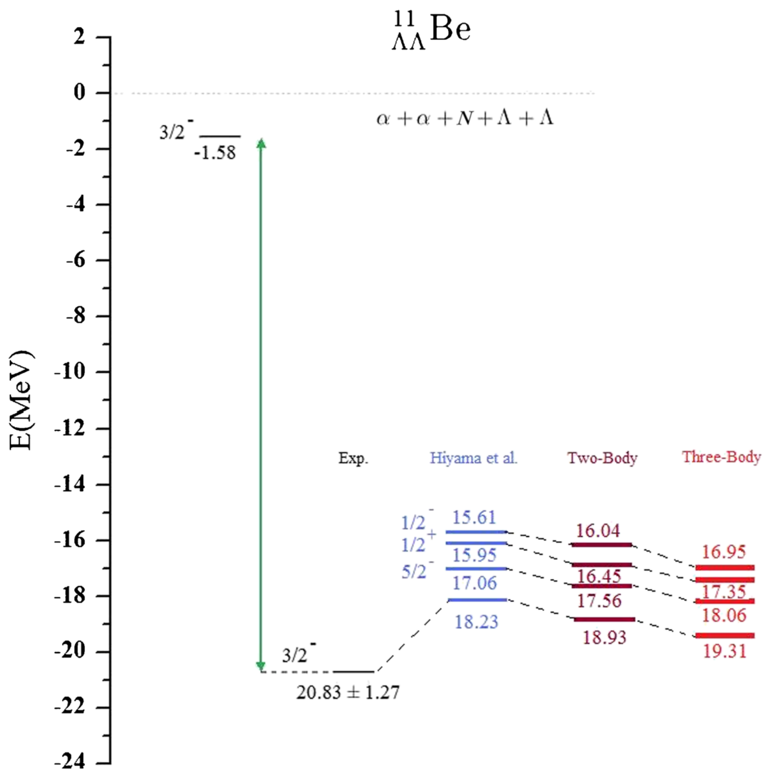

Figure 2. The calculated energy spectra of the low-lying states of $^{11}_{\Lambda\Lambda}$Be together with the results by Hiyama *et al* [5]. Experimental result is taken from [7].

Table 4. Comparison of different $B_{\Lambda\Lambda}$ of ${}^{12}_{\Lambda\Lambda}$Be hypernuclei, calculated using the Skyrme–Hartree–Fock method [9], the shell model calculations [8] and our obtained results by cluster model using only the two-body interaction or both the two- and three-body interactions. The last row shows the experimental data [7].

Type of calculation	MeV	Error
$B^{S^{\Lambda\Lambda 1}}_{\Lambda\Lambda}$ [9]	21.10	5%
$B^{S^{\Lambda\Lambda 2}}_{\Lambda\Lambda}$ [9]	19.97	10%
$B^{S^{\Lambda\Lambda 3}}_{\Lambda\Lambda}$ [9]	17.18	23%
SM [8]	20.71	7%
Present work (two-body)	20.96	6%
Present work (two-body and three-body)	21.45	3%
Experiment	22.23 ± 1.15	[7]

2^+ excited state in ${}^{10}_{\Lambda\Lambda}$Be shows $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\text{Be}(2^+)) = 12.28$ MeV, which agrees with our calculated value. This good agreement suggests that our level structures calculated systematically are predictive and useful for events expected to be found in further analysis of E373 data.

In figure 2, the calculated values of $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\text{Be})$ which is 19.31 MeV, is compared with the calculated result of 18.23 MeV by Hiyama *et al* [5] and the experimental value of 20.83 MeV [7], for the $3/2^-$ ground state. This is a correction of about 7%, much smaller than the results by Hiyama *et al* [5]. We can see the consistency between the experimental data, other theoretical results and our calculated results of ${}^{10}_{\Lambda\Lambda}$Be and ${}^{11}_{\Lambda\Lambda}$Be. We, therefore, discuss the level structures of double-Λ hypernuclei in more detail. As seen in figures 1 and 2 and tables 2–4, the Λ particle behaves glue-like so that the whole system is strongly bound. This effect in a double-Λ hypernucleus is more enhanced than that in the corresponding single-Λ nucleus.

4. Summary and conclusions

We have presented the results of three-body Faddeev-type calculations for systems of three clusters interacting through short-range nuclear as well as the long-range Coulomb interactions. Applications of three-body theory to three-cluster nuclear reactions became possible only in recent years, when a reliable and practical momentum-space treatment of the Coulomb interaction has been developed. For the recent observation of the Hida event for a new double-Λ hypernucleus, we have succeeded in performing a five-body calculation of ${}^{11}_{\Lambda\Lambda}$Be using an $\alpha\alpha N\Lambda\Lambda$ cluster model. The calculated $\Lambda\Lambda$ binding energy is in good agreement with other theoretical methods. More precise data are needed to test our present result, together with the three-body force, quantitatively. Many data for double-Λ hypernuclei are expected to be found in the new emulsion experiment E07 at J-PARC in the near future. Then, our predictions will be tested.

The experimental values of $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\text{Be}) = (11.90 \pm 0.13)$, $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\text{Be}) = (20.49 \pm 1.15)$ and $B_{\Lambda\Lambda}({}^{12}_{\Lambda\Lambda}\text{Be}) = (22.23 \pm 1.15)$ MeV seem to be more compatible with our calculated values of $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\text{Be}) = 14.04$, $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\text{Be}) = 19.31$ and $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}\text{Be}) = 21.45$ MeV in comparison with the other calculated results.

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