PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 83, No. 1
July 2014
pp. 161–164

Nonadiabatic corrections to a quantum dot quantum computer working in adiabatic limit

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MS received 21 March 2013; revised 16 September 2013; accepted 28 January 2014
DOI: 10.1007/s12043-014-0772-z; ***e*Publication:** 2 July 2014

Abstract. The time of operation of an adiabatic quantum computer must be less than the decoherence time, otherwise the computer would be nonoperative. So far, the nonadiabatic corrections to an adiabatic quantum computer are merely theoretical considerations. By the above reason, we consider the particular case of a quantum-dot-confined electron spin qubit working adiabatically in the nanoscale regime (e.g., in the MeV range of energies) and include nonadiabatic corrections in it. If the decoherence times of a quantum dot computer are $\sim$100 ns [J M Kikkawa and D D Awschalom, *Phys. Rev. Lett.* **80**, 4313 (1998)] then the predicted number of one qubit gate (primitive) operations of the Loss–DiVincenzo quantum computer in such an interval of time must be $>10^{10}$. However, if the quantum-dot-confined electron spin qubit is very excited (i.e., the semiclassical limit) the number of operations of such a computer would be approximately the same as that of a classical computer. Our results suggest that for an adiabatic quantum computer to operate successfully within the decoherence times, it is necessary to take into account nonadiabatic corrections.

Keywords. One qubit; all geometrical gate; quantum dots; adiabatic quantum-computing.

PACS Nos **03.67.−a; 03.67.Lx**

The idea of using geometric phases accumulated by an adiabatic time-dependent Hamiltonian to realize quantum gates has been previously considered in [1–3]. The efforts for describing the evolution of a quantum state from the adiabatic quantum computing approach has been considerable. However, so far, the formalism has not been implemented on a particular physical system. The advancements of such a formalism has the status of merely theoretical considerations. It is well known that adiabatic evolution is the basis of the adiabatic quantum computation [4]. On the other hand, it is worth mentioning that there is a competence between the adiabatic scales of time and the scales of time where the quantum computation promises many advantages, such as speeding up for processing the quantum algorithms [1]. That is, in an operative quantum computer, the quantum algorithms must be executed within scales of time smaller than the decoherence time of the quantum computer device. However, it is evident that there is the risk that the adiabatic scales of time could be larger than the decoherence times. In the present paper, such a situation is studied for the particular case of a quantum-dot-confined electron spin qubit commonly known as a quantum dot quantum computer, which was originally proposed by Loss and DiVincenzo [5]. Such a device satisfies the five requirements imposed by DiVincenzo [6] to a quantum computer in order to be operative. In this paper we demonstrate that if nonadiabatic corrections are taken into account in the quantum-dot-confined electron spin qubit operating in the adiabatic limit, then this one works successfully within its decoherence time. In other words, such a quantum computer executes more than 10^{10} one qubit gates in a scale of time less than its decoherence time. Such results compel us to conclude that nonadiabatic corrections to an adiabatic quantum computer are necessary for this one to become an operative device.

In the original model of a quantum dot, the electron is confined to a point with scarce tunnelling. Theoretically, one expects that the electron must be confined to a region of size approximately equal to the Compton's electron wavelength ($\sim 10^{-12}$ m) [7]. However, in practice, the available energies for quantum dots are in the range of MeV [8]. Consequently, from the experimental point of view, the confinement testing reduces to a region of size of the order of nanometres ($L \sim 10^{-9}$ m). In the present work, we shall assume that the electron is contained in the X-axis and that the confinement potential is a harmonic oscillator potential having the following form:

$$V = \frac{1}{2} C x^2, \tag{1}$$

where $C \sim 500$ MeV$/L^2 \sim 5 \times 10^{20}$ MeV/m$^2 \gg 1$ in such a way that the potential is very thin as illustrated in figure 1. The respective wave functions and energies are [9]

$$\phi_n(x) = \frac{1}{2^n n!} \left(\frac{\kappa}{\pi}\right)^{1/4} \mathrm{e}^{-\kappa x^2/2} H_n(\sqrt{\kappa} x), \tag{2}$$

$$H_n(\sqrt{\kappa} x) = (-1)^n \frac{2^n}{\kappa^n} \mathrm{e}^{\kappa^2 x^2} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \mathrm{e}^{-\kappa^2 x^2}, \tag{3}$$

where $\kappa \equiv \sqrt{Cm}/\hbar$ and m is the electron mass. The corresponding eigenenergies will be (by using the value of C we obtain that $\hbar\sqrt{(C/m)} \sim 10$ keV)

$$E_n = \hbar \sqrt{\frac{C}{m}} \left(n + \frac{1}{2}\right). \tag{4}$$

The corresponding instantaneous eigenfunction will be $\psi_n(x,t) = \phi_n(x) \mathrm{e}^{-iE_n t/h}$.

The adiabatic evolution has served as a basis for the adiabatic quantum algorithms [4]. For a quantum computer to be an operative device, clearly the elapsed time must be less than the decoherence time. Consequently, it is necessary to take into account nonadiabatic correction to the formalism. In this way, for a slowly varying Hamiltonian $H(t)$, Shi and Wu [1] have shown that the perturbative expression for a one qubit gate is (such a result follows if the wave function is expanded as $|\Psi(t)\rangle = \sum_n a_n(t) |\psi_n(t)\rangle \exp(i\eta_n t)$ where $\eta_n(t) = (-i/\hbar) \int_0^t E_n(\tau) \mathrm{d}\tau$ is the dynamical phase. Then the Schrödinger equation

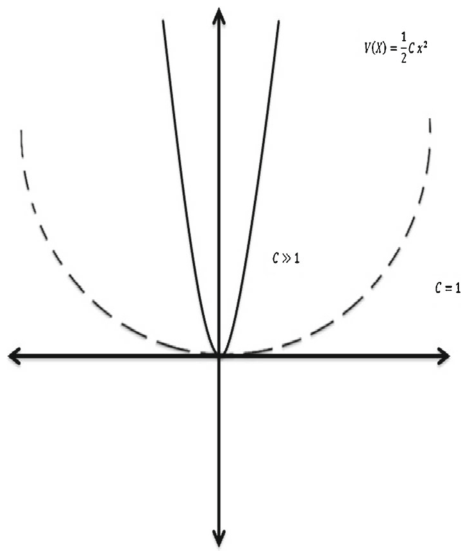

Figure 1. Quantum dot confinement potential. For $C \gg 1$ the confinement potential is very thin.

$i\hbar\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$ leads to $\partial_t a_n(t) = -\sum_m a_m \langle\psi_n|\partial_t\psi_m(t)\rangle \exp[i\eta_m(t) - i\eta_n(t)]$, which, together with the initial condition $a_n(0) \simeq \langle\psi_n(0)|\Psi(0)\rangle$, determines $|\Psi(t)\rangle$. When a nonadiabatic correction is considered, the exact state should be the solution of the differential equation for $a_n(t)$. Such considerations yield the result)

$$U(t)|\psi_n(t=0)\rangle = \exp[i\gamma_n(t) + i\eta_n(t)]\Bigg[|\psi_n(t)\rangle + \hbar\sum_{m\neq n}\frac{|\psi_m(t)\rangle\langle\psi_m(t)|\partial_t\psi_n(t)\rangle}{E_m(t) - E_n(t)} + \cdots\Bigg], \tag{5}$$

where $\eta_n(t) = -i/\hbar\int_0^t E_n(\tau)\mathrm{d}\tau$ is the dynamic phase and $\gamma_n(t) = \int_0^t\langle\psi_n(t)|\partial_\tau\psi_n(\tau)\rangle\mathrm{d}\tau$ is the geometric or Berry phase [10]. For the stationary states of eq. (3) one has $\eta_n(t) = \gamma_n(t) = -iE_n t$.

In order to estimate the time required to execute a one qubit quantum gate, it is necessary to calculate the transition frequency between the initial and final states. This can be accomplished if one knows the matrix element $\langle\psi_a(t=0)|U(t)|\psi_n(t=0)\rangle$. Using eq. (5), one can obtain, within the adiabatic approximation, the time of execution of a one quantum dot gate as

$$\begin{aligned}\mathcal{T}_{n\to a} &= \frac{2\pi}{|\langle\psi_a(t=0)|U(t)|\psi_n(t=0)\rangle|}\\ &\simeq \mathrm{e}^{-E_n t/\hbar}\frac{2\pi\hbar}{E_a} + \cdots\\ &\sim \frac{10^{-17}}{a+1/2}\cdot\mathrm{e}^{-10^{19}(n+1/2)t} + \cdots\quad,\end{aligned} \tag{6}$$

where the orthonormality of the eigenfunction of the harmonic oscillator potential has been used.

We have considered implementing the adiabatic limit in a quantum dot quantum computer. Naturally, there is a competence between the time of operation of the adiabatic quantum computer and its decoherence time. If the time of operation is greater than the decoherence time, the quantum computer is not operative. On the other hand, the nonadiabatic corrections to an adiabatic quantum computer have the status of being merely a theoretical consideration. Therefore, we have considered here the particular case of a quantum-dot-confined electron spin qubit technology in the context of adiabatic limit including nonadiabatic corrections. Our main result is that if nonadiabatic corrections are taken into account in such a computer, this one becomes operative. In other words, by assuming that the quantum-dot-confined electron spin qubit works adiabatically in the nanoscales regime (e.g., in the MeV range of energies) then by considering the nonadiabatic corrections we have proved that the number of one quantum gate operations performed by the quantum dot computer is greater than 10^{10}. Such well-known results suggest that for an adiabatic quantum computer to become operative, it is necessary to take into account nonadiabatic corrections.

Acknowledgement

The author thanks SNI-Conacyt grant.

References

[1] Y Shi and Y-S Wu, *Phys. Rev. A* **69**, 024301 (2004)
[2] J A Jones *et al*, *Nature* (*London*) **403**, 869 (1999)
[3] P Zanardi and M Rasetti, *Phys. Lett. A* **264**, 94 (1999)
[4] E Farhi *et al*, *Science* **292**, 472 (2001)
[5] D Loss and D P DiVincenzo, *Phys. Rev. A* **57**, 120 (1998)
[6] D P DiVincenzo, Mesoscopic electron transport, *NATO Advanced Study Institute, Series E: Applied Sciences* edited by L Sohn, L Kouwenhoven and G Schoen (Kluwer, Dordrecht, 1997) vol. 345
[7] CODATA, NIST (2006)
[8] C B Murray, C R Kagan and M G Bawendi, *Ann. Rev. Mater. Res.* **30**, 545 (2000)
[9] C C Tanoujdi, B Diu and F Laloe, *Quantum mechanics* (Wiley-Interscience, New York, 1992)
[10] M V Berry, *Proc. R. Soc. London Ser. A* **392**, 45 (1984)