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Shadowing corrections to the derivative of the reduced cross-section at small *x*

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Abstract. We analyse the derivative of the reduced cross-section $\frac{\partial\sigma_r^s}{\partial \ln y}|_x$, using the nonlinear Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (NLDGLAP) evolution equation at small x. The small x behaviour of the structure functions are obtained by solving the Gribov, Levin, Ryskin, Mueller and Qiu (GLR-MQ) evolution equation with the nonlinear shadowing term incorporated. We show that the strong rise corresponding to the linear QCD evolution equations, can be tamed by screening effects.

Keywords. Shadowing correction; reduced cross-section; nonlinear Dokshitzer–Gribov–Lipatov–Altarelli–Parisi.

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1. Introduction

Precise measurements of the inclusive scattering cross-section at the *ep* collider HERA are important for understanding proton substructure. In the one-photon exchange approximation, the neutral current double differential cross-section, $\mathrm{d}^2\sigma/\mathrm{d}x\,\mathrm{d}Q^2$, is given by the expression

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2 Y_+}{Q^4 x}\sigma_\mathrm{r}, \tag{1}$$

where the reduced cross-section is defined as

$$\sigma_\mathrm{r} \equiv F_2(x, Q^2) - \frac{y^2}{Y_+}F_\mathrm{L}(x, Q^2) \tag{2}$$

and $Y_+ = 1 + (1 - y)^2$. Here Q^2 is the squared four-momentum transfer, x denotes the Bjorken scaling variable, $y = Q^2/sx$ is the inelasticity, with s the *ep* centre of mass energy squared and α the fine structure constant [1].

The structure functions F_2 and F_L are related to the cross-sections σ_T and σ_L for the interaction of transversely and longitudinally polarized virtual photons with protons [2].

In the quark parton model F_L is predicted to be zero for spin-1/2 partons [3], while in QCD, F_L acquires a nonzero value [4] due to gluon radiation, which is proportional to the strong coupling constant α_s [5,6]. Due to the positivity of the cross-sections for longitudinally and transversely polarized photon's scattering off protons, the two proton structure functions F_2 and F_L obey the relation $0 \leq F_L \leq F_2$ [7].

Thus, the contribution of the longitudinal structure function F_L to the cross-section can be sizeable only at large values of the inelasticity y, and in most of the kinematic range the relation $\sigma_r \approx F_2$ holds to a very good approximation [8].

The reduced cross-section depends on two independent structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$. In the region of moderate x ($x \geq 10^{-2}$), well-established methods of operator expansion and renormalization group equations have been applied successfully. The DGLAP equations [9], which are based on the sum of QCD ladder diagrams, are the evolution equations in this kinematical region. As observed, the longitudinal structure function can be related to the gluon and sea-quark distribution. HERA shows [10–15] that the gluon distribution function has a steep behaviour in this region. At small x, the problem is more complicated as recombination processes between gluons in a dense system have to be taken into account. As the density of gluons and quarks becomes very high and a new dynamical effect is expected to stop further growth of the structure functions, it has to be tamed by screening effects. These screening effects are provided by multiple gluon interactions, which lead to nonlinear terms in the DGLAP equation. These nonlinear terms reduce the growth of the gluon distribution in this kinematic region where α_s is still small but the density of partons becomes very large. Gribov, Levin, Ryskin, Mueller and Qiu (GLR-MQ) [16,17] performed a detailed study of this region. They argued that the physical processes of interaction and recombination of partons become important in the parton cascade at a large value of the parton density, and that these shadowing corrections could be expressed in a new evolution equation (the GLR-MQ equation). The main characteristic of this equation is that it predicts a saturation of the gluon distribution at very small x [18,19]. This equation was based on two processes in a parton cascade:

(i) The emission induced by the QCD vertex $G \to G + G$ with the probability that is proportional to $\alpha_s \rho$, where $\rho (= \frac{xg(x,Q^2)}{\pi R^2})$ is the density of gluon in the transverse plane, πR^2 is the target area, and R is the size of the target that the gluons populate.

(ii) The annihilation of a gluon by the same vertex $G + G \to G$ with the probability that is proportional to $\alpha_s^2 R^2 \rho^2$, where α_s is the probability of the processes.

The value of R depends on how the gluon ladders couple to the proton, or on how the gluons are distributed within the proton. R will be of the order of the proton radius ($R \simeq 5\,\text{GeV}^{-1}$) if the gluons are spread throughout the nucleon, or much smaller ($R \simeq 2\,\text{GeV}^{-1}$) if gluons are concentrated in hot-spot within the proton. As $x \to 0$ the value of the gluon density becomes so large that the annihilation of gluons becomes important. So, this singular behaviour is tamed by the shadowing effects. We assume that this behaviour is similar to the Regge-like behaviour. This phenomenon is usually described by assuming a power-like behaviour of parton distribution functions as $xf_k(x, Q^2) = f_k(Q^2)x^{-\lambda}$, that the singlet parts of the parton distribution functions are controlled by pomeron exchange at low-x, where λ is the pomeron intercept minus one. This behaviour for the gluon distribution at low-x is well known in literatures

[20,21]. Therefore, we extended this behaviour to the shadowing corrections of the gluon distribution function as we have

$$G^{\mathrm{sh}}(x, Q^2) = A_g x^{-\lambda_g^{\mathrm{sh}}}, \tag{3}$$

where at $x > x_0 = 10^{-2}$, shadowing and unshadowing corrections to the gluon distribution behaviours are equal, and λ_g^{sh} is the shadowed pomeron exponent [22]. The structure of this article is as follows. In §2 we present the basic formalism of our approximation method with a brief review of the calculational steps. The connection of the shadowing corrections to the structure functions with hard (Lipatov) pomeron intercept is also given. In §3 we give our analysis of the shadowing corrections to the derivative of the reduced cross-section at small and large y. These results are discussed in §4.

2. Shadowing corrections to DGLAP evolution equations

The shadowing corrections to the DGLAP evolution equations modify the evolution equations by adding a term proportional to $(\alpha(Q^2)/Q^2)\rho$ and quadratic in $G(x, Q^2)$. This picture allows us to write an equation for the parton density in a phase-space cell of volume $\Delta \ln\frac{1}{x}\Delta \ln Q^2$ as

$$\frac{\partial^2 \rho}{\partial \ln \frac{1}{x} \partial \ln Q^2} = \frac{\alpha_{\mathrm{s}} N_{\mathrm{c}}}{\pi}\rho - \frac{\alpha_{\mathrm{s}}^2 \gamma \pi}{Q^2}\rho^2,$$

where N_{c} is the number of colours and γ has been calculated in PQCD [16–19]. Therefore, the GLR-MQ corrections [16,17] which arise from the fusion of two gluon ladders modify the parton evolution equations; as for the gluon distribution we have

$$\frac{\partial G^{\mathrm{sh}}(x, Q^2)}{\partial \ln Q^2} = \left.\frac{\partial G(x, Q^2)}{\partial \ln Q^2}\right|_{\mathrm{DGLAP}} - \frac{81\alpha_{\mathrm{s}}^2}{16R^2Q^2}\int_0^{1-\chi}\frac{\mathrm{d}z}{1-z}\left[G\left(\frac{x}{1-z}, Q^2\right)\right]^2 \tag{4}$$

and for proton structure function

$$\frac{\partial F_2^{\mathrm{sh}}(x, Q^2)}{\partial \ln Q^2} = \left.\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2}\right|_{\mathrm{DGLAP}} - \frac{5}{18}\frac{27\alpha_{\mathrm{s}}^2}{160R^2Q^2}[xg(x, Q^2)]^2, \tag{5}$$

where the first terms are the standard DGLAP equations, linear in the parton distribution functions (PDFs). Here $\chi = x/x_0$, where x_0 is the boundary condition that the gluon distribution joins smoothly onto the unshadowed region. We neglect the quark–gluon emission diagrams as it is of little importance in the gluon-rich low-x region. The nonsinglet contribution is also negligible and can be ignored [22].

To determine the appropriate behaviour we need to specify the shadowing corrections to the parton distributions. With respect to the pomeron behaviour toward the gluon density we solve the GLR-MQ equations with shadowing corrections in differential form given in eqs (4) and (5). Inserting the distribution (3) the equations have the explicit forms [22]

$$\frac{\partial G^{\text{sh}}(x,Q^2)}{\partial \ln Q^2} = \frac{3\alpha_s}{\pi} G^s \frac{1-\chi^{\lambda^{\text{sh}}}}{\lambda^{\text{sh}}} - \frac{81\alpha_s^2}{16R^2Q^2}(G^s)^2 \frac{1-\chi^{2\lambda^{\text{sh}}}}{2\lambda^{\text{sh}}} \tag{6}$$

and

$$\frac{\partial F_2^{\text{sh}}(x,Q^2)}{\partial \ln Q^2} = \frac{5\alpha_s}{9\pi} T(\lambda_g) G^s - \frac{5}{18}\frac{27\alpha_s^2}{160R^2Q^2}(G^s)^2, \tag{7}$$

where $T(\lambda_g) = \int_\chi^1 z^{\lambda_g^s}[z^2+(1-z)^2]\mathrm{d}z$ and G^s is the shadowing gluon distribution function.

In perturbative QCD, the longitudinal structure function can be written as

$$x^{-1}F_L = C_{L,\text{ns}} \otimes q_{\text{ns}} + \langle e^2\rangle (C_{L,q}\otimes q_s + C_{L,g}\otimes g) + x^{-1}F_L^{\text{heavy}}, \tag{8}$$

where q_i and g represent the number distributions of quarks and gluons, respectively, in the fractional hadron momentum. q_s stands for the flavour-singlet quark distribution, $q_s = \sum_{u,d,s}(q+\bar{q})$, and q_{ns} is the corresponding nonsinglet combination. The average squared charge ($=\frac{5}{18}$ for active flavours) is represented by $\langle e^2\rangle$. The symbol $\otimes$ represents the standard Mellin convolution and is given by

$$A(x)\otimes B(x) = \int_0^1 \frac{\mathrm{d}y}{y} A(y) B\left(\frac{x}{y}\right). \tag{9}$$

The perturbative expansion of the coefficient functions can be written [23,24] as

$$C_{L,a}(\alpha_s, x) = \frac{\alpha_s}{4\pi} c_{L,a}(x). \tag{10}$$

Therefore, evolution of the longitudinal structure function at low-x with respect to $\ln Q^2$ is given as

$$\begin{aligned}\frac{\partial F_L^{\text{sh}}(x,Q^2)}{\partial \ln Q^2} = {} & \frac{\mathrm{d}\alpha_s}{\mathrm{d}\ln Q^2}\frac{1}{\pi}\left(\frac{4}{3}\int_x^1 \frac{\mathrm{d}y}{y}\left(\frac{x}{y}\right)^2 F_2^{\text{sh}}(y,Q^2)\right. \\ & \left. +2\sum_q e_q^2 \int_\chi^1 \frac{\mathrm{d}y}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right) G^{\text{sh}}(y,Q^2)\right) \\ & +\frac{\alpha_s(Q^2)}{\pi}\left(\frac{4}{3}\int_\chi^1 \frac{\mathrm{d}y}{y}\left(\frac{x}{y}\right)^2 \frac{\partial F_2^{\text{sh}}(y,Q^2)}{\partial \ln Q^2}\right. \\ & \left. +2\sum_q e_q^2 \int_\chi^1 \frac{\mathrm{d}y}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right)\frac{\partial G^{\text{sh}}(y,Q^2)}{\partial \ln Q^2}\right),\end{aligned} \tag{11}$$

where the shadowing corrections to the derivative of the parton structure functions are given by eqs (6) and (7). After substitution and rearranging [22] we find that the shadowing correction to the longitudinal structure function is proportional to the shadowing corrections to the gluon distribution function as

$$F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2) = \alpha_{\mathrm{s}} \int \frac{F_{0\mathrm{L}}^{\mathrm{sh}}(x, Q^2)}{\alpha_{\mathrm{s}}} \mathrm{d} \ln Q^2, \tag{12}$$

where

$$\begin{aligned} F_{0\mathrm{L}}^{\mathrm{sh}}(x, Q^2) = & \frac{\alpha_{\mathrm{s}}(Q^2)}{\pi} \frac{4}{3} \int_{\chi}^{1} \frac{\mathrm{d}y}{y} \left(\frac{x}{y}\right)^2 [\text{eq. (5)}] \\ & + \frac{\alpha_{\mathrm{s}}(Q^2)}{\pi} \frac{20}{9} \int_{\chi}^{1} \frac{\mathrm{d}y}{y} \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) [\text{eq. (4)}]. \end{aligned} \tag{13}$$

3. Shadowing corrections to the derivative of the reduced cross-section

To determine the shadowing corrections to the reduced cross-section we imply the shadowing behaviour of the structure functions to eq. (2). With respect to these nonlinear behaviours, the reduced cross-section behaviour can be tamed. In principle, we should replace eq. (2) with the nonlinear shadowing terms by

$$\sigma_{\mathrm{r}}^{\mathrm{sh}} = F_2^{\mathrm{sh}}(x, Q^2) - \frac{y^2}{Y_+} F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2). \tag{14}$$

The quantity $\sigma_{\mathrm{r}}^{\mathrm{sh}}$ has been independently measured as a function of both x and Q^2. Hence, the slope of $\sigma_{\mathrm{r}}^{\mathrm{sh}}$ with respect to x or Q^2, with the other variable kept fixed, can be determined. In order to estimate its derivative, we take the derivatives of eq. (14) with respect to $\ln\, y$ for each value of constant x (i.e., $(\mathrm{d}\sigma_{\mathrm{r}}/\mathrm{d} \ln y)|_x$). So we obtain the following result:

$$\left.\frac{\partial \sigma_{\mathrm{r}}^{\mathrm{sh}}}{\partial \ln y}\right|_x = \frac{\partial F_2^{\mathrm{sh}}(x, Q^2)}{\partial \ln Q^2} - f_1 F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2) - f_2 \frac{\partial F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2)}{\partial \ln Q^2}, \tag{15}$$

where the factors $f_2 = y^2/Y_+$ and $f_1 = 2y^2(2 - y)/Y_+^2$ are significant only for large y. H1 analysis shows that σ_{r} is a fairly linear function of $\ln\, y$ at all y. So the derivative of the reduced cross-section can be obtained from straight line fits to the σ_{r} data. Here we want to show this behaviour with respect to the shadowing corrections to the gluon distribution at low-x. Therefore, combine the terms and define the shadowing corrections to the derivatives of the reduced cross-section as

$$\begin{aligned} \left.\frac{\partial \sigma_{\mathrm{r}}^{\mathrm{sh}}}{\partial \ln y}\right|_x = & \frac{\partial F_2^{\mathrm{sh}}(x, Q^2)}{\partial \ln Q^2} [\text{eq. (5)}] - \left[f_1 - \frac{f_2}{\ln Q^2} \right] \\ & \times F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2) [\text{eq. (12)}] - f_2 F_{0\mathrm{L}}^{\mathrm{sh}}(x, Q^2) [\text{eq. (13)}]. \end{aligned} \tag{16}$$

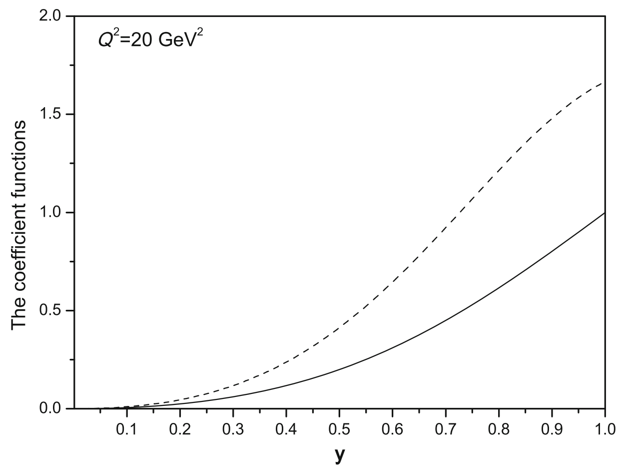

Figure 1. The behaviour of the coefficient functions $f_1 - \frac{f_2}{\ln Q^2}$ (dash line) and f_2 (solid line) as a function of y at small and large y.

The y-derivative of $\sigma_{\mathrm{r}}^{\mathrm{sh}}$ (at constant x) is insensitive to $F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2)$ and $F_{0\mathrm{L}}^{\mathrm{sh}}(x, Q^2)$ when y is small. Therefore, the behaviour of $\frac{\partial \sigma_{\mathrm{r}}^{\mathrm{sh}}}{\partial \ln y}|_x$ at low-y values directly constrains the behaviour of $\frac{\partial F_2^{\mathrm{sh}}(x,Q^2)}{\partial \ln Q^2}$ and this behaviour is smaller than the behaviour $\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}|_{\mathrm{DGLAP}}$ (where the linear DGLAP equation in the PDFs is dominant) with respect to the shadowing corrections.

In the large-y region, $F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2)$ and $F_{0\mathrm{L}}^{\mathrm{sh}}(x, Q^2)$ can no longer be neglected. In particular, $f_1 \sim 1.4$ when $y = 0.8$; furthermore, $f_1 \sim 2 f_2$ over the entire y range. Hence the contribution $\frac{\partial F_2^{\mathrm{sh}}(x,Q^2)}{\partial \ln Q^2}$ is sensitive to $f_2[(2 - \frac{1}{\ln Q^2}) F_{\mathrm{L}}^{\mathrm{sh}}(x, Q^2) - F_{0\mathrm{L}}^{\mathrm{sh}}(x, Q^2)]$ at large-y. We observe this behaviour in figure 1.

Therefore, the derivatives of the reduced cross-section $\frac{\partial \sigma_{\mathrm{r}}^{\mathrm{sh}}}{\partial \ln y}|_x$ is extracted as a function of x in the range 0.0005−0.0008. These values are listed in table 1 for $Q^2 = 20$ GeV^2 at the hot-spot point $R = 2$ GeV^{-1} and compared to the values measured (derivatives of the structure function) by the H1 Collaboration [12]. The same results are shown in table 2 at $R = 5$ GeV^{-1}. We observed that shadowing corrections are dependent on the size of the target that the gluons populate in that region. At hot-spot point, we show that the derivatives of the structure function and reduced cross-section are tamed as x decreases.

Table 1. The values of the shadowing derivative of the reduced cross-section at hot-spot point $R = 2$ GeV^{-1} by solving the GLR-MQ evolution equation that is compared with H1 Collab. data [12] on the derivative of the structure function.

Q^2 (GeV^2)	x	y	$\frac{\partial F_2}{\partial \ln Q^2}\|_x$ [12]	λ_g^{s}	G^{s}	$F_{\mathrm{L}}^{\mathrm{s}}$	$\frac{\partial F_2^{\mathrm{s}}}{\partial \ln Q^2}\|_x$	$\frac{\partial \sigma_{\mathrm{r}}^{\mathrm{s}}}{\partial \ln Q^2}\|_x$
20	0.0008	0.277	0.286 ± 0.062	0.375	13.867	0.100	0.265	0.256
20	0.0005	0.443	0.348 ± 0.090	0.347	15.947	0.144	0.316	0.272

Table 2. The same as in table 1 at $R = 5$ GeV^{-1}.

Q^2 (GeV^2)	x	y	$\frac{\partial F_2}{\partial \ln Q^2}\|_x$ [12]	λ_g^s	G^s	F_L^s	$\frac{\partial F_2^s}{\partial \ln Q^2}\|_x$	$\frac{\partial \sigma_r^s}{\partial \ln Q^2}\|_x$
20	0.0008	0.277	0.286 ± 0.062	0.437	28.905	0.152	0.542	0.528
20	0.0005	0.443	0.348 ± 0.090	0.421	38.728	0.249	0.744	0.668

Consequently, this shadowing correction suppresses the rate of growth in comparison with the standard DGLAP result at hot-spot point.

4. Conclusion

Therefore, we expect the derivative of the reduced cross-section to be sensitive to the shadowing corrections in the HERA kinematic region with respect to the size of the target that the gluons populate. Our data show that shadowing derivative of the reduced cross-section are tamed with respect to nonlinear terms at GLR-MQ equation at hot-spot point. We estimated the shadowing corrections to the derivative of the reduced cross-section into shadowing correction to the longitudinal and singlet distribution functions with respect to GLR-MQ equations at low and high inelasticities. The behaviour of this observable is directly dependent on the behaviour of the longitudinal and singlet distribution functions and, therefore, strongly sensitive to the shadowing corrections. Results obtained show that the shadowing derivative of the reduced cross-section at small-x is strongly modified at hot-spot point by shadowing corrections as this growth is tamed by the shadowing effects.

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