PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 82, No. 5
May 2014
pp. 931–941

Evidences for magicity in superdeformed shapes

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DOI: 10.1007/s12043-014-0745-2; ***e*Publication:** 30 April 2014

Abstract. Many empirical evidences that point to the existence of preferred magic nucleon numbers for superdeformed (SD) shapes are presented in this paper. We use a simple premise based on the 4-parameter formula fitted using observed γ-rays of SD bands. In particular, plots of γ-ray energy ratios, nuclear softness parameter values and the number of SD bands for given N and Z are used to pinpoint the magicity (N, Z numbers) that are most favoured as the SD magic numbers. This analysis also leads to several new predictions on the occurrence of SD bands specially in neutron-rich nuclei.

Keywords. Superdeformed bands; 4-parameter formula; nuclear softness parameter; $R(I)$ ratio.

PACS Nos **26.10.−n; 21.10.Re; 21.10.Pc; 27.70.+q**

1. Introduction

The phenomenon of superdeformation represents one of the most remarkable discoveries in nuclear physics during the last decade of the 20th century. A large number of superdeformed (SD) bands have now been observed in the mass regions $A \approx 30$, 60, 80, 130, 150, 165 and 190. The cascades of the SD bands, which are often unusually long, are connected by electric quadrupole ($E2$) transitions. However, the spin assignments of most of these bands carry an uncertainty of 1–2$\hbar$ [1]. This is because of the near absence of information on linking transitions to levels of normal deformation (ND) except in a few cases. One of the main use of phenomenological expressions to fit the SD bands has so far been to make a calculated guess of angular momentum to the observed levels in an SD band. Some of the relations used in the literature are, the Harris expansion, three-parameter model, Lipas–Ejiri relation and its extension, $SU_q(2)$ model and VMI-inspired IBM [2–7]. Most of these models/formulae have been used to fit only some of the SD bands in the mass of 150 and 190 regions. In ref. [8], we use a simple 4-parameter formula based on the prescription of Bohr and Mottelson to discuss the band moment of inertia ($\Im_0$) and nuclear softness parameter (σ) for the SD bands in all the mass regions covering the complete Segre chart.

We also look at some possible empirical signatures of SD magic numbers [8]. Many types of empirical evidences support the existence of the spherical magic numbers (2, 8, 20, 28, etc.), which have played instrumental roles in the development of nuclear physics [9]. The idea of magic numbers has now been generalized to non-spherical shapes and the relationship of shapes and shell structure has played a vital role in the development of nuclear structure physics. It is now well known that shell closure at 2:1 shape leads to a good understanding of the observed SD bands and fission isomers. We present such evidences in ref. [8] and identify the SD magic numbers. Observation of hundreds of SD bands in the mass regions predicted by shell correction-based models testify to the success of the concept. It is however, remarkable that no direct empirical evidence has been presented so far supporting the existence of SD magic numbers and what these numbers are. We present such evidences in §3 and identify the SD magic numbers.

2. Softness parameter and moment of inertia for SD bands

Bohr and Mottelson [9,10] pointed out that the rotational energy of a $K=0$ band in an even–even nucleus can be expanded in a power series of $I(I+1)$:

$$E(I) = A(I(I+1))^1 + B(I(I+1))^2 + C(I(I+1))^3 + D(I(I+1))^4. \quad (1)$$

The expansion for a $K\neq 0$ band can take a form similar to eq. (1), but includes a term for the band-head energy, and $I(I+1)$ has to be replaced by $I(I+1)-K^2$. It has now been well established that the extensive data on ND bands can be described rather well by eq. (1) except in the band crossing region where phenomena like back-bending occur. The energy may also be written in a different form as [11]

$$E(I) = 1/2\Im_0[(I(I+1)-1/2\sigma(I(I+1))^2+\sigma^2(I(I+1))^3-3\sigma^3(I(I+1))^4], \quad (2)$$

where the softness parameter $\sigma = 1/2S(\Im_0)^3$ [12] is a small parameter of expansion with S as the stiffness constant and $\Im_0$ is the ground-state moment of inertia. On rearranging eq. (1), we obtain

$$E(I) = A[(I(I+1) + B/A(I(I+1))^2 + C/A(I(I+1))^3 + D/A(I(I+1))^4]. \quad (3)$$

A comparison of eqs (2) and (3) suggests that

$$A = \frac{1}{2\Im_0}, \qquad \frac{B}{A} = -\frac{\sigma}{2}, \qquad \frac{C}{A} = \sigma^2, \qquad \frac{D}{A} = -3\sigma^3.$$

2.1 *The 4-parameter formula for SD bands*

This formula is equivalent to expression (1). For SD nuclei, where $K\neq 0$, eq. (1) can be re-written as

$$E(I) = E_0 + A((I(I+1) - (I_0(I_0+1)) + B((I(I+1))^2 - (I_0(I_0+1))^2) + C((I(I+1))^3 - (I_0(I_0+1))^3) + D((I(I+1))^4 - (I_0(I_0+1))^4), \quad (4)$$

where E_0 is band-head energy and I_0 is the band-head spin. For SD bands, the band-head energies are generally not known; only the stretched $E2$ transitions are known. Thus, instead of using the band-head energy, we choose to fit the $E2$ transitions.

$$E_\gamma(I) = E(I) - E(I-2). \tag{5}$$

Using eq. (5), we obtain

$$\begin{aligned} E_\gamma(I) = {} & A((I(I+1) - (I-2)(I-1)) + B((I(I+1))^2 - ((I-2)(I-1))^2) \\ & + C((I(I+1))^3 - ((I-2)(I-1))^3) \\ & + D((I(I+1))^4 - ((I-2)(I-1))^4). \end{aligned} \tag{6}$$

The parameters A, B, C and D can be determined by fitting the $E2$ transitions for SD cascades. One may then obtain the band moment of inertia ($\Im_0$) and the softness parameter (σ) [13].

2.2 *Discussion and results*

The 4-parameter formula has been used for the first time to fit the γ transition energies of the SD bands. We have considered only those SD bands for which estimates of spin assignments are available in the table of SD bands [1]. A total of 220 SD bands were fitted from all the mass regions; this includes the six cases which were fitted by excluding the backbending region and the six cases including the backbending region. Excellent fits were obtained in nearly all the cases. There are 57 cases where the coefficient B is positive, which goes against the suggestions of Bohr and Mottelson [9]. There are at least 21 cases where the spin of the band-head had to be reduced by $2\hbar$ units or exclude backbending region to obtain the best fit/correct sign of the coefficient B. The fitting provides us the parameters A, B, C and D, which in turn gives us the band moment of inertia ($\Im_0$) and the nuclear softness parameter (σ).

We have summarized in [13,14], the expressions for the moment of inertia ($\Im_0$) and the nuclear softness parameter (σ) from the various models discussed above. By using these expressions, it is possible to compare the numerical values for σ and $\Im_0$ from different models with those from the 4-parameter approach. We note that the values obtained for $\Im_0$ and σ for the same SD band are very much similar although seemingly different expressions were used in the fitting. This gives us considerable confidence in the simple 4-parameter approach used.

In brief, 220 SD bands have been found to provide excellent fits. The band moment of inertia ($\Im_0$) and the softness parameter (σ) values have been obtained for all the 220 bands. The band moment of inertia scaled with rigid sphere value [15] is shown in figure 1. The nuclear softness parameters for all the SD nuclei are shown in figure 2. The nuclear softness parameters (σ) for SD bands lie in the range $10^{-4} \leq \sigma \leq 10^{-6}$ compared to ND bands [16] having a range of $10^{-2} \leq \sigma \leq 10^{-4}$. The nuclear softness parameters is related to the rigidity of the nucleus. Thus, the SD bands are much more rigid than the ND bands. Also, as the deformation of the SD nuclei increases, there is a distinct rise in the rigidity. This suggests a clear correlation between the pairing correlations and the deformation of SD bands.

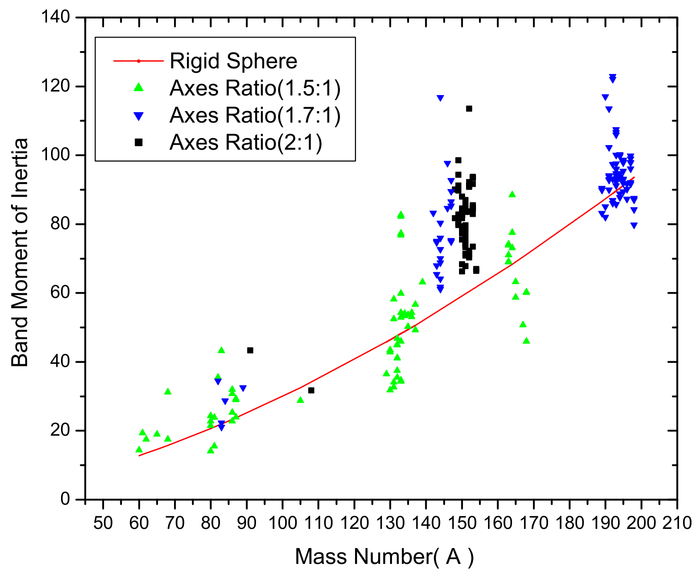

Figure 1. Band moment of inertia vs. mass number for all the SD bands.

3. Superdeformed magic numbers: Empirical evidence

The premise of our evidence is as follows. SD bands display a near rigid rotor behaviour. Since the magic numbers correspond to the positions of least level density, we expect the SD bands near magic nucleon numbers to be more close to exact rigid rotor. The usual

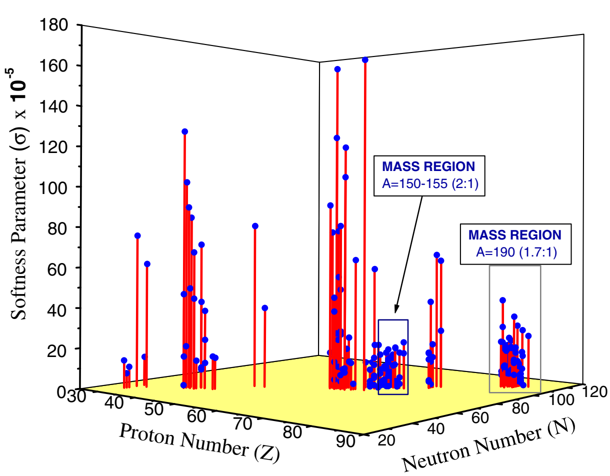

Figure 2. Softness parameter vs. proton number vs. neutron number for all the SD bands.

parameter to look for the extent of rigidity is the $(E4/E2)$ energy ratio. As the SD bands are high-spin bands and band-heads are rarely known, instead of using the energy ratio, we use γ-ray energy ratios. The values for γ energy are taken from the latest collection of SD bands in nuclear data sheet [1]. We have used four types of approaches to identify the SD magic numbers.

3.1 *γ-ray energy ratios*

We calculate typical ratios like $E_\gamma(53/2 \rightarrow 49/2)/E_\gamma(49/2 \rightarrow 45/2)$ and plot them for a set of bands in a given mass region where these spins are known. This ratio must approach the rigid rotor value more closely at the SD magic numbers. This approach of identifying the SD magic numbers is very simple yet very robust.

3.2 *$R(I)$ ratios*

This idea may be generalized to define a ratio $R(I)$ calculated for the whole SD bands [1].

$$R(I) = \frac{E_\gamma(I) - E_\gamma(I-2)}{E_\gamma(I-2) - E_\gamma(I-4)}. \tag{7}$$

Such ratios are then compared with the rigid rotor values for the same spins. Only some of the SD bands are found to be nearly identical to the rigid rotors.

3.3 *Softness parameter*

We also use the softness parameter values obtained by using the 4-parameter model [13] for SD bands in different mass regions. The softness parameter is least for those SD bands whose neutron and proton numbers are magic for the SD shapes.

3.4 *Number of SD bands at magic numbers*

It is well known that nuclei with spherical magic numbers of neutrons and protons exhibit maximum number of isotones/isotopes. We use the same approach by plotting the number of bands vs. neutron/proton number. The number of SD bands corresponding to the magic neutron or proton number is more than that for its neighbours.

3.5 *Discussion and results*

We show in figures 3 and 4, some typical graphs of γ-ray energy ratios for the deformations (1.5 : 1) from mass region 57–174. Only a few nuclei come really close or lie really onto the line defining the rigid rotor values. Figure 3, plotted for the mass regions 57–174 and deformation 1.5 : 1, suggests that the neutron numbers 46, 58–59, 73–75 and 92 are almost on the rigid rotor line. Similar plot for proton numbers, shown in figure 4, suggests that the proton numbers 40–41, 46, 58–59 and 71 (for deformation 1.5 : 1) are near the rigid rotor value. To compare the $R(I)$ values with the rigid rotor behaviour, we use the rigid rotor expression from [15]. Figure 5 shows the ratio $R(I)$ for Ce_{73} along

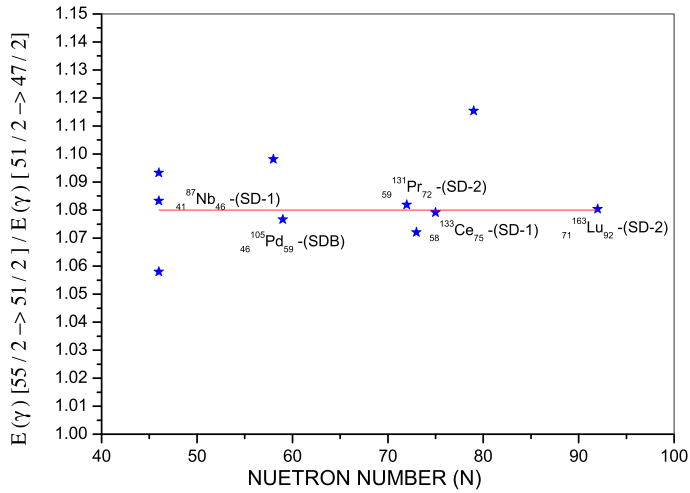

Figure 3. γ-ray energy ratio vs. neutron numbers for deformation (1.5 : 1). The line represents the rigid rotor value for the ratio on Y-axis. Nuclei approaching or lying on the rigid rotor line are labelled.

with the rigid rotor values as seen through the whole band; the two match very closely. Similarly, in figure 6, the behaviour of Lu isotope (Z=71) with neutron number 92 is very close to the rigid rotor behaviour for all angular momentum values compared to neutron numbers 94 and 96. We may point out that this comparison was carried out for all the

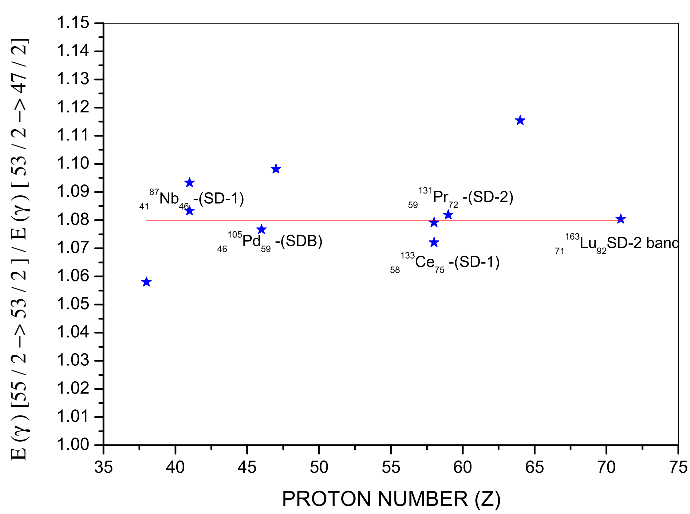

Figure 4. γ-ray energy ratio vs. proton numbers with deformation (1.5 : 1). The line represents the rigid rotor value for the ratio on Y-axis. Nuclei approaching or lying on the rigid rotor line are labelled.

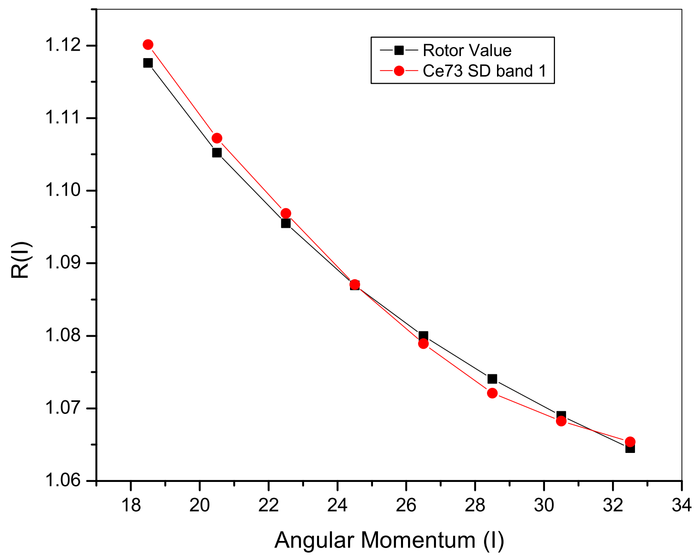

Figure 5. $R(I)$ vs. the angular momentum (I). The rigid rotor behaviour of the SD band in Ce_{73} is evident from the figure.

SD bands. The Gd_{80} emerges as the best magic nucleus from these studies [17]. In the softness parameter approach described in [13], we try to look for the minima in softness parameter values (figures 7 and 8); we get minima at neutron numbers 32, 44, 46, 60,

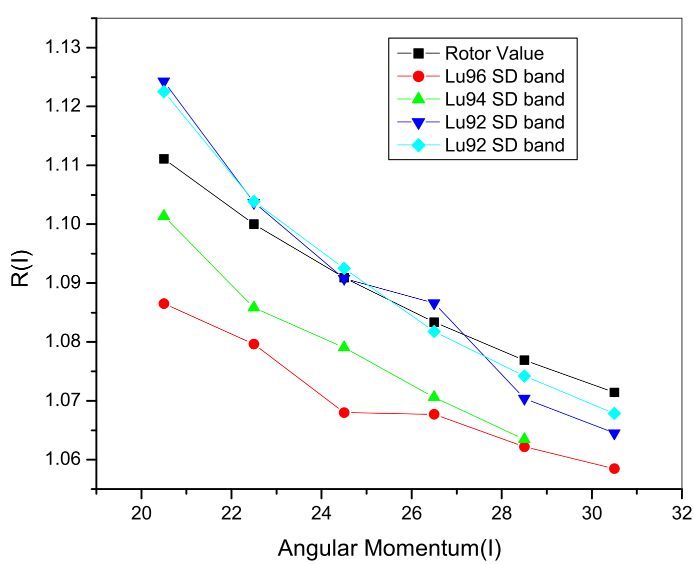

Figure 6. $R(I)$ vs. the angular momentum (I). The behaviour of Lu isotope (Z=71) with neutron number 92 is very close to the rigid rotor behaviour for all angular momentum values.

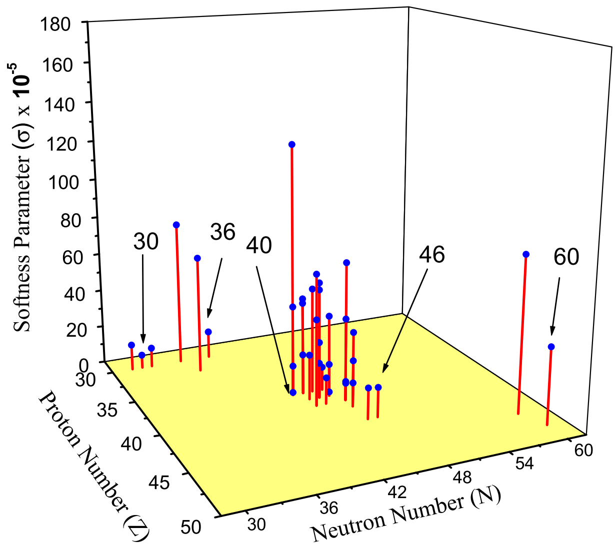

Figure 7. Softness parameter vs. proton number vs. neutron number for the SD bands.

73–74, 80, 82, 84, 86, 92 and 114–115. Finally, figures 9 and 10 point out that the neutron numbers 30, 35, 44, 46, 60, 74, 80, 86, 93, 96, 102, 113–115 and the proton numbers 30, 38, 40, 58, 60, 64, 71 and 82 are most likely to be the magic numbers having more number of SD bands. These numbers match with the values obtained above.

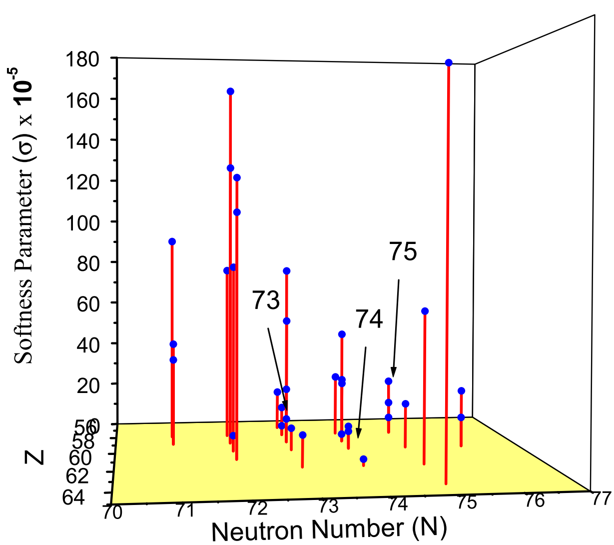

Figure 8. Softness parameter vs. proton number vs. neutron number for the SD bands.

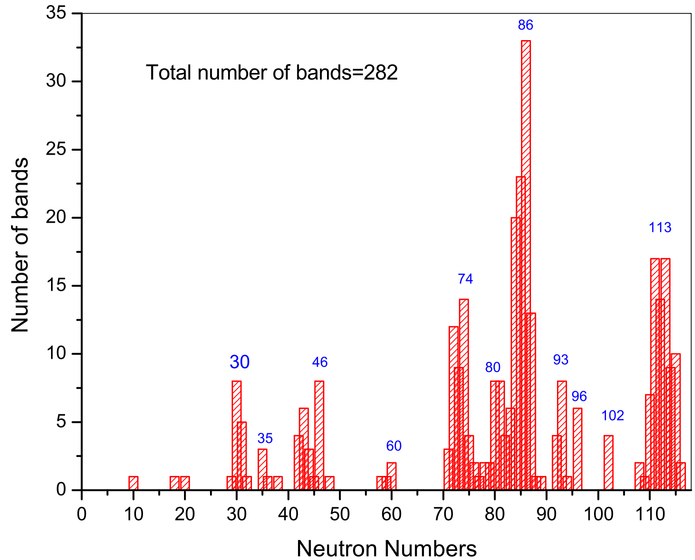

Figure 9. Number of SD bands vs. neutron number for all the SD bands.

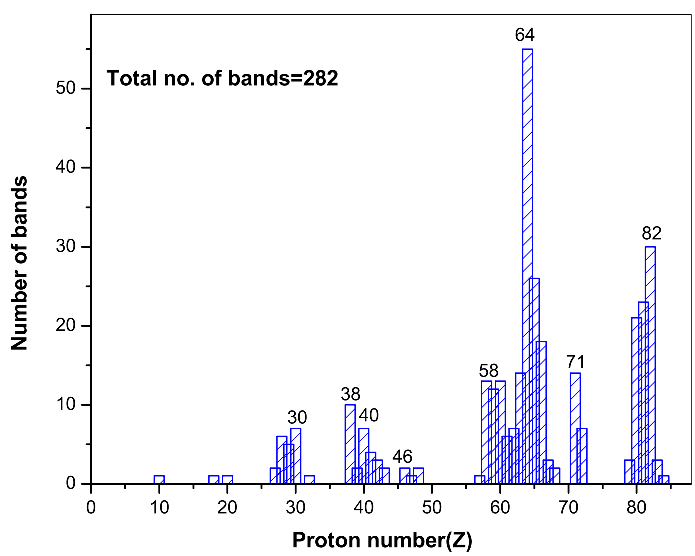

Figure 10. Number of SD bands observed vs. proton number for all the SD bands.

4. Conclusions

The 4-parameter formula was found to provide excellent fits to SD bands. In brief, 220 SD bands were fit and the band moment of inertia ($\Im_0$) and the softness parameter (σ) values were obtained for all the 220 bands. The nuclear softness parameters for SD bands lie in the range $10^{-4} \leq \sigma \leq 10^{-6}$ as compared to ND bands [12] having a range of

Table 1. Comparison of the empirical SD magic numbers with spherical (1 : 1) shape and SD (2 : 1) shape magic numbers. Theoretical spherical and SD (2 : 1) magic numbers are obtained by isotropic harmonic oscillator potential with and without $\vec{l}\cdot\vec{s}$ term (in brackets). The empirical results from this work provided the magic numbers for SD shapes, and the resulted magic numbers are given in columns 3, 4 and 5, respectively.

Deformation($\omega_z : \omega_x$)							
1 : 1 (Theory)	2 : 1 (Theory)	1.5 : 1 (Empirical)		1.7 : 1 (Empirical)		2 : 1 (Empirical)	
		Z	(N)	Z	(N)	Z	(N)
2	4						
8	10						
	16						
20(28)	28	30	(32,36)				
40(50)	40(40–46)	40	(46)				
	60(60–68)	58–59,62	(58–59)	64		66	
70(82)	80(80–90)	71	(73–75)	80,82	(80)		(86)
112(126)	110(110–118)		(92)		(113–115)		

$10^{-2} \leq \sigma \leq 10^{-4}$. The nuclear softness parameter is related to the rigidity of the nucleus. Thus, the SD bands are much more rigid than the ND bands. Also, as the deformation in SD nuclei increases, the SD bands become more and more rigid. This suggests a clear correlation between pairing correlations and deformation.

On the basis of these evidences, we identify $Z = 30$, 40, 46, 58–59, 62, 71 and $N = 32$, 35, 46, 58–59, 73–75, 92 as the SD magic numbers in the mass region 57–174 with deformation 1.5 : 1 and $Z = 64$, 80–82 and $N = 80$, 113–115 in the mass region 144–198 with deformation 1.7 : 1 and $Z = 66$, $N = 86$ in the mass region 150 with deformation 2 : 1 as the most likely SD magic numbers. These magic numbers are listed in table 1 along with the magic numbers for normal nuclei 1 : 1 and deformed nuclei with deformation 2 : 1. While most of the magic numbers are the same, we noticed significant changes in several predicted magic numbers and those obtained from our analysis. We predicted that many more SD bands might be observed in the mass region $A = 102$–112 corresponding to $Z = 44$–48 and $N = 58$–64. We also predicted that many SD bands could exist for the combination of (N, Z) values $Z = 58$ and $N = 80$. Similarly, one can search the SD bands in neutron-rich nuclei around $Z = 64$, $N = 96$ and $Z = 71$ and $N = 112$ (towards the neutron-rich side). Any nuclear model for the SD nuclei should, therefore, be adjusted so as to reproduce these magic numbers.

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