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Transverse momentum spectra of the produced hadrons at SPS energy and a random walk model

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Abstract. The transverse momentum spectra of the produced hadrons have been compared to a model, which is based on the assumption that a nucleus–nucleus collision is a superposition of isotropically decaying thermal sources at a given freeze-out temperature. The freeze-out temperature in nucleus–nucleus collisions is fixed from the inverse slope of the transverse momentum spectra of hadrons in nucleon–nucleon collision. The successive collisions in the nuclear reaction lead to gain in transverse momentum, as the nucleons propagate in the nucleus following a random walk pattern. The average transverse rapidity shift per collision is determined from the nucleon–nucleus collision data. Using this information, we obtain parameter-free result for the transverse momentum distribution of produced hadrons in nucleus–nucleus collisions. It is observed that such a model is able to explain the transverse mass spectra of the produced pions at SPS energies. However, it fails to satisfactorily explain the transverse mass spectra of kaons and protons. This indicates the presence of collective effect which cannot be accounted for, by the initial state collision broadening of transverse momentum of produced hadrons, the basis of random walk model.

Keywords. Relativistic heavy-ion collisions; transverse momentum distribution.

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1. Introduction

In high-energy heavy-ion collisions it is observed that the average transverse momenta of the produced hadrons depend strongly on the mass of hadrons [1,2]. Further, it is found to be substantially larger than in nucleon–nucleon collisions at a given energy. One of the possible physical effects responsible for this is the transverse flow [3]. The heavier the mass of the particle, more is the gain in momentum and hence, more transverse is the flow. This has been shown by extracting the effective temperature (effective because it includes the true freeze-out temperature and the transverse flow) from transverse momentum (p_{T}) distribution of various hadrons. It is observed that protons have larger effective temperature than kaons, which in turn has larger effective temperature than pions. The success of hydrodynamic models in explaining the transverse momentum distributions of various hadrons at least up to lower momentum, is an indication of the presence of radial flow [4]. Figure 1 depicts this effect for *AA* collisions at SPS energies. However, similar effect is also observed in nucleon–nucleus (*pA*) collisions (also shown in figure 1). Hence, broadening of p_T even in *pA* collisions, where one does not expect any collective effect like radial flow, indicates that there exists a 'normal' p_T broadening observed in all heavy-ion reactions. This needs to be understood properly before making any quantitative conclusions regarding the radial flow. Such broadening of p_T is thought to arise due to successive collisions in nuclear reactions. It is basically an initial state collision broadening of p_T. This effect at high p_T (typically above 2 GeV) is referred to as Cronin effect [5]. Apart from the commonly produced hadrons, the Drell–Yan dileptons and quarkonium resonances also show in *pA* collisions a broadening of p_T compared to that observed in *pp* collisions. The virtual photons in Drell–Yan production do not interact in the nuclear medium and the quarkonia leave the medium before the collective effects develop. This makes the case for the p_T broadening as an initial state effect stronger [6,7].

In [6,8], it has been shown that such an effect can explain the transverse mass (m_T) distribution of the produced hadrons at SPS energies, thereby, questioning the presence of true collective effects (such as radial flow) in relativistic heavy-ion collisions. The aim of the present work is to have a detailed study of the effect of p_T gain through successive collisions in nuclear reactions within the framework of a random walk model [6,8] (discussed in §2). In this paper, we systematically analyse the available *pp*, *pA* and *AA* data and come to the conclusion that collective effects do exist in relativistic heavy-ion collisions. We have chosen to concentrate on SPS energies, although data are currently

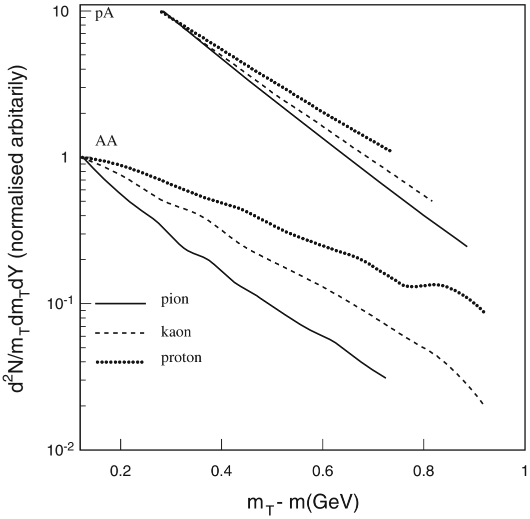

Figure 1. Experimental transverse mass spectra of produced pions, kaons and protons for nucleon–nucleus (*pA*) and nucleus–nucleus (*AA*) collisions showing the effect p_T broadening with mass of hadrons. The spectra have been arbitrarily normalized to 10 and 1 at the lowest momentum bin for *pA* and *AA* collisions, respectively. The *pA* data correspond to AGS energy while the *AA* data to SPS energy.

available at RHIC and LHC energies. These will be addressed in a similar approach in the near future.

This paper is organized as follows: in §2 we discuss the random walk model. In §3 we fix the freeze-out temperature for *AA* collisions from the inverse slope of the p_{T} spectra from the *pp* collisions. Section 4 deals with the estimation of gain in the average transverse momentum per collision from *pA* collision data. We compare the results of the random walk model calculation with the SPS *AA* collision data in §5. In §6 we give a discussion of the results. Finally, a summary of the work is given in §7.

2. Random walk model

Consider the nucleus–nucleus collision to be a superposition of nucleon–nucleon collision. In this picture, let us assume that in each successive interaction of a nuclear collision, one creates a fireball just like that formed in a nucleon–nucleon collision. If a nucleon starts with zero p_{T}, after the first collision the next one will generally occur at some non-vanishing transverse velocity. Thus, there is a gain in transverse momentum through successive collisions. The propagation of nucleon through successive collisions is assumed to follow a random walk pattern. It is interesting to see if such a model can explain the p_{T} spectra of the produced hadrons in *AA* collisions and more specifically, if it can account for the observed p_{T} broadening with increase in mass of the hadrons. The random walk pattern is modelled by a Gaussian [6] in the present calculation as

$$f_{pA}(\rho) = \left[\frac{4}{\pi\delta^2_{pA}}\right]^{1/2} \exp\left(-\frac{\rho^2}{\delta^2_{pA}}\right), \tag{1}$$

where ρ is the transverse rapidity and

$$\delta^2_{pA} = (N_A - 1)\delta^2, \tag{2}$$

denotes the kick per collision δ as determined from *pA* interactions. N_A is the number of nucleons which the incident proton encounters on its path through the target nucleus. It is given by [6]

$$N_A \sim (3/4)(2\pi r_0^2 R_A) n_0, \tag{3}$$

where $r_0 \sim 0.8$ fm is the nuclear radius, R_A ($= 1.12A^{1/3}$ fm) is the radius of the nucleus and $n_0 = 0.17\ \mathrm{fm}^3$ is the nuclear density.

The corresponding distribution for an AB collision can be shown to be of the form

$$f_{AB}(\rho) = \left[\frac{4}{\pi\delta^2_{AB}}\right]^{1/2} \exp\left(-\frac{\rho^2}{\delta^2_{AB}}\right), \tag{4}$$

with

$$\delta^2_{AB} = (N_A + N_B - 2)\delta^2. \tag{5}$$

The final expression for the transverse mass distribution is given as [8]

$$\left(\frac{\mathrm{d}N}{\mathrm{d}y m_{\mathrm{T}} \mathrm{d}m_{\mathrm{T}}}\right)_{y=0} = \frac{gV}{2\pi^2}\left[\frac{4}{\pi\delta^2_{AB}}\right]^{1/2} \int \mathrm{d}\rho \exp\left(-\frac{\rho^2}{\delta^2_{AB}}\right) m_{\mathrm{T}} I_0 \times \left(\frac{p_{\mathrm{T}} \sinh\rho}{T}\right) K_1\left(\frac{m_{\mathrm{T}}\cosh\rho}{T}\right), \tag{6}$$

where I_0 and K_1 are modified Bessel functions, T is the freeze-out temperature and $m_{\mathrm{T}} = \sqrt{p_{\mathrm{T}}^2 + m^2}$ is the transverse mass.

It should be noted that the volume in eq. (6) refers to the volume of the system as observed in a pp collision, as each collision in the random walk produces a pp type of fireball. If we now introduce a boost-invariant distribution of fireballs along the longitudinal rapidity axis, we finally obtain by integrating over the fireball distributions

$$\left(\frac{\mathrm{d}N}{\mathrm{d}y m_{\mathrm{T}} \mathrm{d}m_{\mathrm{T}}}\right)_{y=0} = \frac{gV}{2\pi^2}\left[\frac{4}{\pi\delta_{AB}^2}\right]^{1/2}\int \mathrm{d}\rho \exp\left(-\frac{\rho^2}{\delta_{AB}^2}\right) \times \int_{-Y_L}^{Y_L} \mathrm{d}Y m_{\mathrm{T}} \cosh Y I_0\left(\frac{p_{\mathrm{T}}\sinh\rho}{T}\right) \times K_1\left(\frac{m_{\mathrm{T}}\cosh Y\cosh\rho}{T}\right). \tag{7}$$

The above expression has two parameters, T and δ. The temperature is assumed to be the same for the whole fireball. The temperature is fixed from the pp collisions and δ is calculated from pA collisions as mentioned before. Fixing these two parameters from pp and pA collisions, we shall attempt to describe the data for AA collisions.

3. Transverse momentum spectra in *pp* collisions

As discussed in the previous section, we would fix the freeze-out temperature for the AA collisions from the inverse slope of the hadronic transverse momentum spectra originating from pp collisions. At high energies, the transverse momentum spectra of the produced hadrons can be described by the following expression:

$$\left(\frac{\mathrm{d}^2N}{\mathrm{d}y\mathrm{d}p_{\mathrm{T}}^2}\right)^{pp} \sim \text{const. } m_{\mathrm{T}} K_1\left(\frac{m_{\mathrm{T}}}{T}\right). \tag{8}$$

Using the expression in eq. (9), we fitted the experimental data of pp collisions of pions, kaons and protons at different centre-of-mass (CMS) energies varying from 23 to 63 GeV [9]. The results are shown in figure 2. The extracted temperature for the various hadron species are shown in figure 3. The error bars shown correspond to the error on the slope parameter due to the fitting procedure adopted. One observes that there is very little variation in T for all the produced hadrons in the range of CMS energy for which data are available and analysed. We shall consider two values of temperature for our study of AA data, one corresponding to 140 MeV and the other corresponding to 150 MeV.

4. Transverse momentum spectra in *pA* collisions

Having fixed the freeze-out temperature for AA collisions from the inverse slope of p_{T} spectra of the produced hadrons in pp collisions, we now try to get the average transverse rapidity shift per collision, δ. We shall determine this empirically from transverse mass spectra of the produced hadrons in pA collisions. To get a proper understanding of this

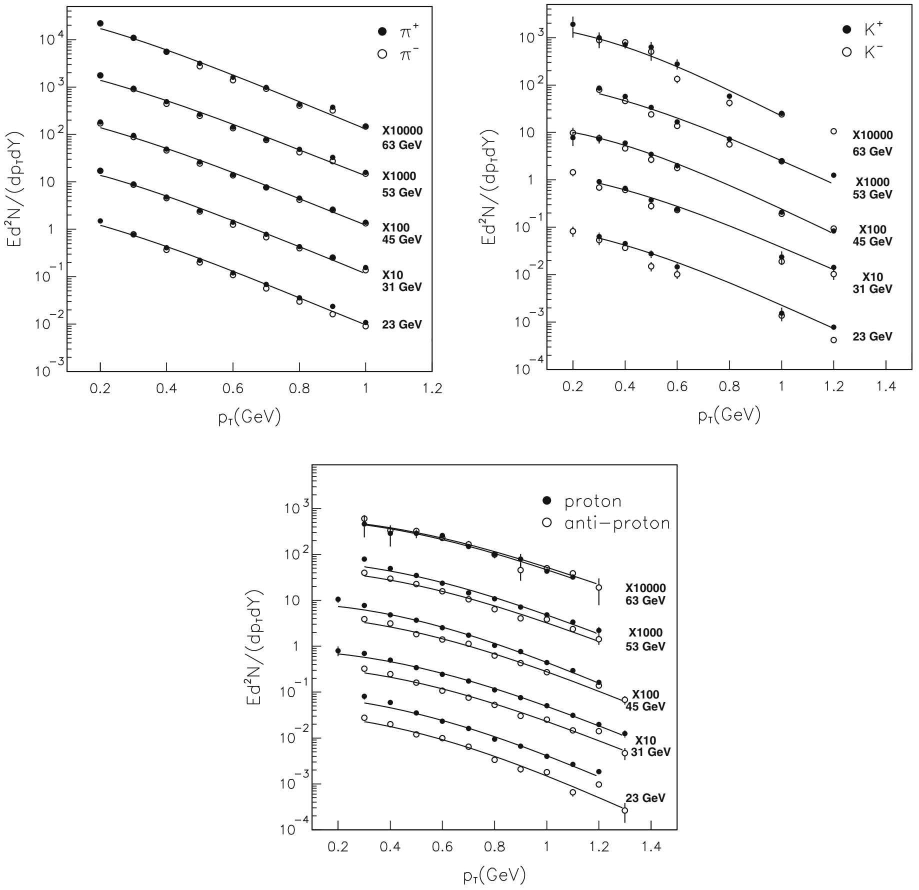

Figure 2. Transverse momentum spectra of pions, kaons, protons and antiprotons for various CMS energies. The data are scaled appropriately for clarity of presentation. The solid lines correspond to results obtained from eq. (9). The fits for π^- and K^- are not shown in the figure.

parameter, we shall study *pA* data for various available beam energies, different species of produced hadrons and several targets. Once this parameter is fixed, along with the fixed temperature from *pp* collisions, we shall be able to predict the initial state transverse momentum broadening in *AA* collisions as discussed in §2. The results will be presented in the next section.

The transverse momentum spectra for different species of the produced hadrons for *pA* collisions can be obtained using eq. (7), where δ_{pA} is the only parameter, with T being fixed from *pp* collisions. The results are shown in figures 4 and 5. Figure 4 shows the comparison of transverse mass distribution for pion, kaon and proton at 14.6 GeV for various targets [10] with those obtained from model calculations. One finds that the data are reasonably well explained by the model calculations. The values of δ_{pA} obtained for pion, kaon and proton are 0.05, 0.25 and 0.25, respectively. The corresponding δ values

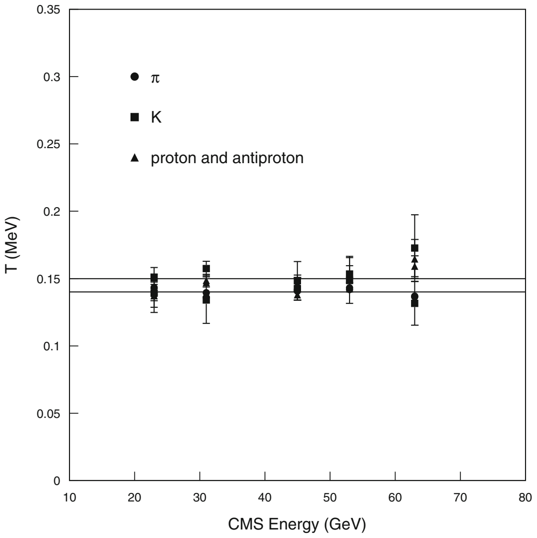

Figure 3. Temperature obtained by fitting the transverse momentum spectra presented in figure 2, for various hadrons. The solid lines correspond to 140 and 150 MeV. The error bars correspond to the error in temperature due to the fitting.

for the p+Cu collisions obtained using eq. (2) are 0.04, 0.22, 0.22 for pions, kaons and protons, respectively. Also, shown in figure 4 is the transverse mass spectra for pions at 14.6 GeV for various targets. We find that there is very little atomic mass dependence of δ_{pA}. It was found to be 0.05 for the various targets shown in the figure. However, one observes from eq. (5), that the average transverse rapidity shift per collision, δ decreases

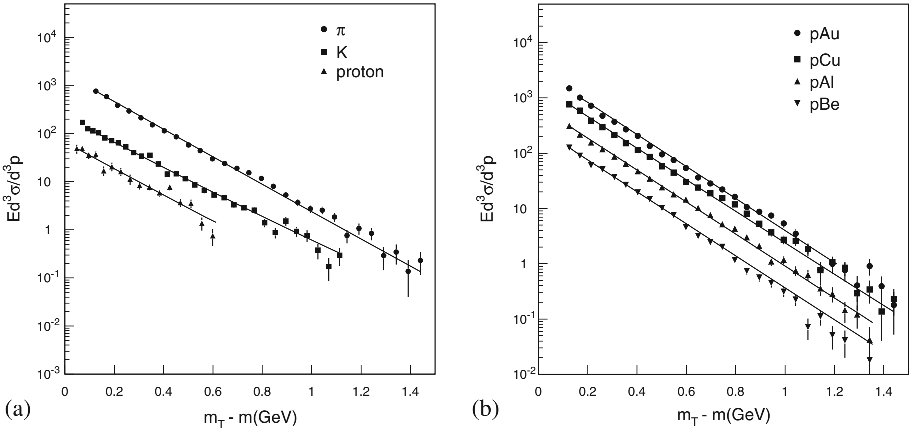

Figure 4. (**a**) The transverse momentum spectra for pions, kaons, protons and antiprotons at 14.6 AGeV for p+Be reaction. (**b**) The corresponding distribution of pions for various targets. The solid lines correspond to results obtained from random walk model.

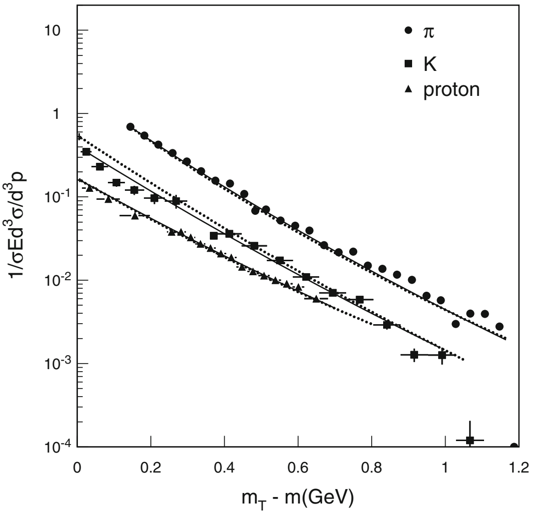

Figure 5. Transverse momentum spectra for pions, kaons, protons and antiprotons at 450 A GeV for p+Pb reaction. The solid lines correspond to results obtained from the random walk model with $T = 150$ MeV and the dotted lines correspond to $T = 140$ MeV.

with increase in atomic mass as we go from beryllium to gold target. Figure 5 shows the transverse mass spectra for pion, kaon and proton for p+Pb collisions at 450 A GeV [11]. The solid lines are the result of model calculation taking $T = 150$ MeV. The resulting δ_{pA}, which satisfactorily fits the data as shown, turns out to be of the order of 0.45 for three hadronic species. This is a clear increase from the lower energy value, indicating that δ_{pA} is energy dependent. The corresponding δ_{pA} value for $T = 140$ MeV (dotted lines) lies between 0.35 and 0.4.

For the description of *AA* data at SPS, we shall consider two values of temperature, of 140 and 150 MeV fixed from the *pp* data. Since the collisions at SPS are lead-on-lead target, we shall use $\delta_{p\text{Pb}}$ values as discussed above for *p*Pb collisions at 450 A GeV. We shall also consider a case with $\delta_{p\text{Be}}$ at 14.6 A GeV in order to get an idea of the effect at lower energies.

5. Transverse momentum spectra in *AA* collisions

Having fixed the temperature from the *pp* spectra and knowing the value of δ from the available *pA* data, we now try to see if the random walk model is able to explain the transverse mass spectra of pions, kaons and protons at SPS energies. The results of the model calculation using eq. (7) along with the available data [12] are shown in figure 6.

The results for pions are shown in figure 6a. The lines correspond to the random walk model calculations for two different temperatures of 140 and 150 MeV. One observes that the model can explain the pion spectra satisfactorily over the energy range of 40–158 A GeV for both 140 and 150 MeV, the χ^2/ndf being less than 1.0 for all the

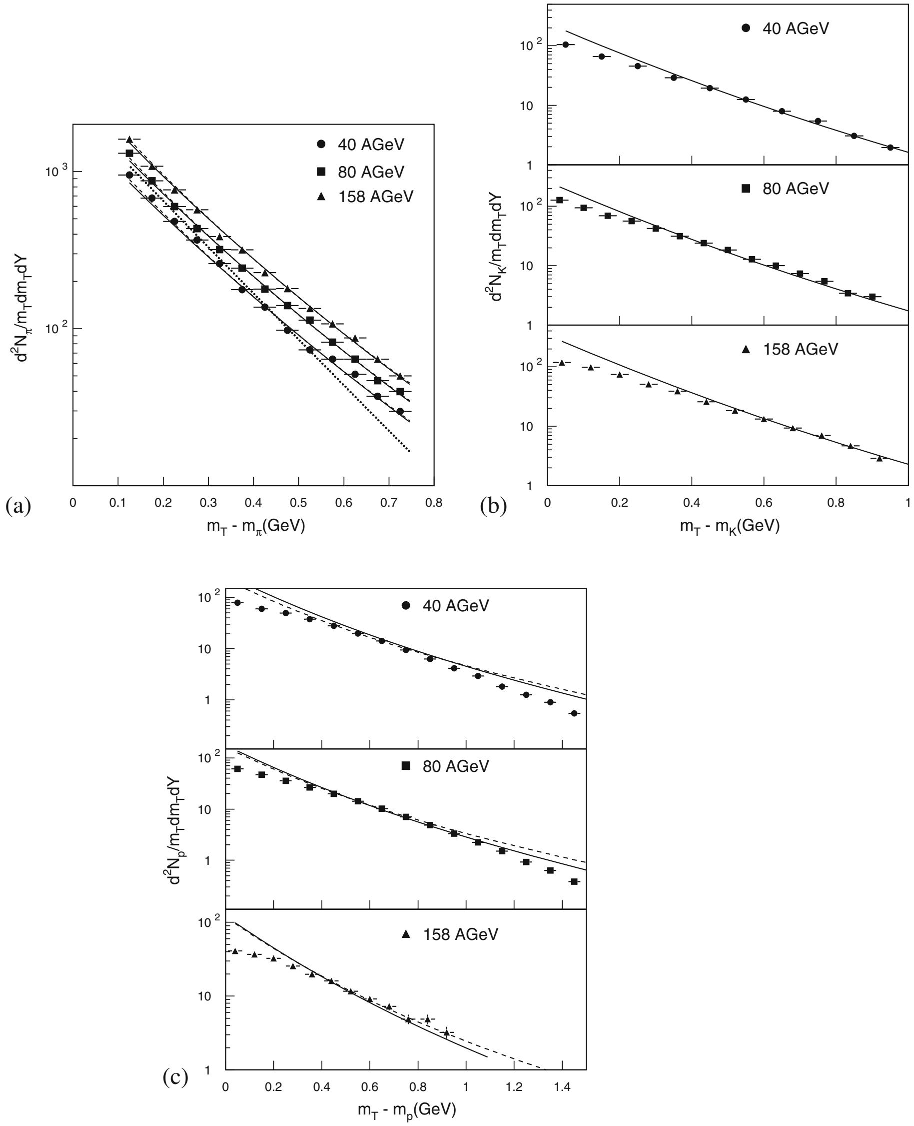

Figure 6. Transverse mass spectra for pions, kaons and protons at 40, 80 and 158 A GeV beam energies. The solid lines correspond to calculations from the random walk model at $T = 150$ MeV. The dashed lines correspond to calculations at $T = 140$ MeV. The δ parameter is set from *p*Pb collision data at 450 A GeV. The dotted line shown for 40 AGeV pion spectra corresponds to a similar calculation at $T = 150$ MeV and δ obtained from *p*Be data at 14.6 GeV.

cases. Taking a δ value corresponding to *pA* results from AGS energies (14.3 A GeV) fails to explain the data. This is shown as dotted line for 40 A GeV beam energy. Similar results are obtained for higher beam energies which are not shown in the figure for clarity of presentation. It may be mentioned that the absolute normalization is adjusted to get the best possible fit. For the kaons (figure 6b), one finds that the model fails to explain the lower transverse momentum region. Further, one notices that χ^2/ndf worsens as we go to higher beam energies. It varies from 3.5 at 40 A GeV to 10 at 158 A GeV. There is not much difference between the results for the model calculation for temperatures of 140 and 150 MeV, hence the spectra corresponding to 140 MeV is not shown in the figure. The model also fails to explain the observed transverse mass spectra of protons at both lower and higher transverse momenta. The results for two different temperatures of 140 and 150 MeV are shown in figure 6c. The χ^2/ndf for these three cases lies between 3.5 and 7.5.

6. Discussion

The results indicate the following: (i) the random walk model fails to explain the transverse mass spectra as the mass of the hadron increases where the effect of possible transverse flow will be more and (ii) the χ^2/ndf indicates that the disagreement of the model calculation with the data for kaons and protons increases with increase in beam energy. In this section we discuss the limitations of the model and try to estimate the relative contribution of initial state p_{T} broadening and transverse flow.

6.1 *Random walk pattern*

The random walk pattern was chosen to be a Gaussian eq. (1). In principle this can be any other type of statistical distribution, such as a Lorentzian. We have studied the sensitivity of the result to such a distribution. We find that the p+Pb data for pion, kaon and proton are well explained by $\delta_{p\mathrm{Pb}}$ of 0.2, 0.15 and 0.35, respectively. The values are lower compared to those obtained by considering a Gaussian random walk pattern. Extrapolating these values to Pb+Pb collisions and taking the value of T to be 150 MeV, we obtain m_{T} spectra for the available SPS data at the highest energy. The results are shown in figure 7. We observe that the model fails to explain the data satisfactorily, specially at low m_{T} for kaons and protons. However, it agrees fairly well with the observed pion spectra. It seems that the results are insensitive to the choice of random walk pattern.

6.2 *Parameters of random walk model*

One of the disadvantages of the present work is the absence of pp and pA data in literature corresponding to the exact SPS energies for which AA collision data are available. However, one must mention that the near-constant temperature exhibited by the available pp data for a wide range of CMS energy more or less fixes the temperature parameter of the model. Contrary to this, one observes a considerable variation of δ with beam energy. In view of these, it is obvious to ask, if the SPS kaon and proton transverse mass spectra can be explained within the framework of random walk model for arbitrary values of temperature and δ. The results are shown in figure 8. One observes that the kaon spectra for the three different beam energies are well explained by the model by taking a common temperature of 235 MeV and δ_{PbPb} of 0.25. The χ^2/ndf is less than 1.0 for all the three cases. It may be mentioned that this temperature, although very high compared

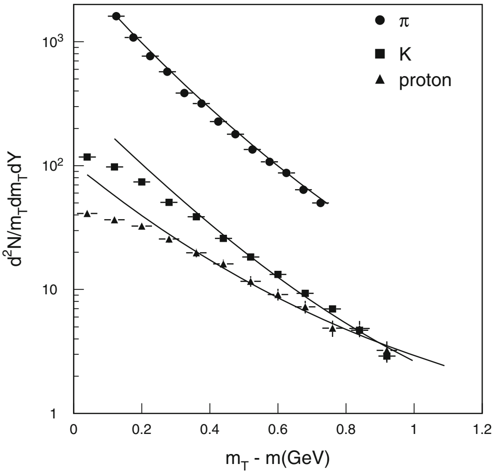

Figure 7. Transverse mass spectra for kaons and protons at 158 A GeV Pb+Pb collisions. The solid lines correspond to results obtained from random walk model with the random pattern following a Lorentzian distribution. The model parameters T and δ are not fixed from pp and p+Pb collisions respectively. The normalization is adjusted to give the best possible agreement with the experimental data.

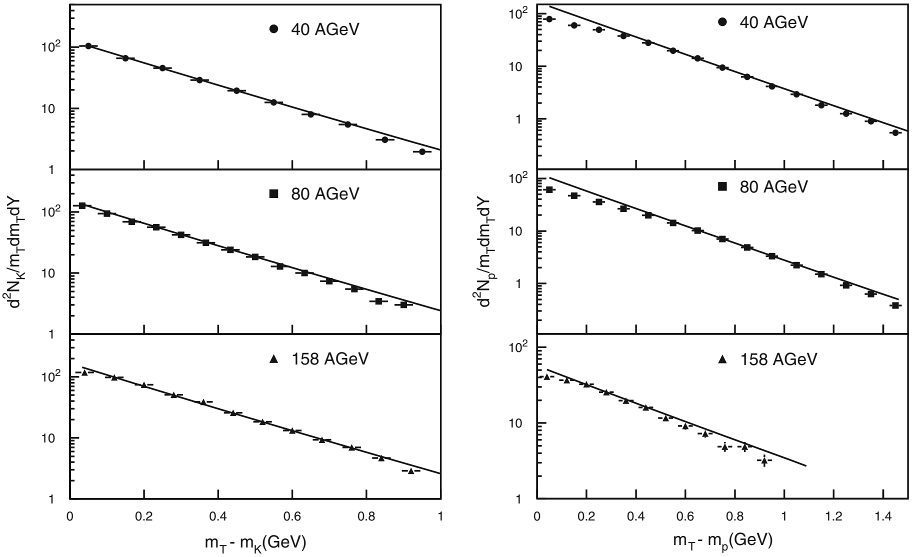

Figure 8. Transverse mass spectra for kaons and protons at SPS energies. The solid lines correspond to the results obtained from random walk model. The model parameters T and δ are not fixed from pp and pA collisions, respectively but are adjusted to give the best possible agreement with the experimental data.

to that obtained from pp (§3), is very close to the temperature obtained by taking radial flow into account [12]. For the protons the spectra are satisfactorily reproduced over a large range of transverse momentum. For this, we needed a common δ_{PbPb} value of 0.15

but the temperature for 40 and 80 A GeV is 260 MeV, while that for 158 A GeV energy is 360 MeV.

6.3 *Transverse flow*

The failure of random walk model to fully explain the transverse momentum spectra of the produced hadrons indicate that initial-state p_T broadening alone cannot account for experimentally measured transverse mass spectra. One observes that the model fails to explain the lower momentum part for kaons and protons. This may be attributed to the lack of chemical equilibrium, which can be accounted for by introducing a chemical potential for the hadrons. But the formulation of the model is such that it affects only the normalization and not the slope. The other possibility is that, there may be both initial-state p_T broadening and collective effects like transverse flow. In other words, the resultant spectra of the produced hadrons is the effect of both the above processes. In order to look at this possibility, we express the transverse mass spectra of the produced hadrons as

$$\left(\frac{\mathrm{d}N}{\mathrm{d}y m_\mathrm{T} \mathrm{d}m_\mathrm{T}}\right)_\mathrm{total} = C_1 \left(\frac{\mathrm{d}N}{\mathrm{d}y m_\mathrm{T} \mathrm{d}m_\mathrm{T}}\right)_\mathrm{random} + C_2 \left(\frac{\mathrm{d}N}{\mathrm{d}y m_\mathrm{T} \mathrm{d}m_\mathrm{T}}\right)_\mathrm{flow}, \tag{9}$$

where the contribution from the flow [12,13] is defined as

$$\left(\frac{\mathrm{d}N}{\mathrm{d}y m_\mathrm{T} \mathrm{d}m_\mathrm{T}}\right)_\mathrm{flow} \sim m_\mathrm{T} I_0\left(\frac{p_\mathrm{T}\sinh\rho}{T}\right) K_1\left(\frac{m_\mathrm{T}\cosh\rho}{T}\right),$$

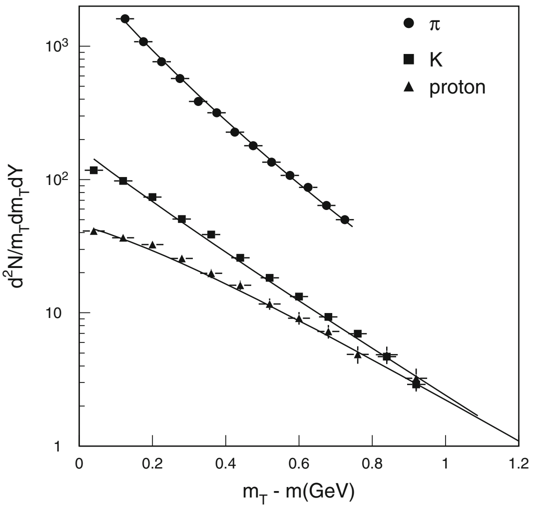

Figure 9. Transverse mass spectra for pions, kaons and protons at 158 A GeV Pb+Pb collisions. The solid lines correspond to results obtained from a combination of random walk model and transverse flow. The model parameters T and δ are fixed from pp, p+Pb collisions, respectively and the transverse flow velocity β_T is taken to be ∼0.45. The normalization is adjusted to give the best possible agreement with the experimental data.

mean transverse velocity β_T is defined through the relation $\rho = a \tanh \beta_T$ and T is the freeze-out temperature which is taken to be 150 MeV as seen from the slope of pp spectra.

We tried to see if such a parametrization can explain the SPS data at 158 A GeV beam energy and get the values of C_1 and C_2, which will tell the relative contribution of the two effects. The results are shown in figure 9. Since pion data is very well explained by the random walk model, we tried not to fit it with the above parametrization. The aim is to see how much contribution of transverse flow in addition to initial-state p_T broadening through the random walk model, can explain the data. From this figure we find that the results for the kaons and protons show that such a parametrization works. We have taken the value of β_T as 0.45 for kaons and 0.48 for protons. The values of C_1 and C_2 are 0.5 for kaons and for protons, $C_1 = 0.2$ and $C_2 = 0.8$. The results indicate that the flow effects dominate with increase in hadron mass.

7. Summary

In summary, a systematic study has been carried out to understand the transverse mass spectra of the produced hadrons in nucleus–nucleus collisions within the framework of a simple random walk model. The model was based on the assumption that the nucleus–nucleus collision is a superposition of nucleon–nucleon collision, where each successive interaction of a nuclear collision created a fireball. The temperature of the fireball was fixed from the available nucleon–nucleon collision data. There was a gain in transverse momentum through successive collisions, the propagation of which was assumed to follow a random walk pattern. The average transverse rapidity shift per collision or the gain in transverse momentum per collision was obtained from the nucleon–nucleus collision data. Having fixed the two parameters of the model, we then applied it to nucleus–nucleus collisions. It was observed that, although the model explained the transverse mass spectra of pions at SPS energies fairly well, it failed to do so for the higher mass hadrons like kaons and protons. We found that the choice of different distribution of random walk pattern yielded the same result. This indicated the presence of true collective effects in the nucleus–nucleus collisions. However, it seemed that a considerable portion of the gain in transverse momentum came from initial state effects. By considering both the initial-state p_T broadening and the transverse flow effect, we were able to explain the SPS data. We found the contribution of transverse flow to increase with increase in hadron mass. Now, the availability of systematic data from pp to pA to AA collision at LHC energy, would further help in quantifying the effect of initial-state broadening of transverse momentum and hence, the true amount of collective effect.

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