PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 82, No. 5
May 2014
pp. 867–878

# Unified approach to alpha decay calculations

C S SHASTRY*, S MAHADEVAN and K ADITYA
Department of Sciences, Amrita Vishwa Vidyapeetham, Coimbatore 641 112, India
*Corresponding author. E-mail: cs.shastry23@gmail.com

**DOI:** 10.1007/s12043-014-0740-7; ***e*Publication:** 2 May 2014

**Abstract.** With the discovery of a large number of superheavy nuclei undergoing decay through $\alpha$ emissions, there has been a revival of interest in $\alpha$ decay in recent years. In the theoretical study of $\alpha$ decay the $\alpha$-nucleus potential, which is the basic input in the study of $\alpha$-nucleus systems, is also being studied using advanced theoretical methods. In the light of these, the Wentzel–Kramers–Brillouin (WKB) approximation method often used for the study of $\alpha$ decay is critically examined and its limitations are pointed out. At a given energy, the WKB expression uses barrier penetration formula for the determination of the transmission coefficient. This approach utilizes the $\alpha$-nucleus potential only at the barrier region and ignores it elsewhere. In the present era, when one has more precise experimental information on decay parameters and better understanding of $\alpha$-nucleus potential, it is desirable to use a more precise method for the calculation of decay parameters. We describe the analytic *S*-matrix (SM) method which gives a procedure for the calculation of decay energy and mean life in an integrated way by evaluating the resonance pole of the *S*-matrix in the complex momentum or energy plane. We make an illustrative comparative study of WKB and *S*-matrix methods for the determination of decay parameters in a number of superheavy nuclei.

**Keywords.** *S*-Matrix; *Q*-values; half-life; Wentzel–Kramers–Brillouin.

**PACS Nos 24.10.−i; 11.55.−m; 21.Tg; 23.60.+e**

## 1. Introduction

Alpha decay process occupies a prominent place in the history of physics. On the one hand, it played a key role in the Rutherford scattering of $\alpha$-particles by nuclei, leading to the firm establishment of nuclear model of the atom. On the other hand, the theoretical explanation of $\alpha$ decay by Gamow [1] as a quantum tunnelling process was one of the first great successes of quantum mechanics developed in the 1920s. Further, the $\alpha$ decay played a key role in the development of nuclear physics in general. During the last few decades, with the discovery of a large number of unstable nuclei undergoing decay by emission of protons, neutrons and heavier nuclei, $\alpha$ decay gets grouped in a class of nuclear decay phenomena known as cluster decay. In addition, there have been the discovery of a large number of superheavy nuclei [2–4], and the latest one has proton number 118. These superheavy nuclei undergo decay by $\alpha$ and $\beta$ emissions. In establishing decay chains of superheavy nuclei, intermediate $\alpha$ decay processes with experimentally determined decay energies and lifetimes play crucial roles. As a result, there has been a significant revival of interest in the theoretical study of $\alpha$ decay.

Most of the theoretical approaches to study $\alpha$ decay are centred around the concept of quantum tunnelling formulated first by Gamow [1]. Alpha decay can be visualized as a process involving the formation of daughter nucleus–$\alpha$ system from the original parent nucleus and the survival of this system as a quasibound state for a while, depending on its lifetime before it undergoes decay resulting in the unbound daughter nucleus–$\alpha$ system with positive relative energy signifying the decay energy. In this mechanism, the formation of daughter nucleus–$\alpha$ system from parent nucleus is governed by the structural aspects of the parent nucleus and in the common approaches to $\alpha$ decay, this aspect is taken into account by a parameter known as cluster pre-formation probability $P_\alpha$. Once the parent nucleus $(A, Z)$ having mass number $A$ and proton number $Z$ forms the two-body system consisting of daughter nucleus $(A-4, Z-2)$ and the $\alpha$-particle, the decay process is governed by the Schrödinger equation governing this two-body system. In the centre of mass system, this reduces to a Schrödinger equation in three dimensions for a single particle having reduced mass $m = 4(A-4)/A$ amu and governed by the potential $U(r)$ between daughter nucleus and the $\alpha$-particle. When the potential $U$ is spherically symmetric, the problem further reduces to the solution of radial Schrödinger equation having appropriate orbital angular momentum quantum number $l$ and governed by the effective potential $U_{\text{eff}}(r) = U(r) + \hbar^2 l(l+1)/(2mr^2)$ which is the sum of $U(r)$ and the centrifugal term.

The potential $U(r)$ is a sum of short-range attractive nuclear potential $U_{\text{N}}(r)$ and the Coulomb potential $U_{\text{c}}(r)$ acting between $\alpha$-particle and the daughter nucleus. The Coulomb term $U_{\text{c}}(r)$ is assumed to be a potential between point charge $2e$ and a uniformly charged sphere of radius $R_{\text{c}}$ having total charge $(Z-2)e$. It is given by

$$U_{\text{c}}(r) = \frac{2(Z-2)e^2(3R_{\text{c}}^2 - r^2)}{2R_{\text{c}}^3}. \tag{1}$$

The parameter $R_{\text{c}}$ is of the order of nuclear radius. For the present discussion, the potential $U_{\text{N}}(r)$ can be approximated by the empirical Woods–Saxon potential

$$U_{\text{N}}(r) = \frac{-U_0}{1 + \exp(\frac{r-R}{a})}, \qquad U_0 > 0, \tag{2}$$

where $U_0$, $R$, $a$, respectively represent the usual strength, radius and diffuseness parameters. If one uses these expressions, for sufficiently small $l > 0$, the total effective potential

$$U_{\text{eff}}(r) = U_{\text{N}}(r) + U_{\text{c}}(r) + \hbar^2 \frac{l(l+1)}{2mr^2} \tag{3}$$

is dominated by the centrifugal barrier near the origin, a Coulomb barrier peaked at barrier radius $R_{\text{b}} \sim R_{\text{c}}$ and a potential pocket between the centrifugal and Coulomb barriers.

Beyond $R_b$ the potential is dominated by the Coulomb tail (figure 1). The potential pocket is crucial in generating the quasibound state of the daughter nucleus–$\alpha$ system. The decay of this state by tunnelling across the barrier describes the physical phenomena of $\alpha$ decay. With increasing $l$, the pocket region diminishes and eventually vanishes beyond a critical value of $l$.

The reasonably smooth varying nature of the effective potential has prompted researchers to use the comparatively simple WKB approximation (see e.g., [5–7]) to determine $\alpha$ decay lifetime $\tau$. In this approximation, the centrifugal term is taken to be $\hbar^2(l+1/2)^2/(2mr^2)$ to generate the correct behaviour of the wave function near the origin. This term generates a centrifugal barrier near the origin even when $l=0$. The pocket region can be used to generate the energy $E$ of the quasibound state by adopting the well-known WKB formula for bound states. Similarly, the transmission across the effective Coulomb barrier at energy $E$ gives transmission coefficient $T$ and hence, can be used to calculate decay constant $\lambda$ and mean lifetime $\tau$. With the availability of accurate $\alpha$-nucleus potentials to study the decay problem, use of more accurate methods than the WKB method is desirable. The reasons are as follows: it should be noted that the quasibound state energy $E$ and its width $\Gamma$ (and hence $\tau$) are generated in principle by the entire potential $U_{\text{eff}}(r)$ using the complete solution of the radial Schrödinger equation with appropriate boundary conditions. On the contrary, WKB method uses a restricted part of the pocket region to generate $E$ and the corresponding restricted part of the barrier to generate $\tau$ in a decoupled way. In addition, $\tau$ is a very sensitive function of $E$ and hence a small error in $E$ can cause a much bigger error in $\tau$. Due to this reason, in many analyses of $\alpha$ decay one uses experimentally measured $E$ rather than the theoretically calculated one, even though a more satisfactory theoretical approach should generate both $E$ and $\tau$ within a unified framework. Further, calculation of transmission probability is carried out using the WKB formula in one dimension which completely ignores the potential except in the domain of the effective barrier at energy $E$. Model studies [8] have shown that

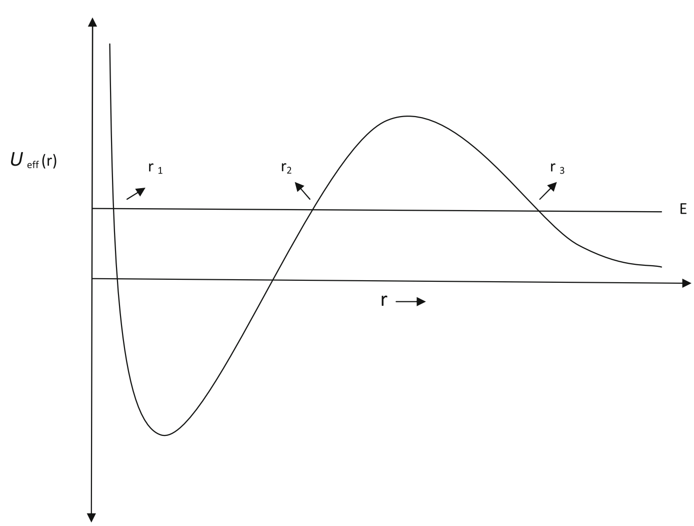



**Figure 1.** Schematic diagram of the effective $\alpha$ daughter nucleus potential.

such a procedure is not very satisfactory. In view of these observations, we have proposed the simultaneous calculations of both $E$ and $\tau$ based on the complete solution of radial equation and analytic $S$-matrix method.

This paper is organized as follows: in §2 we outline the WKB method for the calculation of decay energy $E$ and mean life $\tau$. The unified approach for the calculation of $E$ and $\tau$ is described in §3. Section 4 contains typical set of numerical results and §5 deals with summary and conclusions.

## 2. WKB method

In solving the Schrödinger equation for the radial wave function $R_l(r)$ it is convenient to use modified Schrödinger equation satisfied by $\phi_l(r) = rR_l(r)$:

$$\frac{\mathrm{d}^2\phi_l(r)}{\mathrm{d}r^2} + [k^2 - V_{\mathrm{eff}}(r)]\phi_l(r) = 0, \tag{4}$$

where

$$k^2 = \frac{2mE}{\hbar^2}, \qquad V_{\mathrm{eff}}(r) = \frac{2m}{\hbar^2}U_{\mathrm{eff}}(r). \tag{5}$$

At a given energy $E$ below the Coulomb barrier, the roots $r_1, r_2, r_3$ of the equation

$$\kappa^2 = k^2 - V_{\mathrm{eff}}(r) = 0 \tag{6}$$

define the turning points and we choose $r_1 < r_2 < r_3$. The region $r_1 < r < r_2$ defines the classically allowed pocket region and similarly $r_2 < r < r_3$ defines the classically forbidden barrier region. The WKB approximation [5,6] for bound states can be used to approximately calculate the energies of quasibound states in the pocket region using the relation

$$\int_{r_1}^{r_2} \kappa(r)\mathrm{d}r = (n + 1/2)\pi. \tag{7}$$

It can be seen from (7) that $n$ is the number of nodes of WKB wave function between the turning points and this approximation is good only if the turning points are several wavelengths apart; that is, when $n$ is large compared to unity. Similarly, one can calculate the transmission coefficient $T$ using the WKB approximation. We have

$$T = \exp\left[-2\int_{r_2}^{r_3} [-\kappa^2(r)]^{1/2}\mathrm{d}r\right]. \tag{8}$$

Representing by $A_f$, the assault frequency factor signifying the number of attempts the $\alpha$-particle makes before under going decay, we get the WKB approximation based formula for the decay constant $\lambda$

$$\lambda = P_f A_f T. \tag{9}$$

The simplest approximate estimate of assault frequency is $A_f = \nu/[2(r_2 - r_1)]$ where $\nu$ is the average velocity. The pre-formation parameter $P_f$ is chosen suitably; often, it is set to unity. The formulae connecting $\lambda$, $\Gamma$, $\tau$ and half-life $\tau_{1/2}$ are

$$\lambda = \Gamma/\hbar = \tau^{-1} = (\ln 2)/\tau_{1/2}. \tag{10}$$

In the calculation of both $E$ and $\tau_{1/2}$ based on WKB approach, the possible error $\Delta E$ in the determination of $E$ causes much more enhanced error in the calculation of $\tau_{1/2}$ because in expression (8) for $T$, $E$ is present in the exponential function. Hence, instead of the theoretically computed $E$, often one uses the corresponding experimental $Q$-values. In such an approach, the WKB method completely avoids the use of pocket region of the potential.

## 3. *S*-matrix method for unified determination of $E$ and $\tau$

We now briefly outline the SM method for the calculation of quasibound state energy and its lifetime. For this purpose, we consider the modified radial Schrödinger equation governed by well-behaved short-range potentials. This is adequate for understanding the idea behind the method. However, this procedure has to be appropriately modified for the treatment of $\alpha$ decay where the potential is a sum of short-range nuclear potential and long-range Coulomb potential. In the case of well-behaved short-range potentials, the regular solution of the Schrödinger equation (4) satisfies the condition

$$\phi_l(r) \xrightarrow[r\to 0]{} r^{l+1}. \tag{11}$$

Now we use the methods of analytic SM. The details of analytic *S*-matrix can be found, in e.g., [9–11]. The radial equation has two linearly independent irregular solutions and they are known as Jost solutions $f_l(\pm k, r)$. For large $r$ they satisfy the asymptotic conditions

$$f_l(\pm k, r) \xrightarrow[r\to\infty]{} \exp(\mp ikr). \tag{12}$$

One can express $\phi_l(r)$ as a linear combination of $f_l(k, r)$ and $f_l(-k, r)$ using what are known as Jost functions $F_l(\pm k)$ defined by the Wronskians $W[\phi_l(r), f_l(\pm k, r)]$

$$F_l(\pm k) = W[\phi_l(r), f_l(\pm k, r)] \tag{13}$$

$$\phi_l(r) = [F_l(-k) f_l(k, r) - F_l(k) f_l(-k, r)]/(2ik). \tag{14}$$

This combination given by (14) makes several physical features explicit. Whereas, the condition (11) guarantees the required behaviour near origin, if $F_l(-k) = 0$ for some $k = k_p$ as $r \to \infty$,

$$\phi_l(r) = [-F_l(k) f_l(-k, r)]/2ik \to O(\exp^{ikr}). \tag{15}$$

It is clear that if $k_p = i\beta$, $\beta > 0$, the corresponding wave function $\phi_l(r)$ asymptotically varies as $\exp(-\beta r)$ signifying a bound state. This indicates that the zeros of $F_l(-k)$ can have physical significance. Before dealing with this we note that for real positive energy $E$ the function $\phi_l(r)$ signifies scattering solution behaving asymptotically as

$$\phi_l(r) \xrightarrow[r\to\infty]{} \sin(kr - l\pi/2 + \delta_l), \tag{16}$$

where $\delta_l$ is the phase shift for the $l$th partial wave. Comparing (14) and (16), one gets the following expression for partial wave $S$-matrix $S_l(k)$:

$$S_l(k) = \exp(i\pi l)\frac{F_l(k)}{F_l(-k)}. \tag{17}$$

Equation (17) provides an important relation between Jost functions and $S_l(k)$. We note that the zero of $F_l(-k)$ at $k = i\beta$, $\beta > 0$ or equivalently a pole of $S_l(k)$ on the positive imaginary axis of the complex $k$-plane signifies a bound state. By extensively studying the analytic properties of $F_l(\pm k)$ in the entire complex $k$-plane, the physical significance of zeros of the Jost function and poles of $S_l(k)$ have been established. In particular, for short-range Yukawa-type potential, decreasing exponentially for large $r$, $F_l(\pm k)$ is an analytic function of $k$ except for branch cut in part of the imaginary $k$-axis. Further, in the case of potentials like square well which vanish identically beyond a given $r = R_0$, $F_l(\pm k)$ are entire functions of $k$. The symmetry relation $F_l(k) = [F_l(-k^*)]^*$ satisfied by $F_l(\pm k)$ establishes that $F_l(\pm k)$ cannot be zero on real axis unless $\phi_l(r)$ identically vanishes. Further, the zeros $F_l(\pm k)$ are distributed symmetrically with respect to imaginary $k$-axis. These features are used in identifying the nature of poles of $S_l(k)$ as illustrated for Yukawa-type potential as shown in figure 2. It can be readily seen that the pole $k = k_p$ just below the real axis in the fourth quadrant with small imaginary part $|\text{Im}\, k_p| < |\text{Re}\, k_p|$ signifies a resonance state. Consider $E_p = \hbar^2 k_p^2/(2m)$ associated with a resonant pole at $k = k_p$. We have

$$k_p = \text{Re}\, k_p + i\, \text{Im}\, k_p, \qquad \text{Re}\, k_p > 0, \quad \text{Im}\, k_p < 0 \tag{18}$$

$$E_p = \frac{\hbar^2}{2m}[(\text{Re}\, k_p)^2 - (\text{Im}\, k_p)^2 + 2i\, \text{Re}\, k_p\, \text{Im}\, k_p] = E_r - i\Gamma_r/2 \tag{19}$$

clearly,

$$E_r = \frac{\hbar^2}{2m}[(\text{Re}\, k_p)^2 - (\text{Im}\, k_p)^2] \tag{20}$$

is the positive energy of the resonant state and

$$\Gamma_r = \frac{\hbar^2}{2m} 4\, \text{Re}\, k_p |\text{Im}\, k_p| \tag{21}$$

is the corresponding width. In our approach to $\alpha$ decay calculation we treat the daughter nucleus–$\alpha$ system as a resonant or quasibound state with energy $E_r$ and width $\Gamma_r$. The

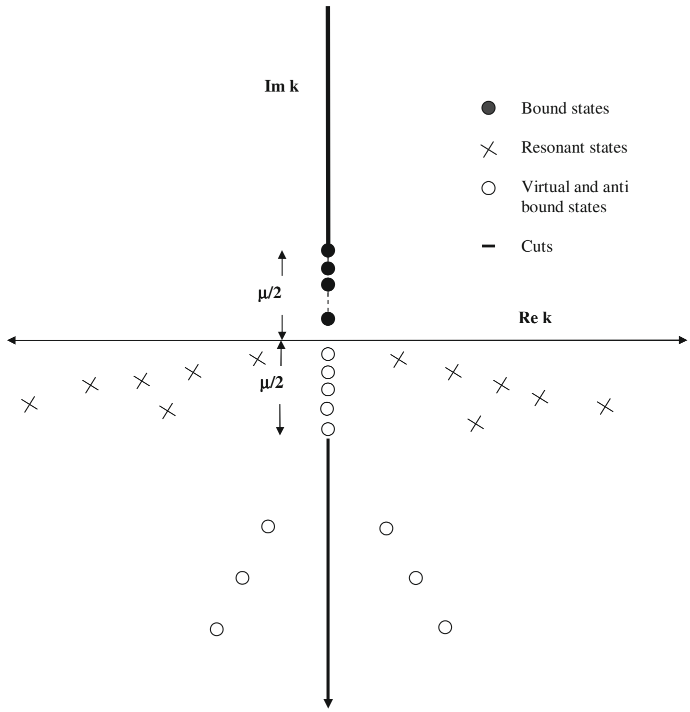



**Figure 2.** Illustration of singularities of $S_l(k)$ in the $k$ plane of Yukawa potential of range $\mu - 1$.

Jost function is the result of the complete-solution of the Schrödinger equation and hence the resonant pole position is calculated in an integrated way. Thus, $E_r$ and $\Gamma_r$ can be determined in a correlated manner and unlike WKB method there is no scope for decoupling of their computation to different regions of the potential. The pole at $k = -\text{Re}\ k_p + i\ \text{Im}\ k_p$, which is the symmetric counterpart of pole at $k = k_p$ does not represent a different resonant state; but, only the time-reversed counterpart of the state associated with $k_p$. From (14) it is clear that the quasibound state wave function is regular at origin and varies as $\exp(ik_pr)$ asymptotically having an exponentially diverging term $\exp(|\text{Im}\, k_p|r)$. This shows that unlike a bound state, quasibound or resonant state is not normalizable. However, within the interaction region it is quite similar to bound state in its behaviour. If one considers the time-dependent part $\exp(-iE_pt/\hbar)$ of the resonant state, it is clear that it decays in time as $\exp(-\Gamma t/2\hbar)$ and indicates a mean life $\tau = \hbar/\Gamma_r$ for the resonance state.

The above brief description summarizes the basic approach one takes in the calculation of $\alpha$ decay energy and width in a unified way using the well-known results of analytic $S$-matrix theory. However, in actual calculations suitable changes are needed to take into account the long-range Coulomb potential present in the $\alpha$ daughter nucleus system in addition to the short-range nuclear potential. In such a case the calculation of $S$-matrix has to be done using Coulomb distorted asymptotic plane wave and spherical waves. The generalization of analytic $S$-matrix theory is described in [12]. The key parameter governing the long-range Coulomb effect is the Rutherford parameter $\eta = Z_1 Z_2 e^2 m/\hbar^2 k$, where $Z_1$ and $Z_2$ are the charges of the two interacting particles. Thus, in the case of Coulomb nuclear problem the corresponding Jost solutions $f_l(\eta, \pm k, r)$ satisfy the following condition as $r \to \infty$:

$$\lim[f_l(\eta, \pm k, r) \exp(\pm ikr) \exp(\mp i \ln(2kr))] = \exp(-\eta\pi/2). \tag{22}$$

It is convenient to write the $S$-matrix $S_l(k)$ as a product of the Coulomb $S$-matrix $S_c(l, k)$ and the so-called nuclear $S$-matrix $S_N(l, k)$:

$$S_l(k) = S_c(l, k) S_N(l, k), \tag{23}$$

where $S_c(l, k) = \exp(2i\sigma_l)$ with $\sigma_l = \arg(\Gamma(l + 1 + i\eta))$. As in the case of short-range potential [12], one can define Jost functions for the Colulomb nuclear problem. The resonance structure of $S_l(k)$ is contained in $S_N(l, k)$ and it has to be computed using the regular solution $\phi_l(\eta, kr)$ of the modified radial Schrödinger equation for the Coulomb nuclear system and the regular and irrregular Coulomb wave functions $F_l(\eta, kr)$ and $G_l(\eta, kr)$, respectively. The expression for the nuclear $S$-matrix becomes

$$S_N(l, k) = \frac{F^N(l, k)}{F^N(l, -k)}, \tag{24}$$

where the nuclear Jost functions $F^N(l, \pm k)$ are given by

$$\begin{aligned} F^N(l, \pm k) &= (\text{const})[\rho G_l(\eta, kr) - G_l'(\eta, kr)] \\ &\quad \mp i[F_l'(\eta, kr) - \rho F_l(\eta, kr)]|_{r=R_m} \end{aligned} \tag{25}$$

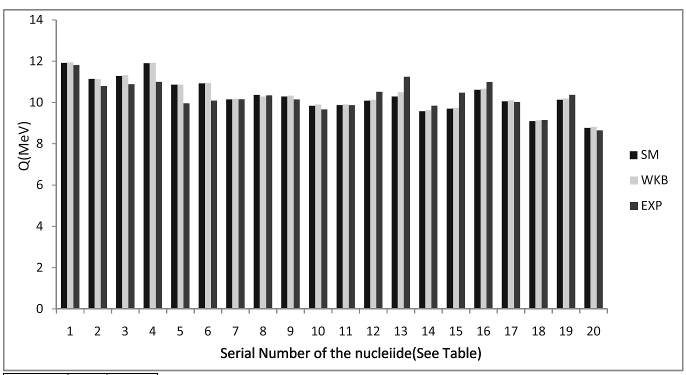
evaluated at $r = R_M$ well beyond the range of nuclear potential. The superscript$'$ indicates the derivative with respect to $r$. The term $\rho$ is given by

$$\rho = \left.\frac{\phi_l'(\eta, kr)}{\phi_l(\eta, kr)}\right|_{r=R_m}. \tag{26}$$

The parameters of quasibound states or resonances can be obtained by computing the zeroes of $F^N(l, -k)$ in the fourth quadrant of the complex $k$ plane just below the real $k$-axis. This has to be done by taking into account the complications involved in the computation of Coulomb wave functions. Even though the present procedure of evaluation of quasibound state parameters is somewhat difficult, it becomes manageable by noting that the calculation of $S_l(k)$ in nuclear optical model code is quite well established and it has to be only extended for complex $k$. The main accomplishment of the $S$-matrix approach is the unified and more accurate treatment of the calculation of decay parameters.

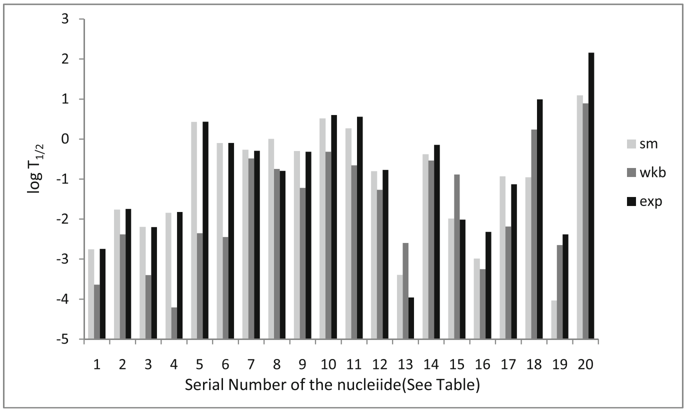
## 4. Numerical results

To carry out the calculations one requires $\alpha$ daughter nuclear potentials. We generate these potentials numerically in the double-folding framework. Explicit calculation involves the following steps: First, the relativistic mean field (RMF) [13–15] calculations are carried out for the relevant nuclei by employing the most widely used NL3 [15] set of Lagrangian parameters. The details related to RMF method can be found in [13]. These spherical densities both for protons and neutrons along with the well-known density-dependent M3Y (DDM3Y) nucleon–nucleon interaction are then used in the doublefolding ($t\rho\rho$) approximation to generate [16,17] the $\alpha$ daughter nucleus potential. It should, however, be noted that the spherically symmetric potential thus generated is really an approximation to the actual interaction between the $\alpha$ and the deformed daughter nuclei. We restrict this approximation, because incorporating deformation terms within the simple potential scattering approach will be too complicated and the simplicity of the present method will be lost. In all calculations, the exact nuclear potentials were used. The results (both $Q$-values and half-lives) are obtained by using a fixed $r_c = 1.24$ f and it is shown in the histograms (figures 3 and 4). Substantial part of the results shown in the histograms are given in [18]. We propose to give a more exhaustive set of results based on $S$-matrix methods in the near future. Further, difficulties that arise in the calculation of decay parameters involving long lifetimes using $S$-matrix method are discussed in [19] and hence, not repeated.




| Serial No | A | Z |
|---|---|---|
| 1 | 294 | 118 |
| 2 | 292 | 116 |
| 3 | 291 | 116 |
| 4 | 290 | 116 |
| 5 | 289 | 114 |
| 6 | 288 | 114 |
| 7 | 287 | 114 |
| 8 | 286 | 114 |
| 9 | 284 | 113 |
| 10 | 283 | 112 |
| 11 | 280 | 111 |
| 12 | 279 | 111 |
| 13 | 273 | 110 |
| 14 | 276 | 109 |
| 15 | 275 | 109 |
| 16 | 266 | 109 |
| 17 | 267 | 108 |
| 18 | 272 | 107 |
| 19 | 262 | 107 |
| 20 | 271 | 106 |

**Figure 3.** Comparison of $Q$-values computed using SM, WKB methods with the experimental values.




| Serial No | A | Z |
|---|---|---|
| 1 | 294 | 118 |
| 2 | 292 | 116 |
| 3 | 291 | 116 |
| 4 | 290 | 116 |
| 5 | 289 | 114 |
| 6 | 288 | 114 |
| 7 | 287 | 114 |
| 8 | 286 | 114 |
| 9 | 284 | 113 |
| 10 | 283 | 112 |
| 11 | 280 | 111 |
| 12 | 279 | 111 |
| 13 | 273 | 110 |
| 14 | 276 | 109 |
| 15 | 275 | 109 |
| 16 | 266 | 109 |
| 17 | 267 | 108 |
| 18 | 272 | 107 |
| 19 | 262 | 107 |
| 20 | 271 | 106 |

**Figure 4.** Comparison of $\tau_{1/2}$ values computed using SM, WKB methods with the experimental values.

## 5. Summary and conclusions

The main findings are summarized now: unlike the WKB-based approximations, SM method treats the $\alpha$ decay as the decay of the long-lived resonance state of the $\alpha$ daughter nucleus two-body system and utilizes the full potential to generate the $Q$-values and half-lives in a unified and correlated way. This method is fully quantum mechanical and exact in principle. Further, in order to generate the $\alpha$-nucleus potential, we have used microscopic RMF-based $\alpha$-nucleus nuclear potentials. The present calculations show that these $\alpha$-nucleus potentials are quite useful for a more accurate study of decay parameters for a wide range of systems. The Coulomb potential used in our calculations is for a uniformly charged daughter nucleus with radius $R_c$. Most of the results obtained using SM method are marginally better than the corresponding WKB results. However, a more exhaustive calculation taking into account a much larger set of $\alpha$ decaying nuclei is desirable. In addition, the method used in our calculations comes broadly within the framework of $\alpha$ nucleus Coulomb nuclear potential scattering and hence, provides a common framework for the study of $\alpha$ daughter nucleus systems. Hence, this opens up the possibility of using RMF-generated potentials with confidence in the study of $\alpha$-scattering and decay in a unified way. The present set of calculations opens up the possibility of using a reasonably realistic global $\alpha$-nucleus potential for a comprehensive study of all decay data.

In this paper, we have mainly discussed the *S*-matrix and WKB approaches to $\alpha$ decay. The main merit of the former, is that it is based on the full solution of the radial Schrödinger equation and the decay of the quasibound or resonance state of the daughter nucleus–$\alpha$ system. However, we wish to point out that there are other approaches based on the full solution of the radial equation. In the approach adopted by Ni and Ren [20], $\alpha$ decay is visualized as the quantum transition of the $\alpha$ daughter nucleus quasibound state to scattering or continuum state and the width $\Gamma$ is described by the matrix element $2\pi|\langle\Psi|U_i - U_f|\Phi\rangle|^2$, where $|\Psi\rangle$ symbolizes the quasibound state governed by the total effective potential $U_i$ and $U_f$ is the sum of the Coulomb potential and the centrifugal potential governing the continuum state $|\Phi\rangle$ which is described by the appropriate relevant Coulomb wave functions. Another approach used successfully by Sahu and collaborators [21,22] is to use the expression $\Gamma = (\mu/\hbar)|G_l(\eta, k, R)|^2$, where $P = |u(R)|^2/[\int_0^R |u(r)|^2 \mathrm{d}r]$. In this expression, $u(r)$ is the modified radial wave function for the quasibound state. Thus, at present, including the *S*-matrix method, we have three approaches which fully use the quasibound state wave function for the $\alpha$ decay calculations. It will be very interesting to have a systematic comparative study of these three approaches along with the traditional WKB approach.


## Acknowledgements

The authors thank Prof. Y K Gambhir and Prof. A Bhagawat for providing RMF-based potentials and Dr P Prema for her assistance in the preparation of the manuscript.


## References


[1] Z Gamov, *Z. Phys.* **51**, 204 (1928)
[2] G Munzenberg, *J. Phys. G: Nucl. Part. Phys.* **25**, 717 (1999)
[3] S Hoffman and G Munzenberg, *Rev. Mod. Phys.* **72**, 733 (2000)
[4] Yu Ts Oganessian, *Nucl. Phys. A* **685**, 17c (2001)
[5] L I Schiff, *Quantum mechanics* 3rd ed. (McGraw Hill, New York, 1968)
[6] S Flugge, *Practical quantum mechanics* (Springer, Berlin, 1994) Vol. 1
[7] N Froman and P Froman, *JWKB approximation: Contribution to theory* (North Holland, Amsterdam, 1965)
[8] S Mahadevan, *Analytical S-matrix approach for the study of α decay of super heavy elements*, Ph.D. Thesis (Amrita Vishwa Vidyapeetham, 2009)
P Prema, S Mahadevan, C S Shastry, A Bhagawat and Y K Gambhir, *Int. J. Mod. Phys. E* **17**, 611 (2008)
[9] R G Newton, *Scattering theory of waves and particles* (McGraw Hill, New York, 1966)
[10] C J Jochain, *Quantum collision theory* (North Holland Publishing Company, Amsterdam, 1965)

[11] R J Eden, P V Landshoff, D Lolive and J C Polkinghome, *The analytic S matrix* (Cambridge University Press, Cambridge, 2002)
[12] S Mukherjee and C S Shastry, *Nucl. Phys. B* **3**, 1 (1967)
[13] Y K Gambhir, P Ring and A Thimet, *Ann. Phys. (N.Y.)* **198**, 132 (1990)
[14] G A Lalazissis, M M Sharma, P Ring and Y K Gambhir, *Nucl. Phys. A* **608**, 202 (1996)
[15] G A Lalazissis, J Konig and P Ring, *Phys. Rev. C* **55**, 549 (1997)
[16] Y K Gambhir, A Bhagawat, M Gupta and Arun K Jain, *Phys. Rev. C* **68**, 044316 (2003)
[17] Y K Gambhir, A Bhagwat and M Gupta, *Phys. Rev. C* **71**, 037301 (2005); *Ann. Phys. (N.Y.)* **320**, 429 (2005)
[18] P Prema, S Mahadevan, C S Shastry and Y K Gambhir, *Int. J. Mod. Phys. E* **19**, 2033 (2010)
[19] S Mahadevan, P Prema and C S Shastry, *Phys. Rev. C* **74**, 57601-1-4 (2006)
[20] D Ni and Z Ren, *Phys. Rev. C* **80**, 014314 (2009); *Phys. Rev. C* **82**, 024311 (2010)
[21] B Sahu, R Paira and B Rath, *NPA* **908**, 40 (2013)
[22] B Sahu, *Phys. Rev. C* **85**, 057601 (2012)