PRAMANA — journal of physics

Vol. 82, No. 5
May 2014
pp. 859–865

Nucleon–nucleon scattering in the light of supersymmetric quantum mechanics

J BHOI[1,*], U LAHA[1] and K C PANDA[2]

[1]Department of Physics, National Institute of Technology, Jamshedpur 831 014, India
[2]School of Physics, Sambalpur University, Jyoti Vihar, Burla 768 019, India
*Corresponding author. E-mail: jskbhoi@gmail.com

DOI: 10.1007/s12043-014-0739-0; ***e*Publication:** 2 May 2014

Abstract. By exploiting supersymmetry-inspired factorization method together with a judiciously chosen deuteron ground-state wave function, approximate higher partial wave nucleon–nucleon potentials are generated. In this context, a minor modification is also introduced to the generated potentials. The n–p scattering phase shifts are computed and analysed via the phase function method.

Keywords. Supersymmetry and factorization; nucleon–nucleon potential; phase function method; n–p scattering.

PACS Nos 03.65.Nk; 24.10.–i; 13.75.Cs

1. Introduction

The invariance of the supersymmetric Hamiltonian under translation in superspace corresponds to the existence of supercharges that commute with the supersymmetric Hamiltonian and leads to definite relations between bosonic and fermionic spectra. For any Hamiltonian with one degree of freedom, a comparison Hamiltonian can be constructed such that the resulting system as a whole is supersymmetric [1–3]. The Hamiltonian hierarchy problems in supersymmetric quantum mechanics (SQM) lead to the addition of appropriate centrifugal barriers and consequently, the higher partial wave potentials are generated fairly accurately [4]. In the case of Coulomb potential the so-called accidental degeneracy is recovered as a natural consequence. At small values of radial coordinate r, the Hulthen potential behaves like a Coulomb potential, whereas for large 'values of r' it decreases exponentially so that its capacity for bound state is smaller than that of the Coulomb potential. Also, the Hulthen potential serves as a model for the interaction between nucleons in deuteron. Because of the above similarity and points of contrasts between the Coulomb and the Hulthen potentials, it may be of considerable

interest to generate supersymmetric partners of the latter and study their partner potentials, related physical observables etc., which have important applications in quantum scattering theory.

Arnold and Mackellar [5] parametrized Hulthen potential to fit the deuteron binding energy and S-wave scattering lengths. Recently, by exploiting this deuteron ground-state wave function in conjunction with supersymmetry-inspired factorization method, we have generated higher partial wave nucleon–nucleon potentials (triplet states only) [6] and computed the related phase shifts. Following SQM formalism, here, we shall generate higher partial wave nucleon–nucleon potentials for both singlet and triplet states and introduce another set of new higher partial wave potentials, namely, 'modified' potentials. We shall also report the results for the corresponding phase shifts by taking recourse to the use of the phase-function method (PFM) [7].

The PFM represents an efficient approach to evaluate the scattering phase shifts for quantum mechanical problems involving local [7] and non-local interactions [8,9] and is based on the separation of radial wave function of the Schrödinger equation into an amplitude part $\alpha_l(k, r)$ and an oscillating part with variable phase $\delta_l(k, r)$. Physically, this amounts to factorizing out the two effects of the potential which manifest themselves in deforming the wave function and in producing the scattering phases [10]. The function $\delta_l(k, r)$ called the phase function, has at each point the meaning of phase shift of the wave function for scattering by the potential truncated at a distance r. A completely amputated potential will not produce any phase shift. Thus, $\delta_l(k, 0) = 0$. In §2 we discuss the basic formalism of supersymmetry quantum mechanics and generate higher partial wave potentials to calculate phase shifts via PFM. Finally, in §3 we discuss results and present some concluding remarks.

2. Supersymmetry and higher partial wave interactions

In the supersymmetric quantum mechanics, any Hamiltonian of the form

$$H_0 = -\frac{\partial^2}{\partial x^2} + V_0(x), \tag{1}$$

which has a ground state $\Psi_0^{(0)}$ and $E_0^{(0)}$ can be factorized as

$$H_0 = A_0^{(+)} A_0^{(-)} + E_0^{(0)} \tag{2}$$

with

$$A_0^{(\pm)} = \pm\frac{\partial}{\partial x} + \frac{\partial}{\partial x}\ln\Psi_0^{(0)}. \tag{3}$$

The supersymmetric partner H_1 with potential V_1 (P-wave potential) of the Hamiltonian H_0 is given by

$$H_1 = -\frac{\partial^2}{\partial x^2} + V_1(x) \tag{4}$$

with

$$V_1(x) = V_0(x) - \frac{\partial^2}{\partial x^2}\ln\Psi_0^{(0)}. \tag{5}$$

The applications of the above relations to the Hulthen potential are now in order.

The Yamaguchi potential [11] is written as

$$V(r, s) = \lambda e^{-\beta(r+s)}, \tag{6}$$

where λ is the strength and β is the inverse range parameters. The bound or antibound state energy is obtained from [5] $\lambda = -2\beta(\beta + \alpha)^2$, where the wave number α is positive or negative depending on whether the state is bound or antibound. The bound state wave function for the Yamaguchi potential is identical to the wave function for the first bound state of the Hulthen potential with range $(\beta - \alpha)^{-1}$ and depth $-(\beta^2 - \alpha^2)$. A Hulthen potential with these parameters, hereby designated as nuclear Hulthen potential, is rewritten as

$$V_{0N}(r) = -\left(\beta^2 - \alpha^2\right) \frac{e^{-\beta r}}{e^{-\alpha r} - e^{-\beta r}}. \tag{7}$$

For the potential in eq. (7) the ground state solution $\Psi_0^{(0)}$ of the Schrödinger equation is given by

$$\Psi_0^{(0)} \sim e^{-\alpha r} - e^{-\beta r}. \tag{8}$$

In view of eqs (5) and (7), the wave function in eq. (8) leads to the supersymmetric partner potential

$$V_{1N}(r) = V_{0N}(r) + \frac{(\beta - \alpha)^2 e^{-(\alpha+\beta)r}}{\left(e^{-\alpha r} - e^{-\beta r}\right)^2}. \tag{9}$$

Equation (9) is regarded as the approximate P-wave nuclear Hulthen potential, the second term in which simulates the effect of centrifugal barrier apart from a factor of 2. To simulate the proper effect of the centrifugal term, we redefine eq. (5) in the following form:

$$V_1(x) = V_0(x) - C \frac{\partial^2}{\partial x^2} \ln \Psi_0^{(0)}, \tag{10}$$

where C is a normalization constant with its numerical value 2. With this normalization constant the modified P-wave potential reads as

$$V_{1NM}(r) = V_{0N}(r) + \frac{2(\beta - \alpha)^2 e^{-(\alpha+\beta)r}}{\left(e^{-\alpha r} - e^{-\beta r}\right)^2}. \tag{11}$$

Here, the subscript M indicates the modified potential.

3. Results and discussions

For a local potential the phase function $\delta_l(k, r)$ satisfies a first-order non-linear differential equation given by [7]

$$\delta_l'(k, r) = -k^{-1} V(r) \left[\hat{J}_l(kr) \cos \delta_l(k, r) - \hat{\eta}_l(kr) \sin \delta_l(k, r)\right]^2 \tag{12}$$

with $\hat{J}_l(kr)$ and $\hat{\eta}_l(kr)$, the Riccati Bessel functions. We shall follow the phase convention of Calogaro [7] with Hankel function of the first kind written as $\hat{h}_l(x) = -\hat{\eta}_l(x)$

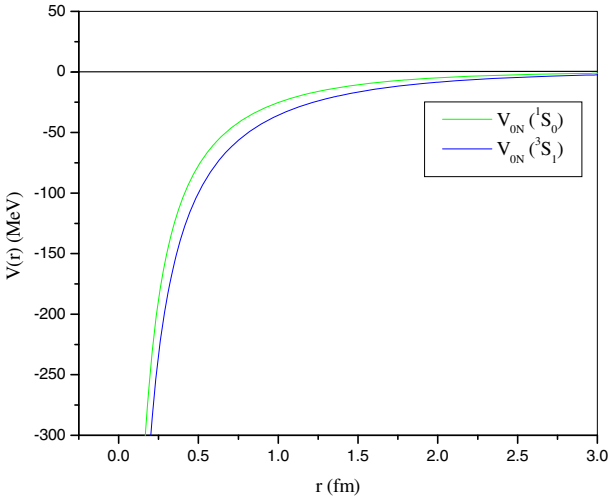

Figure 1. $^1S_0$ and $^3S_1$ potentials as a function of r.

$+ i\hat{J}_l(x)$. The scattering phase shift $\delta_l(k, r)$ is obtained by solving the equation from origin to asymptotic region with the initial condition $\delta_l(k, 0) = 0$. During the solution of the phase equation, $\delta_l(k, r)$ is built up by the potential as one moves away from the origin and it reaches its asymptotic value as soon as one gets out of the range of the potential. Obviously, $\delta_l(k) = \lim_{r\to\infty} \delta_l(k, r)$.

In figures 1–3 we portray the n–p potentials as a function of distance for S- and P-waves with $\lambda = -5.237$ fm^{-3} and $\beta = 1.4054$ fm^{-1} for $^1S_0$ scattering and, $\lambda = -7.533$ fm^{-3}

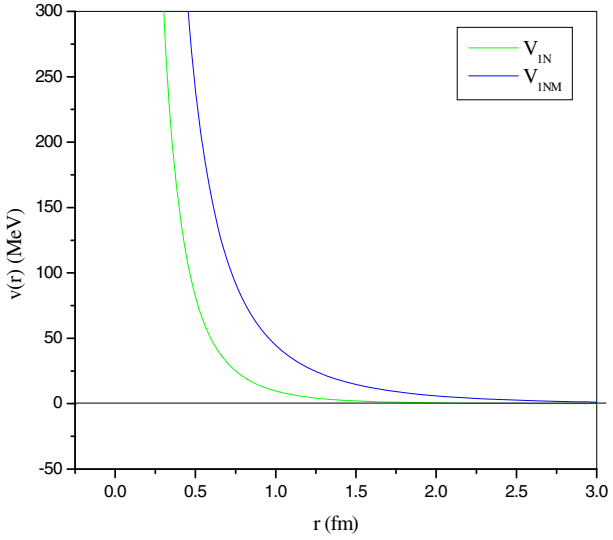

Figure 2. $^1P_1$ potentials as a function of r.

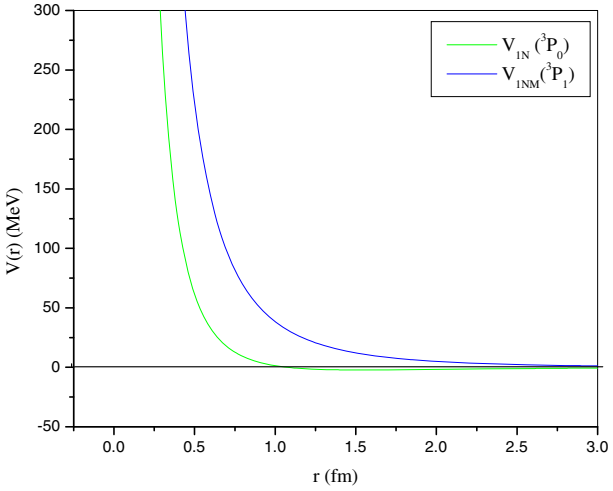

Figure 3. $^3P_0$ and $^3P_1$ potentials as a function of r.

and $\beta = 1.4054\ \text{fm}^{-1}$ for $^3S_1$ scattering [5]. It is observed that in figures 2 and 3 repulsive cores develop in the generated potentials. These potentials, generated from their $^1S_0$ and $^3S_1$ parts, correspond to $^1P_1$, $^3P_0$ and $^3P_1$ states, respectively. The corresponding singlet and triplet state phase shifts for S- and P-waves have been computed using PFM and presented in figures 4–6 as a function of laboratory energy up to 300 MeV along with the values of Arndt *et al* [12] for comparison. The $^1S_0$ phase shifts δ_{0N} agree well with that of Arndt *et al* [12] for $E_{\text{Lab}} \leq 25$ MeV. Beyond 25 MeV the phase shifts differ significantly with energy. Thus, it is expected that our generated higher partial wave singlet potentials give reasonable fit to n–p scattering phases at low and intermediate energies. Among the two sets of phase shift values δ_{1N} and δ_{1NM} for $^1P_1$ state (figure 5),

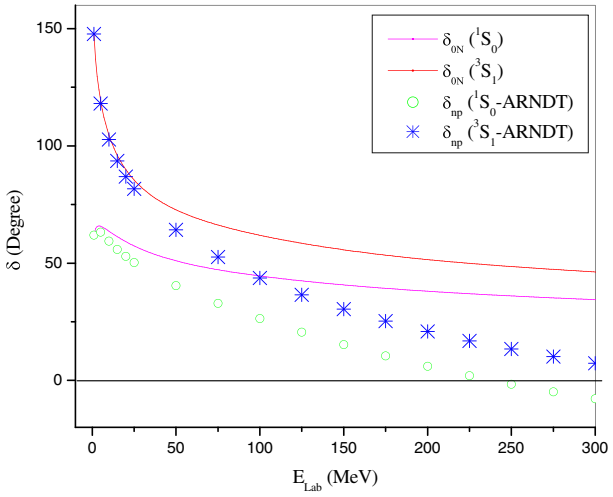

Figure 4. $^1S_0$ and $^3S_1$ phase shifts as a function of E_{Lab}.

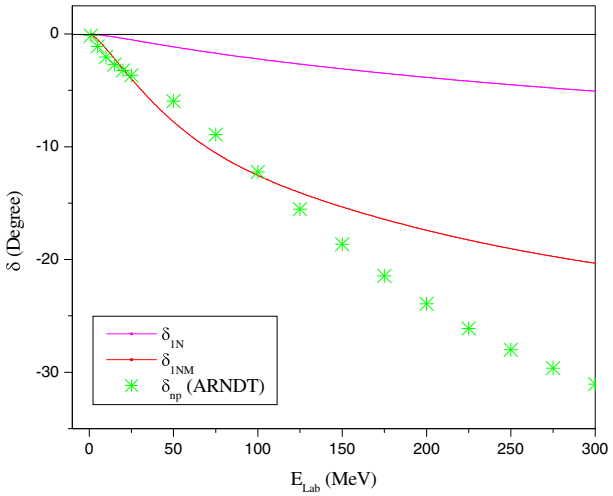

Figure 5. $^1P_1$ phase shifts as a function of $E_{\rm Lab}$.

δ_{1NM} is the most consistent one and permits comparison with the work of Arndt *et al* [12] up to $E_{\rm Lab} = 125$ MeV. Our phase shift values for $^3S_1$ state with the parameters of Arndt and Mackellar [5] agree with that of [12] up to $E_{\rm Lab} = 25$ MeV. Looking closely at figure 6 we notice that δ_{1N} contribute positive values of phase shifts up to 250 MeV. Beyond that they change sign. These phase shifts correspond to $^3P_0$ state but with lower numerical values. The other phase shifts δ_{1NM}, however, produce correct nature for $^3P_1$ state and are quite capable for comparison with Arndt *et al* [12] up to $E_{\rm Lab} = 100$ MeV.

The higher partial wave potentials that are generated here belong to Eckart class of potentials, the second term in them behaves as centrifugal barrier. Among these two sets

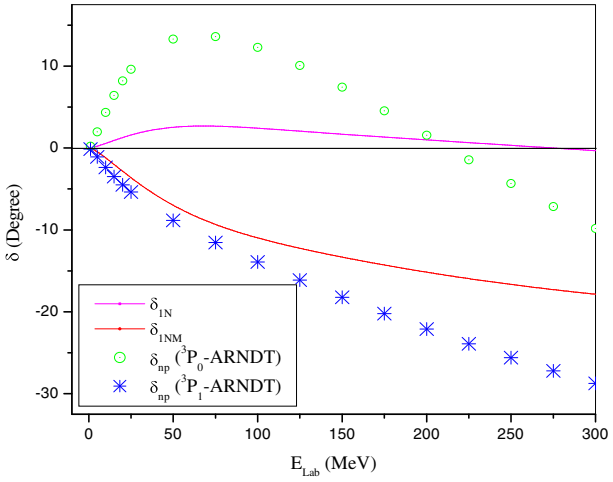

Figure 6. $^3P_0$ and $^3P_1$ phase shifts as a function of $E_{\rm Lab}$.

of potentials the modified interaction is the most effective and efficient one that justifies our modification for addition of a normalization constant. By comparing our results for phase shifts with those of Arndt *et al* [12] it can be concluded that this simple-minded combined approach of SQM and PFM for dealing with nucleon–nucleon scattering at low and intermediate energies will be considerably interesting to a large number of physicists.

References

[1] E Witten, *Nucl. Phys. B* **188**, 513 (1981)
[2] F Cooper and B Freedman, *Ann. Phys.* **146**, 262 (1983)
[3] C V Sukumar, *J. Phys. A: Math. Gen*. **18**, L57 (1985); 2917 (1985)
[4] U Laha, C Bhattacharyya, K Roy and B Talukdar, *Phys. Rev. C* **38**, 558 (1988)
[5] L G Arnold and A D Mackellar, *Phys. Rev. C* **3**, 1095 (1971)
[6] J Bhoi and U Laha, *J. Phys. G: Nucl. Part. Phys.* **40**, 045107 (2013)
[7] F Calogero, *Variable phase approach to potential scattering* (Academic, New York, 1967)
[8] B Talukdar, D Chatterjee and P Banerjee, *J. Phys. G: Nucl. Phys.* **3**, 13 (1977)
[9] G C Sett, U Laha and B Talukdar, *J. Phys. A: Math. Gen.* **21**, 3643 (1988)
[10] U Fano, C E Theodosious and J L Dhemer, *Rev. Mod. Phys.* **48**, 49 (1976)
[11] Y Yamaguchi, *Phys. Rev*. **95**, 1628 (1954)
[12] R A Arndt, L D Roper, R A Bryan, R B Clark, B J VerWest and P Sisnell, *Phys. Rev. D* **28**, 97 (1983)