PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 82, No. 5
May 2014
pp. 797–807

Neutron star in the presence of strong magnetic field

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DOI: 10.1007/s12043-014-0730-9; ***e*Publication:** 22 April 2014

Abstract. Compact stars such as neutron stars (NS) can have either hadronic or exotic states like strange quark or colour superconducting matter. Stars can also have a quark core surrounded by hadronic matter, known as hybrid stars (HS). The HS is likely to have a mixed phase in between the hadron and the quark phases. Observational results suggest huge surface magnetic field in certain NS. Therefore, we study here the effect of strong magnetic field on the respective equation of states (EOS) of matter under extreme conditions. We further study the hadron–quark phase transition in the interiors of NS giving rise to HS in the presence of strong magnetic field. The hadronic matter EOS is described based on RMF theory and we include the effects of strong magnetic fields leading to Landau quantization of the charged particles. For quark phase, we use the simple Massachusetts Institute of Technology (MIT) bag model, assuming density-dependent bag pressure and magnetic field. The magnetic field strength increases from the surface to the centre of the star. We construct the intermediate mixed phase using Glendenning conjecture. The magnetic field softens the EOS of both the matter phases. We finally study, the mass–radius relationship for such types of mixed HS, calculating their maximum mass, and compare them with the recent observations of pulsar PSR J1614-2230, which is about 2 solar mass.

Keywords. Stars: neutron stars; magnetic fields; equation of state.

PACS Nos 26.60.Kp; 52.35.Tc; 97.10.Cv

1. Introduction

The central density of neutron stars (NS) exceeds the nuclear saturation density ($n_0 \sim 0.15$ fm^{-3}), thereby giving the idea that compact stars might contain deconfined and chirally restored quark matter in them. Recently [1], the mass measurement of millisecond pulsar PSR J1614-2230 has set a new robust mass limit for compact stars to be $M = 1.97 \pm 0.04 M_{\odot}$. This value, together with the mass of pulsar J1903+0327 of $M = 1.667 \pm 0.021 M_{\odot}$ [2] is much larger than any of the highest precisely measured pulsar mass. These measurements have for the first time set a very strong limit on parameters of the equation of states (EOS), which describe matter under extreme conditions [3,4].

Broadly, there can be two classes of compact stars with quark matter. The first is the so-called (strange) quark stars (SS) of absolutely stable strange quark matter. The second is the so-called hybrid stars (HS). Along with the hadronic matter HSs have quark matter in their interior either in the form of a pure strange quark matter core or colour superconducting matter. Between the quark and the hadronic phases a quark–hadron mixed phase exists. The size of the core depends on the critical density for the quark–hadron phase transition and the EOS describing the matter phases.

The presence of magnetic field in compact stars has an important role in astrophysics. New observations suggest that in some pulsars the surface magnetic field can be as high as 10^{14}–10^{15} G. Also the observed giant flares, SGR 0526-66, SGR 1900+14 and SGR 1806-20 [5], are the manifestations of strong surface magnetic field in those stars. Such stars are separately assigned as magnetars. If we assume flux conservation from a progenitor star, we can expect the central magnetic field of such stars as high as 10^{17}–10^{18} G. These strong fields are bound to affect the NS properties. It can modify either the metric describing the star [6,7] or the EOS of matter of the star. The effect of strong magnetic field, for both nuclear matter [8–12] and quark matter [13–15], has been studied earlier in detail.

We study the hadron–quark phase transition within a compact star with a mixed phase region in between the quark core and nuclear outer region. This paper is organized as follows. In §2, we discuss the relativistic nuclear EOS and the effect of Landau quantization due to magnetic field on the charged particles. In §3, we employ the simple MIT bag model for the quark matter EOS and the effect of magnetic field on the quarks (also due to Landau quantization). In §4, we develop the mixed phase region by Glendenning construction. We show results in §5 for the density-dependent bag constant and varying magnetic field for the mixed HS. Finally, we summarize our results and draw some conclusions in §6.

2. Magnetic field in hadronic phase

The EOS at normal nuclear density includes hadron degrees of freedom. To describe the hadronic phase, we use a non-linear version of the relativistic mean field (RMF) model with hyperons (TM1 parametrization) which is widely used to construct EOS for NS. In this model, the baryons interact with mean meson fields [16–21].

For the beta equilibrated matter, the condition is

$$\mu_i = b_i \mu_\mathrm{B} + q_i \mu_\mathrm{e}, \tag{1}$$

where b_i and q_i are respectively the baryon number and charge (in terms of electron charge) of species i. μ_B is the baryon chemical potential and μ_e is the electron chemical potential. For charge neutrality, the condition is

$$\rho_\mathrm{c} = \sum_i q_i n_i, \tag{2}$$

where n_i is the number density of species i.

For magnetic field we choose the gauge to be $A^{\mu} \equiv (0, -yB, 0, 0)$, where B is the magnitude of the magnetic field. For this particular gauge choice we can write $\vec{B} = B\hat{z}$. Due to the magnetic field, the motion of the charged particles is Landau quantized in perpendicular direction to the magnetic field. The momentum of the x–y plane is quantized and hence, the energy of the nth Landau level is [22]

$$E_i = \sqrt{{p_i}^2 + {m_i}^2 + |q_i|B(2n+s+1)}, \tag{3}$$

where n=0, 1, 2, ..., being the principal quantum numbers for allowed Landau levels, $s = \pm 1$ refers to spin up (+) and down (−) and p_i is the component of particle (species i) momentum along the field direction. Setting $2n + s + 1 = 2v$, where $v = 0, 1, 2...$, we can rewrite the single particle energy eigenvalue in the following form:

$$E_i = \sqrt{{p_i}^2 + {m_i}^2 + 2v|q_i|B} = \sqrt{{p_i}^2 + \tilde{m}_{i,v}^2}, \tag{4}$$

where the $v = 0$ state is singly degenerate. It should be remembered that for baryons the mass is $m_{\rm b}^*$.

The total energy density of the system can be written as [21]

$$\begin{aligned}\varepsilon = &\frac{1}{2}m_{\omega}^2\omega_0^2 + \frac{1}{2}m_{\rho}^2\rho_0^2 + \frac{1}{2}m_{\sigma}^2\sigma^2 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_{\phi}^2\phi_0^2 + \frac{3}{4}d\omega_0^4 + U(\sigma)\\ &+\sum_b \varepsilon_{\rm b} + \sum_l \varepsilon_l + \frac{B^2}{8\pi^2},\end{aligned} \tag{5}$$

where the last term is the contribution from the magnetic field. The general expression for the pressure is given by

$$P = \sum_i \mu_i n_i - \varepsilon. \tag{6}$$

3. Magnetic field in quark phase

Considering the simple MIT bag model for quark matter in the presence of magnetic field, we assume that the quarks are non-interacting. The current masses of u and d quarks are extremely small, e.g., 5 and 10 MeV, respectively, whereas, for s-quark the current quark mass is taken to be 150 MeV.

The thermodynamic potential in the presence of strong magnetic field $B(>B^{(c)}$, the critical value) is given by [23]

$$\Omega_i = -\frac{g_i|q_i|BT}{4\pi^2}\int {\rm d}E_i \sum_v \frac{{\rm d}p_i}{{\rm d}E_i} \ln[1 + \exp(\mu_i - E_i)/T]. \tag{7}$$

For zero temperature, the Fermi distribution is approximated by a step function and by interchanging the order of the summation over v and integration over E,

$$\Omega_i = -\frac{2g_i|q_i|B}{4\pi^2}\sum_v \int_{\sqrt{m_i^2+2v|q_i|B}}^{\mu} {\rm d}E_i\sqrt{E_i^2 - m_i^2 - 2v|q_i|B}. \tag{8}$$

The total energy density and pressure of the strange quark matter are given by [21]

$$\begin{aligned} \varepsilon &= \sum_i \Omega_i + B_G + \sum_i n_i \mu_i \\ p &= -\sum_i \Omega_i - B_G, \end{aligned} \tag{9}$$

where B_G is the bag constant.

4. Phase transition and mixed phase

With the above given hadronic and quark EOS, we can perform the Glendenning construction [24] for the mixed phase, which determines the range of baryon density where both phases coexist. We have considered both the hadron and quark phases to be separately charged, however, the total charge neutrality is preserved in the mixed phase. Thus, the matter can be treated as a two-component system, and can be parametrized by two chemical potentials, usually the pair (μ_e, μ_n), i.e., electron and baryon chemical potentials. To maintain mechanical equilibrium, the pressure of the two phases is equal. Satisfying the chemical and beta equilibrium, the chemical potentials of different species are connected to each other. The Gibbs condition for mechanical and chemical equilibrium at zero temperature between both phases is given by

$$P_{\mathrm{HP}}(\mu_e, \mu_n) = P_{\mathrm{QP}}(\mu_e, \mu_n) = P_{\mathrm{MP}}. \tag{10}$$

This equation gives the chemical equilibrium potentials of the mixed phase corresponding to the intersection of the two phases. At lower densities below the mixed phase, the system is in the charge neutral hadronic phase, and for higher densities above the mixed phase the system is in the charge neutral quark phase. As the two surfaces intersect, one can calculate the charge densities ρ_c^{HP} and ρ_c^{QP} separately in the mixed phase. If χ is the volume fraction occupied by quark matter in the mixed phase, we have

$$\chi \rho_c^{\mathrm{QP}} + (1-\chi)\rho_c^{\mathrm{HP}} = 0. \tag{11}$$

Therefore, the energy density ϵ_{MP} and the baryon density n_{MP} of the mixed phase can be obtained as

$$\epsilon_{\mathrm{MP}} = \chi \epsilon_{\mathrm{QP}} + (1-\chi)\epsilon_{\mathrm{HP}}, \tag{12}$$

$$n_{\mathrm{MP}} = \chi n_{\mathrm{QP}} + (1-\chi) n_{\mathrm{HP}}. \tag{13}$$

5. Results

The introduction of the magnetic field changes the EOS of the matter. The single-particle energy is now Landau quantized, and therefore it changes all the other thermodynamic variables of the EOS, namely, the number density, pressure and the energy density.

We parametrized the bag constant in such a way that it attains a value B_∞, asymptotically at very high densities. The experimental range of B_∞ is given in Burgio *et al* [25], and from which we choose the value $B_\infty = 130$ MeV. With these assumptions, we then construct a Gaussian-type parametrization given as [25]

$$B_{\rm G}(n_{\rm b}) = B_\infty + (B_{\rm g} - B_\infty) \exp\left[-\beta\left(\frac{n_{\rm b}}{n_0}\right)^2\right]. \tag{14}$$

The value B_∞ is the lowest attained at asymptotic high density in quark matter, and is fixed at 130 MeV. The quoted value of bag pressure is at the nuclear and mixed phase intersection point denoted by $B_{\rm g}$ in the equation. The value of $B_{\rm G}$ decreases with increase in density and attains $B_\infty = 130$ MeV asymptotically. The rate of decrease of the bag pressure is governed by parameter β.

We assume that the parametrization of the magnetic field strength depends on the baryon number density. Therefore, we assume a simple density dependent as given by [9]

$$B\,(n_{\rm b}/n_0) = B_{\rm s} + B_0\{1 - {\rm e}^{-\alpha(n_{\rm b}/n_0)^\gamma}\}, \tag{15}$$

where α and γ are two parameters determining the magnetic field profile with given $B_{\rm s}$ and B_0, and $n_{\rm b}$ is the baryon number density. The value of B mainly depends on B_0, and is quite independent of $B_{\rm s}$. Therefore, we vary the field at the centre, whereas surface field strength is taken to be $B_{\rm s} = 10^{14}$ G. We keep γ fixed at 2, and vary α to have the field variation. In the above parametrization, the magnetic field strength depends on the baryon number density. However, at each density the field is uniform and constant.

In figure 1 we have plotted pressure against energy density for a density-dependent bag pressure of 170 MeV only for $\alpha = 0.01$. With such α value, the asymptotic $B_0 = 10^{18}$ G gives a field strength of 6×10^{17} G at $10n_0$ baryon density. For $B_0 = 2 \times 10^{18}$ G the field strength is 1.21×10^{18} G at the same $10n_0$ baryon density. As the variation (α) becomes stiffer, the EOS curve becomes softer. This is seen clearly in figure 2.

We find that for such varying bag constant and magnetic field, the change in the curves from the non-magnetic case is maximum. There is considerable change in the stiffness of the curves and also change in the mixed phase region. Towards the centre, the magnetic

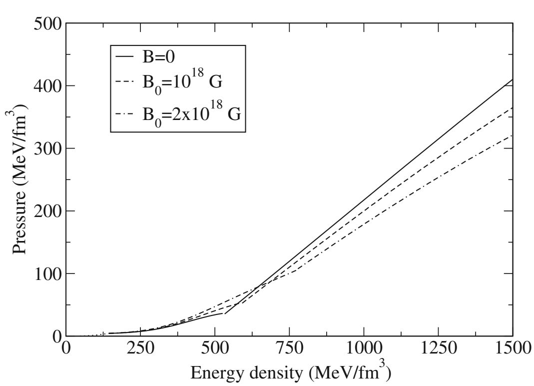

Figure 1. Pressure vs. energy density plot for a density-dependent bag pressure of 170 MeV, without magnetic field and with two different magnetic fields, having $\alpha = 0.01$.

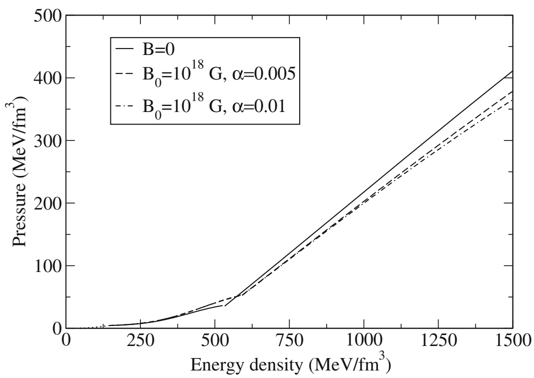

Figure 2. Pressure vs. energy density plot for a density-dependent bag pressure of 170 MeV, without magnetic field and with same magnetic field but different α values.

field increases whereas the bag pressure decreases. On the one hand, the low bag pressure makes the curve stiffer and on the other hand large magnetic field strength makes the curve softer. The low bag constant makes the mixed phase region to shrink, and the larger magnetic field tries to expand the mixed phase region. The effect of bag pressure is greater than the magnetic field and therefore, the mixed phase is smaller than the constant bag pressure case. On the low density side, the effect of magnetic field is insignificant. Therefore, the phase boundary between the nuclear and mixed phases is not much affected.

For a varying bag constant we can have a significant mixed phase region with $B_{\mathrm{g}} = 160$ MeV (figure 3). The curve with 160 MeV bag pressure is stiffer than other curves. This is because the bag pressure of $B_{\mathrm{g}} = 160$ MeV is lower than other higher bag pressure. Therefore, the effective matter pressure for this curve is higher than any other curve, which is reflected in the stiffness of the curve. For 160 MeV bag constants, the mixed phase region starts at $0.15\,\mathrm{fm}^{-3}$ density and ends at $0.38\,\mathrm{fm}^{-3}$.

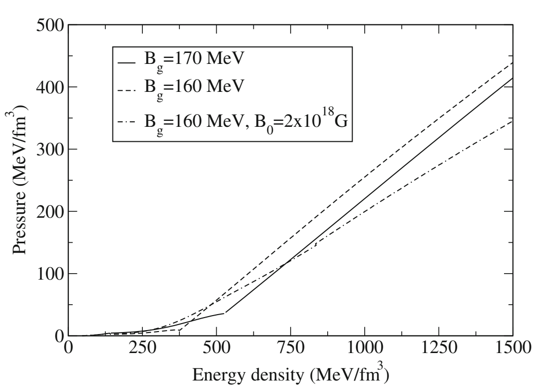

Figure 3. Pressure vs. energy density plot with two different density-dependent bag pressures 160, 170 MeV. We have also plotted the magnetic field-induced (field strength $B_0 = 2 \times 10^{18}$ G) EOS curve for 160MeV bag pressure having $\alpha = 0.01$.

Assuming the star to be non-rotating and is spherically symmetric, the distribution of mass is in hydrostatic equilibrium. The equilibrium configuration solutions are obtained by solving the Tolman–Oppenheimer–Volkoff (TOV) equations [26] for the pressure $P(\epsilon)$ and the enclosed mass m,

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\epsilon(r)}{r^2} \frac{\left[1 + P(r)/\epsilon(r)\right]\left[1 + 4\pi r^3 P(r)/m(r)\right]}{1 - 2Gm(r)/r}, \tag{16}$$

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \epsilon(r), \tag{17}$$

where G is the gravitational constant. Starting with a central energy density $\epsilon(r = 0) \equiv \epsilon_c$, we integrate until the pressure on the surface equals the one corresponding to the density of iron. This gives the stellar radius R and the total gravitational mass

$$M_G \equiv m(R) = 4\pi \int_0^R \mathrm{d}r\, r^2 \epsilon(r). \tag{18}$$

For the description of the NS crust, we have added the hadronic EOS with the ones by Negele and Vautherin [27] in the medium-density regime, and the one by Feynman–Metropolis–Teller [28] and Baym–Pethick–Sutherland [29] for the outer crust.

In figure 4, we plot curves with varying bag constant (170 MeV) for two different values of alpha (0.005 and 0.01), with field strength of $B_0 = 2 \times 10^{18}$ G. Both the magnetic field and bag pressure are density-dependent. The magnetic field makes the mass–radius curve flatter. The magnetic field variation becomes higher, increasing the magnetic field strength as we go inwards, thereby making the EOS flat. As the EOS becomes flat, the mass–radius curve also becomes flat, and the maximum mass decreases. To compare the mass dependence on varying magnetic fields and varying bag pressure, we have plotted curves for two different sets of curves with varying bag pressure 170 and 180 MeV (figure 5). Each set comprises two curves, one without magnetic field and the other with magnetic field, of strength $B_0 = 2 \times 10^{18}$ G. The qualitative nature of the curves remains same due to the reasons discussed earlier. As it has been pointed out, with

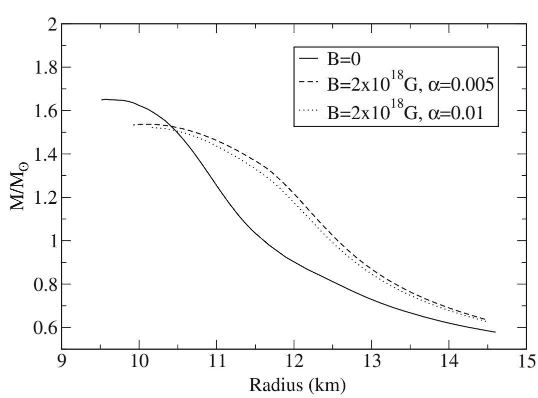

Figure 4. Gravitational mass (in solar mass) with radius having a density-dependent bag pressure of 170 MeV. Curves are plotted without magnetic field and with magnetic field, of field strength $B_0 = 2 \times 10^{18}$ G but different α values.

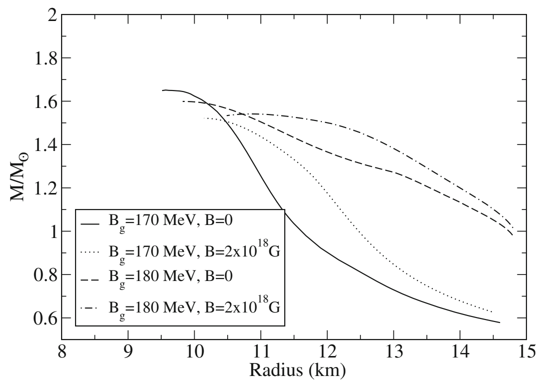

Figure 5. Gravitational mass (in solar mass) vs. radius with two different bag pressures of 170 and 180 MeV. The curves are plotted without magnetic field and with magnetic field, of strength $B_0 = 2 \times 10^{18}$ G having $\alpha = 0.01$.

varying bag constant and magnetic field we can have mixed phase EOS with 160 MeV bag pressure. In figure 6, we have plotted the mass–radius curve for $B_{\rm g} = 160$ MeV, with ($B_0 = 2 \times 10^{18}$ G) and without magnetic field. The magnetic field is varying having α of 0.01. The maximum mass for this case is obtained without the magnetic field effect and the introduction of the magnetic field makes the curve flatter and also reduces the maximum mass. The maximum mass of a mixed HS obtained with such mixed phase region is 1.84 $M_\odot$

The main objective of this paper is to show the effect of magnetic field on the mixed phase EOS and its effect on the maximum mass of a star. We are also interested in showing whether simple EOS (hyperonic nuclear and MIT bag quark) can reach the limit set by PSR J1614-2230. The other most interesting fact of this calculation is that the mixed HS has radius corresponding to the maximum mass, quite different from the nuclear and

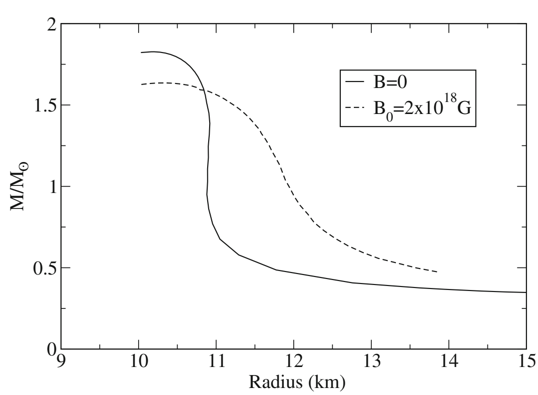

Figure 6. Gravitational mass (in solar mass) vs. radius with a density-dependent bag pressure of 160 MeV. Two curves are plotted, one without magnetic field and the other with magnetic field, having strength $B_0 = 2 \times 10^{18}$ G and $\alpha = 0.01$.

strange star. They are not as compact as SS and their radii lie between the nuclear star and SS. It is also clear from our calculation that, if the magnetic field influences the EOS only through the Landau quantization, it has a negative effect on the matter pressure thereby making the EOS softer, and the star becomes less massive.

6. Summary and discussion

To summarize, we have studied the effect of magnetic field on the nuclear and quark matter EOS. We have taken into account Landau quantization effect on the charged particles of both the EOS. We have considered relativistic mean-field EOS model for the nuclear matter EOS. For the quark matter EOS, we have considered simple MIT bag model with density-dependent bag constant. The nuclear matter EOS is much stiffer than the quark matter EOS, and so the effect of magnetic field is much more pronounced in the quark matter. The magnetic field due to Landau quantization softens the EOS for both the matter phases because magnetic pressure contributes negatively to the matter pressure. Here, we should mention that the effect of magnetization of matter is important for strong magnetic fields. However, it is believed that in NS such magnetization is small [8]. Therefore, in our calculation we have neglected this effect.

We have also considered varying the magnetic field. Observationally, the inferred surface magnetic field of a NS may be as high as 10^{15} G and is believed to increase at the centre. As the density decreases with increasing radial distance, we have taken the parametrization of the magnetic field as a function of density, having maximum field strength at the core. Considering density-dependent bag pressure and magnetic field, we construct mixed phase EOS following Glendenning construction.

We find that the effect of magnetic field is insignificant unless the surface field is of the order of 10^{14} G. Such constant magnetic field value has no effect on the nuclear matter EOS and has very little effect on the mixed and quark matter EOS. For a varying magnetic field whose surface value is 10^{14} G but whose central value is of the order of 10^{17} G, we find significant effect on the stiffness of the EOS and also on the extend of the mixed phase region in the EOS. As the bag pressure increases, the EOS for the quark phase becomes soft, and hence, more is the effect of magnetic field. At the central region, the bag pressure decreases but the magnetic field increases, and so their respective effect on the EOS acts in opposite directions.

The magnetic field increases as we go to much higher densities, and so the boundary between the mixed phase and the quark phase changes with increasing field strength. As the magnetic field increases, the EOS becomes less stiffer and the phase boundary between the mixed and quark phases shifts upwards to the higher density value. Towards the low-density regime of the curve, the effect of magnetic field is less pronounced, as the magnetic field strength is less and also the nuclear matter EOS is much stiffer. Therefore, the phase boundary between the nuclear and mixed phases is less affected.

The maximum mass limit of the mixed phase EOS star is also shown in this paper. We obtain a significant mixed phase region with a central bag constant of 160 MeV having *s*-quark mass of 150 MeV. For higher s-quark mass (300 MeV) we get a small mixed phase region with a bag pressure of 150 MeV. For such a case, we find that the maximum mass for a mixed hybrid star with the given set of EOS is $2.01 M_{\odot}$. The maximum mass is obtained without magnetic field effect and the introduction of the magnetic field reduces the maximum mass. Therefore, the mass limit set by the observation of pulsar PRS J1614-2230 is maintained by the mixed star with varying bag constants. Our calculation also shows that the mixed HS has radius (for the maximum mass) quite different from the neutron or strange star, their radius lying between the neutron and strange star.

The mass–radius relationship for a mixed HS is quite different from the pure neutron or strange star, and so it is likely to have different observational characteristics. It is also clear that magnetars are different from normal pulsars, as they have lesser mass due to flatter EOS. It is to be mentioned here that we have only considered the effect from Landau quantization and found that they have significant effect on the mixed phase region once it is greater than 10^{14} G.

As the interiors of the compact stars are hidden from direct observation, we have to rely only on the observations coming from their surface. Recent developments have been made on accurately measuring the mass of compact stars but an exact measurement of their radii is still not possible [1]. The knowledge of the radius of compact stars can give us the hint of matter components at the star interiors, as we have seen that different EOSs provide different mass–radius relationship.

Acknowledgement

K K Mohanta thanks the Institute of Physics, Bhubaneswar, India for the hospitality and for the facilities provided during the compilation of this work.

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