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Fission dynamics of hot nuclei

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Abstract. Experimental evidence accumulated during the last two decades indicates that the fission of excited heavy nuclei involves a dissipative dynamical process. We shall briefly review the relevant dynamical model, namely the Langevin equations for fission. Statistical model predictions using the Kramers' fission width will also be discussed.

Keywords. Fission; dissipation; Langevin equations; Kramers' fission width; pre-scission neutron multiplicity.

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1. Introduction

Since its discovery in 1939 [1,2], nuclear fission has been a rich source of information regarding the bulk properties of nuclei at extreme deformations. After the advent of heavy ion beams at energies above the entrance channel Coulomb barrier, fusion–fission reactions have further enriched our knowledge about the properties and dynamics of nuclei at high excitation energies. In what follows, we shall briefly review the statistical and the stochastic dynamical models of nuclear fission and discuss a few of their applications to explain the experimental data.

2. The transition state model of fission

N Bohr and J A Wheeler [3] presented a comprehensive theory of nuclear fission following closely its experimental discovery. Considering an ensemble of nuclei in equilibrium, the fission rate according to this theory, often referred to as the statistical model of nuclear fission, is determined by the number of nuclei moving outward at the saddle configuration ('transition state'). The fission probability in the transition state model is obtained as follows [3]. Consider a microcanonical ensemble of nuclei with intrinsic excitation energies between E^* and $E^*+\delta E^*$. Then, the number of quantum states between the above energy interval is given by $\rho(E^*)\delta E^*$ where $\rho(E^*)$ is the density of states at the ground-state configuration. The ensemble is chosen in such a way that the number of nuclei is exactly equal to the number of levels in the selected energy interval and there is one nucleus in each state. Therefore, the number of nuclei which divide per unit time can be represented as

$$R = \frac{\Gamma_{\mathrm{BW}}}{\hbar}\rho(E^*)\delta E^*, \tag{1}$$

where Γ_{BW} is the fission width.

Again, the rate R can be expressed in the following way. The number of fission events is equal to the number of nuclei in the 'transition state' which pass outward over the fission barrier. Now, in a unit distance measured in the direction of fission, there are $(\mathrm{d}p/h)\rho^*(E^* - V_{\mathrm{B}} - \epsilon)\delta E^*$ number of quantum states for which the momentum associated with the fission distortion lies in the interval $(p,\ p + \mathrm{d}p)$ and the kinetic energy is ϵ. The density of states ρ^* is different from ρ in a sense that it does not contain the degree of freedom associated with the fission itself. Therefore, the number of nuclei crossing the saddle point per unit time in the momentum interval $(p,\ p + \mathrm{d}p)$ is given by $v(\mathrm{d}p/h)\rho^*(E^* - V_{\mathrm{B}} - \epsilon)\delta E^*$, where v is the magnitude of speed of the fission distortion. Hence, the rate of fission events R can be written as

$$R = \delta E^* \int v(\mathrm{d}p/h)\rho^*(E^* - V_{\mathrm{B}} - \epsilon). \tag{2}$$

Now, comparing eq. (1) with the above expression, we get

$$\Gamma_{\mathrm{BW}} = \frac{1}{2\pi\rho(E^*)}\int_0^{E^*-V_{\mathrm{B}}} \mathrm{d}\epsilon\rho^*(E^* - V_{\mathrm{B}} - \epsilon), \tag{3}$$

where the relation: $v\mathrm{d}p = \mathrm{d}\epsilon$ is used. The above integration can be performed approximately [4] under the condition $E^* \gg V_{\mathrm{B}}$ to give

$$\Gamma_{\mathrm{BW}} = \frac{T}{2\pi}\exp\left(-V_{\mathrm{B}}/T\right), \tag{4}$$

where the temperature T is related to E^* through the Fermi gas model ($T = \sqrt{E^*/a}$). In 1973, Strutinsky [5] introduced a phase-space factor corresponding to the collective degrees of freedom in the ground-state region and consequently the above approximate form of the Bohr–Wheeler fission width becomes

$$\Gamma_{\mathrm{BW}} = \frac{\hbar\omega_{\mathrm{g}}}{2\pi}\mathrm{e}^{-V_{\mathrm{B}}/T}, \tag{5}$$

where ω_{g} is the frequency of a harmonic oscillator potential which represents the nuclear potential near the ground state (figure 1).

3. A dissipative dynamical model of nuclear fission

One crucial ingredient in the transition state model is the assumption of equilibration at each instant during the fission process. This assumption can be justified when the flux

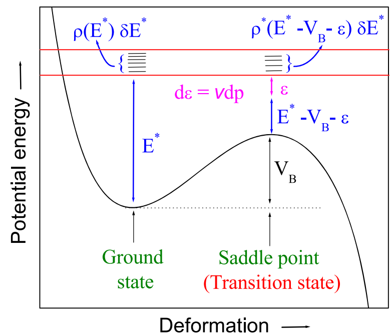

Figure 1. A schematic representation of the Bohr–Wheeler theory of fission.

across the fission barrier is very small or in other words, the fission barrier is much larger than the temperature. However, for systems with lower fission barrier, there may not be sufficient nuclei near the fission barrier after the initial crossings [6]. In order to maintain a steady flux across the barrier, it becomes essential to consider a dynamical model of fission.

Nuclear fission can be picturized as an evolution of the nuclear shape from a relatively compact mononucleus to a dinuclear configuration. In a macroscopic description of this shape evolution, the gross features of the fissioning nucleus can be described in terms of a small number of parameters, also called the collective degrees of freedom. The time development of these parameters is the result of a complicated interplay between various dynamical effects which are similar to that experienced by a massive Brownian particle floating in an equilibrated heat bath which is placed in a potential field. Here, the heat bath comprised of a large number of intrinsic degrees of freedom representing the rest of the nucleus and the potential energy is associated with a given shape of the nucleus. Moreover, the fission degrees of freedom are connected with the heat bath through dissipative interactions and, as a result, the shape evolution is both damped and diffusive. The diffusion happens essentially due to the fluctuating force exerted by the heat bath on the Brownian particle. In most cases, the inertia associated with the fission parameters are large enough so that their dynamics can be treated entirely by the laws of classical physics. The above scenario is illustrated schematically in figure 2.

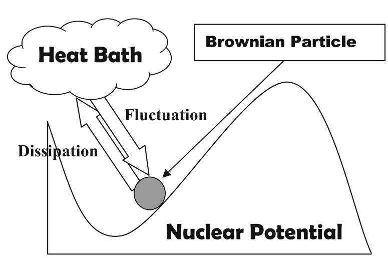

Figure 2. A schematic diagram for the dynamical model of fission.

3.1 *Langevin equation*

The Fokker–Planck and the Langevin equations are two equivalent prescriptions that can be used to describe the motion of a Brownian particle in a heat bath. Langevin approach was first proposed by Abe [7] as a phenomenological framework to portray the nuclear fission dynamics. It deals directly with the time evolution of the Brownian particle while the Fokker–Planck equation deals with the time evolution of the distribution function (in classical phase-space) of Brownian particles. Although the two approaches describe different aspects of the dynamics, they are equivalent with respect to their physical content. In the Langevin dynamical approach, the equation of motion of a Brownian particle is written as

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\vec{p}}{m}$$

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}(t) + \vec{H}(t), \tag{6}$$

where $\vec{F}(t)$ is the externally applied conservative force. It is related to the external potential field $V(\vec{r})$ through the relation $\vec{F}(t) = -\vec{\nabla}V(\vec{r})$. The non-conservative force $\vec{H}(t)$ describes the coupling of the collective motion with heat bath and is given by

$$\vec{H}(t) = -\frac{\eta}{m}\vec{p} + \vec{R}(t). \tag{7}$$

The above equation has two parts: a slowly varying part which describes the average effect of heat bath on the particle and is called the friction force ($\frac{\eta}{m}\vec{p}$) and the rapidly fluctuating part $\vec{R}(t)$ which has no precise functional dependence on t. Since it depends on the instantaneous effects of collisions of the Brownian particle with the molecules of the heat bath, $\vec{R}(t)$ is a random (stochastic) force and it is assumed to have a probability distribution with zero mean value. It is further assumed [8,9] that $\vec{R}(t)$ has an infinitely short time-correlation which means the process is Markovian. Therefore, $\vec{R}(t)$ is completely characterized by the following moments:

$$\langle R_i(t)\rangle = 0,$$

$$\langle R_i(t)R_j(t')\rangle = 2D\delta_{ij}\delta(t-t'), \tag{8}$$

where the suffix i denotes the ith component of the vector $\vec{R}(t)$. D is the diffusion coefficient which is related to the friction coefficient η and the temperature of the heat bath T by the Einstein's fluctuation–dissipation theorem:

$$D = \eta T. \tag{9}$$

It should be noted that, the Langevin equations are different from ordinary differential equations as it contains a stochastic term $\vec{R}(t)$. In order to calculate physical quantities such as the mean values or the distributions of observables from such a stochastic equation, one has to deal with a sufficiently large ensemble of trajectories for a true realization of the stochastic force. The physical description of the Brownian motion is therefore contained in a large number of stochastic trajectories rather than in a single trajectory, as would be the case for the solution of a deterministic equation of motion.

3.2 *Fokker–Planck equation*

The Fokker–Planck equation which is an alternative description of the Brownian motion can be derived from the Langevin equations by constructing analytically the distribution function of the Langevin trajectories in phase space. In 1940, Kramers [6] solved the one-dimensional Fokker–Planck equation to get the stationary current of the Brownian particles over a potential barrier described by two harmonic oscillators. The steady-state Fokker–Planck equation in one dimension is given as

$$p\frac{\partial\rho}{\partial q} - \frac{\mathrm{d}V}{\mathrm{d}q}\frac{\partial\rho}{\partial p} = \eta\frac{\partial\,(p\rho)}{\partial p} + \eta T\frac{\partial^2\rho}{\partial p^2}, \tag{10}$$

where ρ is the distribution function of the ensemble and (q, p) are the collective fission coordinates. The fission width is subsequently obtained as

$$\Gamma_{\mathrm{K}} = \frac{\hbar\omega_{\mathrm{g}}}{2\pi}\mathrm{e}^{-V_{\mathrm{B}}/T}\left\{\sqrt{1+\left(\frac{\eta}{2m\omega_{\mathrm{s}}}\right)^2} - \frac{\eta}{2m\omega_{\mathrm{s}}}\right\}, \tag{11}$$

where ω_{s} denotes the frequency of an inverted harmonic oscillator potential which represents the nuclear potential in the saddle region. The above equation, often referred to as the Kramers' fission width, can be used as the fission width in the dissipative decay of an excited compound nucleus. In the above derivation, it is assumed that the inertia of the collective motion and the dissipation strength are independent of the dynamical variable q. The Kramers' fission width is calculated from eq. (11) using the values of ω_{g}, ω_{s}, V_{B}, η and m given by the different models of the nuclear potential, dissipation and inertia, a brief description of which is given in §6.

4. Decay of a compound nucleus

4.1 *Statistical model calculation*

In the statistical model calculation, evaporation of neutrons, protons, α-particles and statistical giant dipole resonance (GDR) γ-rays are generally considered as the decay channels of an excited compound nucleus (CN) in addition to fission. The particle and GDR γ partial decay widths are obtained from the standard Weisskopf formula [10]. A time-dependent fission width is used in order to account for the transient time, which elapses before the stationary value of the Kramers' modified width is reached [11]. A parametrized form of the dynamical fission width is given as [12]

$$\Gamma_{\mathrm{f}}(t) = \Gamma_{\mathrm{K}}\left[1 - \exp(-2.3t/\tau_{\mathrm{f}})\right], \tag{12}$$

where

$$\tau_{\mathrm{f}} = \frac{\beta}{2\omega_{\mathrm{g}}^2}\ln\left(\frac{10V_{\mathrm{B}}}{T}\right)$$

is the transient time period, $\beta = \eta/m$ is the reduced dissipation coefficient and V_{B} is the spin-dependent fission barrier.

The fate of a compound nucleus is decided by the relative magnitudes of the various decay channels in the statistical model. After each particle or γ emission, the residual nucleus is appropriately redefined and its excitation energy and angular momentum are adjusted. The process continues till either the compound nucleus undergoes fission or an evaporation residue is formed. For a fission event, further emissions can take place during transition from saddle to scission, which would contribute to the pre-scission multiplicities. The saddle-to-scission time interval is given as [13]

$$\tau_{\rm ss} = \tau_{\rm ss}^0 \left\{ \sqrt{1 + \left(\frac{\beta}{2\omega_{\rm s}}\right)^2} + \frac{\beta}{2\omega_{\rm s}} \right\}, \tag{13}$$

where $\tau_{\rm ss}^0$ is the non-dissipative saddle-to-scission time interval and its value is taken from [14]. Thus evaporation processes during this time interval are also considered. In the end, one obtains various observables such as particle or γ multiplicities and fission and evaporation residue cross-sections. The multiplicity of neutrons emitted from the fission fragments can be obtained similarly by assuming symmetric fission.

4.2 *Langevin dynamical model calculation*

The Langevin dynamical model calculation involves numerical integration of eqs (6) over small time steps [8]. The initial fission coordinates and momenta are obtained from sampling the appropriate Boltzmann distribution. A typical Langevin trajectory in two dimensions is shown in figure 3 [15]. Following the time evolution of a Langevin trajectory, the fate of the CN at each time step is decided by a Monte Carlo sampling using the particle and γ widths. In the event of a particle or γ emission, the residual nucleus is appropriately redefined and its excitation energy and angular momentum adjusted through another Monte Carlo sampling procedure. This process continues till either the Langevin trajectory reaches the scission configuration (fission event) or an evaporation residue is formed. The final observables are obtained as an average of the outcome of a large number of Langevin trajectories.

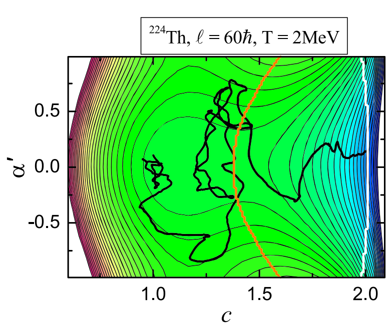

Figure 3. A typical Langevin trajectory (zig-zag line) going to fission. The contours are equipotential lines in elongation (c) and asymmetry (α') coordinates.

5. A few statistical and dynamical model results

We show the results of a statistical model calculation [16] in figure 4 where neutron multiplicities are obtained using the Bohr–Wheeler fission width as given by eq. (5). It is observed that the pre-scission multiplicities are grossly underestimated. However, the total number of neutrons representing energy balance is fairly independent of the fission width as is evident from figure 4. The above observation indicates that the fission time-scale is much larger than that given by the Bohr–Wheeler width in order to allow for a longer time interval for evaporating a larger number of neutrons before fission. The slowing down of the fission process points to a dissipative fission dynamics which can be modelled by either considering the Kramers' fission width (eq. (11)) in a statistical model of compound nuclear decay or using the Langevin equations for the fission dynamics, as described in the preceding sections.

The pre-scission neutron multiplicities calculated by both the statistical and dynamical models [17] are shown in figure 5. Here, the statistical model calculations are first performed by treating β as a free parameter in order to fit the experimental data. Dynamical model calculations are performed next using the best-fit β value obtained from the statistical model calculations and the results are given in figure 5. The calculated values are similar, as they are obtained under similar assumptions. Both the statistical and dynamical models are used to analyse the fusion–fission experimental data.

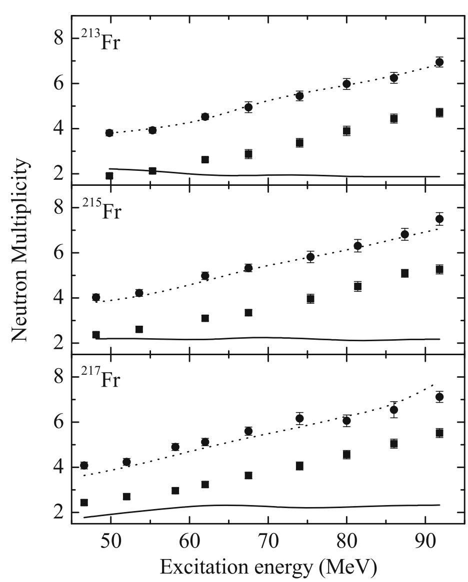

Figure 4. Excitation functions of pre-scission and total (pre-scission + post-scission) neutron multiplicities for different compound nuclei formed in $^{19}F+^{194,196,198}Pt$ reactions. The experimental values for pre-scission and total multiplicities are denoted by the filled squares and filled circles, respectively. Statistical model results obtained with the Bohr–Wheeler width are shown by solid lines and dashed lines for pre-scission and total multiplicities, respectively.

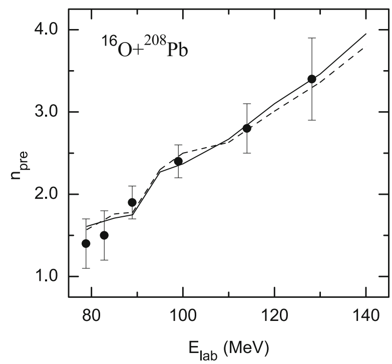

Figure 5. Pre-scission neutron multiplicities from statistical (dashed line) and dynamical (solid line) model calculations with a shape-independent dissipation $\beta = 3.5$ MeV/$\hbar$. Experimental points are from [18].

There are a number of distinguishing features between the statistical model of compound nuclear decay, incorporating the Kramers' fission width and the Langevin dynamical model of fission coupling the particle and the γ decay channels though, both the approaches take into account a dissipative dynamics of fission. The main difference arise from the fact that the Kramers' width is obtained from the Fokker–Planck equation after making a number of simplifying assumptions. A constant or shape-independent inertia and dissipation are assumed in Kramers' fission width though both of them are known to be shape-dependent [17,19]. On the other hand, shape-dependent inertia and dissipation can be easily used in the Langevin equations. Further, the Langevin equations describe the full time evolution of a CN, starting from its formation and the initial transients followed by the establishment of a steady flow across the barrier and finally reaching the scission configuration for a fission event. The Kramers' width however accounts for only the rate of steady flow across the barrier. Thus, effects due to transients and saddle-to-scission dynamics are not included in the Kramers' width and further approximations (eqs (12) and (13)) are to be made to take them into account. It may also be noted that though the Kramers' width, originally derived for collective motion in one dimension was later extended to multi-dimension [20], Langevin equations are more suited to calculate observables such as fission fragment mass and kinetic energy distributions due to its exact treatment of the saddle-to-scission dynamics.

6. What we learn from fission dynamics?

Nuclear fission is a unique process manifesting the interaction between the intrinsic and the collectives coordinates where large-scale collective motion is involved. The dynamics of nuclear fission depends on a number of nuclear properties, namely the potential energy, collective inertia and the dissipation coefficient. They represent the different orders of coupling between the intrinsic and the collective coordinates of the compound nucleus. While the potential corresponds to coupling of order zero (in speed), the dissipation and inertia account for couplings of order one and two, respectively. The nuclear density of states is also an important ingredient since it decides the nuclear temperature.

When discussing fission dynamics, it is necessary to choose a set of collective coordinates. This choice is mainly intutive and is guided by its success in explaining the relevant experimental data. For example, one may choose the following collective coordinates: elongation, mass asymmetry, neck, and the deformation parameters of the two fragments [21]. It therefore becomes necessary to know the above nuclear collective properties, namely the potential, inertia, dissipation and the density of states as functions of nuclear deformation and temperature. In other words, nuclear fission is the ideal testing ground for theoretical models for the above quantities.

6.1 *Potential*

Among the various approaches to calculate nuclear potential, the finite-range liquid drop model has been extensivly used for fission [22]. Present day efforts are centred around obtaining the nuclear potentials where shell effects are included with the aim of using the calculated potential over the entire deformation landscape to study dynamics of fission [23,24].

6.2 *Inertia*

In order to define the collective inertia, we consider the nucleus as a homogeneous fluid and write the total kinetic energy of the system in terms of the velocity field $\vec{v}$ as

$$T = \frac{1}{2}\rho_{\rm m} \int v^2 {\rm d}^3 r, \tag{14}$$

where $\rho_{\rm m}$ is the constant mass density and the integration is over the volume of the nucleus. Now, if the position vector $\vec{r}$ of a fluid element depends only on the collective coordinates $\{q_1, q_2, ..., q_k\}$, then $\vec{v} = \sum(\partial\vec{r}/\partial q_i)\dot{q}_i$, where $\dot{q}_i^{'}$ s are the generalized velocities and the kinetic energy T can be written as

$$T = \frac{1}{2}\sum m_{ij}(\vec{q})\dot{q}_i\dot{q}_j, \tag{15}$$

which defines the inertia tensor m_{ij}. Usually, the collective inertia is calculated by further making the Werner–Wheeler approximation for incompressible irrotational flow [25,26]. For the one-dimensional calculation, with c as the collective coordinate, m is a scaler quantity which may be denoted as $m(c)$. A plot of $m(c)$ for the ^{208}Pb nucleus is given in figure 6. The effect of shape-dependence of inertia on fission width has been investigated in [19]. The collective inertia can also be obtained from microscopic approaches such as the linear response theory [27]. Detailed investigations regarding the consequences of using inertia from microscopic models in fission dynamics are yet to be made.

6.3 *Dissipation*

Dissipation in nuclear bulk dynamics at energies below the Fermi energy domain is usually considered to be a one-body (nucleon–nucleus) rather than a two-body (nucleon–nucleon) phenomenon [28]. The one-body theories of nuclear dissipation mainly consider

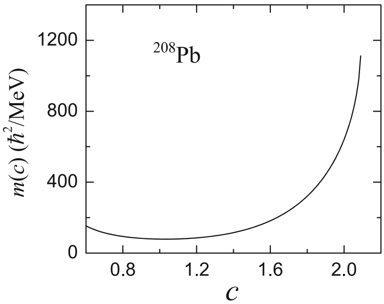

Figure 6. The collective inertia calculated using the Werner–Wheeler approximation.

either the 'wall' or the 'window' mechanism. In the so-called 'wall formula' for one-body dissipation, the damping of collective motion is obtained from the incoherent scattering of nucleons with the moving boundary of the mean-field [29–32]. Exchange of nucleons through the neck in a dinuclear complex is considered to be responsible for dissipation in the window mechanism [29]. In addition, another component of one-body dissipation is also considered which arises due to the mass asymmetry current between the two lobes of an asymmetric dinuclear system [33]. The total dissipation is the sum of all these components and has the general features as shown in figure 7.

An additional feature was later introduced in the wall formula (WF) in order to take into account the degree of irreversibility of energy transfer from the collective motion to nuclear excitation [34,35]. One major assumption of the WF concerns the Markovian nature of the fission process. This implies a fully chaotic nature of the intrinsic motion

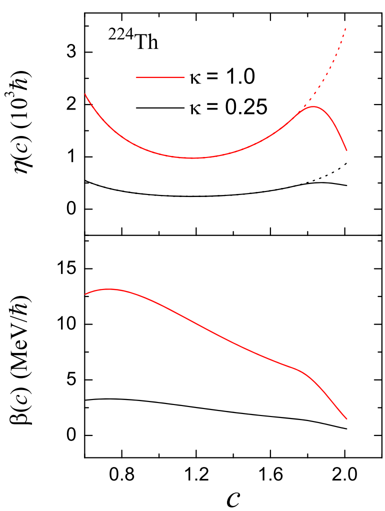

Figure 7. The dissipation coefficient ($\eta(c)$) and the reduced dissipation coefficient ($\beta(c) = \eta(c)/m(c)$) as a function of c. The dotted lines in the upper panel indicate the 'wall'contribution. κ is a reduction factor often required for the wall dissipation for fitting experimental data.

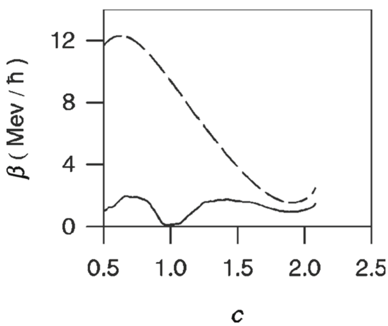

Figure 8. Variation of the reduced dissipation coefficient β with deformation for the CWWF (full line) and WF (dashed line) dissipations.

which makes the energy transfer irreversible. However, the chaotic nature of the intrinsic single-particle motion depends on the symmetrical properties of the mean field and it was shown that a measure of chaos can be introduced in the WF to give rise to the so-called 'chaos weighted wall formula' (CWWF) [34],

$$\eta_{\mathrm{CWWF}} = \mu(\mathrm{shape})\eta_{\mathrm{WF}}, \tag{16}$$

where μ is the chaos factor which depends upon nuclear shape and varies between 0 (spherical) and 1 (for strongly deformed shapes). The chaos factor is obtained from the time dependence of the Lyapunov exponents of single-particle classical trajectories in cavities of various shapes. The chaos factor thus provides a natural explanation for the empirical findings of the need to reduce the strength of the WF to fit the experimental data [36]. Figure 8 compares the dissipation strengths from CWWF and WF [37]. In fact, pre-scission neutron multiplicities obtained from Langevin dynamical calculations using CWWF are found to be closer to the experimental data compared to those obtained from WF [38]. The effect of shape dependence of dissipation on fission width has also been investigated [17]. In statistical model calculations, however, the dissipation strength is usually treated as a shape-independent parameter. The calculated values of pre-scission neutron multiplicity are found to be sensitive to the dissipation strength (e.g., see [39]).

7. Outlook

Though considerable progress has been achieved in understanding the dynamical features of nuclear fission, a vast uncharted ground still remains to be explored. Rigorous testing of the theoretical inputs from microscopic models are yet to be undertaken. Extensive calculations employing the Langevin dynamical model and involving a large number of collective coordinates are essential for an unambiguous mapping of collective nuclear properties in the deformation landscape, as well as, for finding their dependence on shell structure and temperature. Experimental data of fusion–fission experiments using unstable beams are expected to unravell the role of isospin degree of freedom in fission dynamics. This knowledge is crucial for synthesis of superheavy elements as well as for understanding the r-process nucleosynthesis, where fission plays the role of a limiting factor.

The present paper has covered only a few chosen topics and by no means is intended to be an exhaustive review. Evidently, the outlook of the field is much broader than that given in the preceding sections.

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