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# The effect of nonlinearity in relativistic nucleon–nucleon potential

B B SAHU[1,*], S K SINGH[2], M BHUYAN[2] and S K PATRA[2]

[1]Department of Physics, School of Applied Sciences, Kalinga Institute of Industrial Technology (KIIT) University, Bhubaneswar 751 024, India
[2]Institute of Physics, Sachivalaya Marg, Bhubaneswar 751 005, India
*Corresponding author. E-mail: bbsnou@gmail.com

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**Abstract.** A simple form for nucleon–nucleon (*NN*) potential is introduced as an alternative to the popular M3Y form using the relativistic mean field theory (RMFT) with the non-linear terms in $\sigma$-meson for the first time. In contrast to the M3Y form, the new interaction becomes exactly zero at a finite distance and the expressions are analogous with the M3Y terms. Further, its applicability is examined by the study of proton and cluster radioactivity by folding it with the RMFT-densities of the cluster and daughter nuclei to obtain the optical potential in the region of proton-rich nuclides just above the double magic core $^{100}$Sn. The results obtained were found comparable with the widely used M3Y *NN* interactions.

**Keywords.** Relativistic mean field; microscopic *NN* interaction; proton and alpha radioactivity.

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## 1. Introduction

In the nucleonic regime, nuclei behave as sets of interacting nucleons. In order to go beyond some basic nuclear models that provide a global description of the system, one has to include in the picture the elementary interaction between nucleons. One can then explore how the average potential well, in which nucleons evolve, can be built up from this elementary stone and thus gain a more microscopic picture of nuclei as constructed from nucleons. A key idea on which most of the theoretical machinery is founded is the concept of nuclear mean field, which basically relies on the fact that nucleons move quasi-independently from one to another inside a nucleus. Although the mean field underlies many of our discussions, one should not forget the elementary nucleon–nucleon interaction from which it is built. Here, our aim is not to discuss all the works that have been devoted to the *NN* interaction. We thus only recall the shape of the interaction with a few gross properties. We content ourselves by noting that the dominant part of the interaction is central and is strongly repulsive at short range ($\leq$ 0.4 fm, hard core) and attractive at intermediate range ($\sim$1–1.2 fm). This dominant repulsive and attractive shape of the interaction is the typical widely used, well-known M3Y *NN* interaction [1]. The *NN* interaction cannot yet be derived from first principle (QCD). The existing potentials are thus, at least partly, phenomenological and contain a possible large number of parameters and are fitted to deuteron properties and available phase shifts. This fitting procedure does not necessarily ensure a proper reproduction of many-body properties. So, for the first time, we try to give an *NN* interaction analogous to M3Y form derived from the relativistic mean-field (RMF) theory, which leads to an overall agreement with incompressibility and some radioactive properties of proton drip-line nuclei and superheavy region.

The attractive long-range part of the *NN* interaction has long been known to correspond to pion exchange, the $\rho$ and $\omega$ correspond to the shorter range part, etc. But the complex, multimeson contributions are furthermore simulated by effective mesons, such as $\sigma$-meson along with non-linear terms, leading to an overall simple form for the interaction analogous to the widely used M3Y form. Nevertheless, the short-range effects (hard core) have yet to be better understood and properly linked to quark degrees of freedom. It is relevant to mention here that the simplified spin and isospin-independent ($S$=$T$=0) M3Y effective *NN* interaction has been widely used successfully in a number of applications [2–4], whereas effective *NN* interaction is $S$-(and $T$-) dependent [5,6] and generally carries three components as

$$v_{\text{eff}} = V^{\text{C}}(r) + V^{\text{LS}}(r)\vec{L}\cdot\vec{S} + V^{\text{T}}(r)\hat{S}_{12}, \tag{1}$$

where $r$ is the relative distance and $\vec{L}\cdot\vec{S}$ and $\hat{S}_{12}$ are the usual spin–orbit and tensor operators, respectively. The central component [5] is

$$\begin{aligned} V^{\text{C}}(r) &= V_0(r) + V_\sigma(r)\sigma_1\cdot\sigma_2 + V_\tau(r)\tau_1\cdot\tau_2 \\ &\quad + V_{\sigma\tau}(r)(\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2), \end{aligned} \tag{2}$$

with radial and spin-, isospin-, spin–isospin-dependent parts, respectively.

In this paper, rather than using a simple phenomenological prescription [1], we derive the microscopic *NN* interaction from the RMF theory Lagrangian. It is to be noted that in our recently published paper [7] an attempt has been made to simulate the M3Y *NN* interaction from a simple Lagrangian [8,9]. However, the value of incompressibility obtained is quite large, about 600 MeV (though it is difficult to determine empirically, in fact, it is about 200 MeV [10]). Later on, its application to finite nuclei [11] shows that the results also deviate far from the experiment. To overcome the above-mentioned difficulties, we take the Lagrangian of Boguta and Bodmer [12], who have for the first time included the cubic and quartic terms in the scalar field. Boguta and Bodmer [12] studied the empirical properties of nuclear matter and finite nuclei without abnormal solution involving the non-linear terms in the original linear $\sigma$–$\omega$ model of Miller and Green [8] in 1977. Later on, this Lagrangian became extremely successful for both finite as well as infinite nuclear matter [13,14]. Therefore, it is interesting to find an *NN* interaction from this Lagrangian that can simulate the form of M3Y or R3Y, which was attempted in our earlier paper [7]. We employ it here for the study of proton and cluster radioactive decays and compare our results with those based on the phenomenological M3Y-effective *NN* interaction.

## 2. Theoretical framework

### 2.1 *The relativistic mean-field (RMF) theory and the microscopic NN interaction*

The non-linear, relativistic mean-field Lagrangian density for a nucleon–meson many-body system [12,15–17] is

$$
\begin{aligned}
L = {} & \bar{\psi}_i\{i\gamma^{\mu}\partial_{\mu} - M\}\psi_i + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma \\
& -\frac{1}{2}m_{\sigma}^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& -g_{\sigma}\bar{\psi}_i\psi_i\sigma - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_w^2 V^{\mu}V_{\mu} \\
& -g_w\bar{\psi}_i\gamma^{\mu}\psi_i V_{\mu} - \frac{1}{4}\vec{B}^{\mu\nu}\cdot\vec{B}_{\mu\nu} + \frac{1}{2}m_{\rho}^2\vec{R}^{\mu}\cdot\vec{R}_{\mu} \\
& -g_{\rho}\bar{\psi}_i\gamma^{\mu}\vec{\tau}\psi_i\cdot\vec{R}^{\mu} - \frac{1}{2}m_{\delta}^2\delta^2 + g_{\delta}\bar{\psi}_i\delta\vec{\tau}\psi_i,
\end{aligned}
\tag{3}
$$

where the field for $\sigma$-meson is denoted by $\sigma$, that for $\omega$-meson by $V_{\mu}$ and that for the isovector $\rho$ by $\vec{R}_{\mu}$, respectively. The $\psi_i$ are the Dirac spinors for the nucleons. An iso-spin is denoted by $\tau$. Here $g_{\sigma}$, $g_{\omega}$, $g_{\rho}$ and $g_{\delta}$ are the coupling constants for $\sigma$, $\omega$, $\rho$ and $\delta$ mesons, respectively. $M$, $m_{\sigma}$, $m_{\omega}$, $m_{\rho}$ and $m_{\delta}$ are the masses of the nucleons, $\sigma$, $\omega$, $\rho$ and $\delta$ mesons, respectively. $\Omega^{\mu\nu}$ and $\vec{B}_{\mu\nu}$ are the field tensors for $V^{\mu}$ and $\vec{R}_{\mu}$, respectively. In this Lagrangian, the contribution of $\pi$ meson has not been taken into account as, at the mean-field level, its contribution is zero due to its pseudoscalar nature [18,19]. It is essential for quantitative discussions to introduce the self-coupling terms with the coupling constants $g_2$ and $g_3$ for the $\sigma$ meson. The coupling strengths, $g$s, and the meson masses, $m$s, are the parameters of this theory.

We solve the nuclear system under the mean-field approximation using the above Lagrangian and obtain the field equations for the nucleons and mesons as

$$\left(-i\alpha.\nabla + \beta(M + g_{\sigma}\sigma) + g_{\omega}\omega + g_{\rho}\tau_3\rho_3 + g_{\delta}\delta\tau\right)\psi_i = \epsilon_i\psi_i, \tag{4}$$

$$(-\nabla^2 + m_{\sigma}^2)\sigma(r) = -g_{\sigma}\rho_s(r) - g_2\sigma^2 - g_3\sigma^3, \tag{5}$$

$$(-\nabla^2 + m_{\omega}^2)V(r) = g_{\omega}\rho(r), \tag{6}$$

$$(-\nabla^2 + m_{\rho}^2)\rho(r) = g_{\rho}\rho_3(r), \tag{7}$$

$$(-\nabla^2 + m_{\delta}^2)\delta(r) = -g_{\delta}\rho_3(r), \tag{8}$$

respectively, for Dirac nucleons, $\sigma$, $\omega$, $\rho$, $\delta$ mesons.

The interaction between a pair of nucleons when they are embedded in a heavy nucleus is less than when they are in empty space. This suppression of the two-body interactions within a nucleus in favour of the interaction of each nucleon with the average nucleon density means that the non-linearity acts as a smoothing mechanism and hence leads in the direction of the one-body potential and shell structure [20]. Here we deal with the non-linearity in the meson field. This non-linearity can take any form as it is devoted to the neutral scalar meson theory in which the non-linearity corresponds to a point-contact repulsion between mesons [21]. So we take opposite sign to the source term for $\sigma^3$ and $\sigma^4$ terms first by using only classical field theory, and second by choosing the mesons to be of the neutral scalar type. A positive term proportional to $\sigma^4$ must be added to the Hamiltonian density and $\sigma^3$ term to the wave equation. This seems a simple and natural form to use, but it brings a serious problem into the analysis and interpretation of the formalism. So in a simple way, the solution for the second and third terms of eq. (5) is taken as [22]

$$V_\sigma(r) = +\frac{g_2^2}{4\pi}\frac{\mathrm{e}^{-2m_\sigma r}}{r^2}$$

and

$$V_\sigma(r) = +\frac{g_3^2}{4\pi}\frac{\mathrm{e}^{-3m_\sigma r}}{r^3}$$

to get a new *NN* interaction analogous to M3Y form in order to improve the incompressibility and the finite nuclei results, which was the deficiency in our earlier paper [7]. In addition to this, the self-coupling of the $\sigma$ meson (non-linear terms) helps to generate the repulsive part of the *NN* potential at a long distance to satisfy the saturation properties (Coester-band problem) [23]. Thus, we take into account the non-linear terms in $\sigma$ field and are able to obtain a similar type of potential with M3Y form. The resultant effective nucleon–nucleon interaction, obtained from the summation of the scalar and vector parts of the single meson fields, is then defined as [8,19,24]

$$\begin{aligned} v_{\mathrm{eff}}(r) &= V_\omega + V_\rho + V_\sigma + V_\delta \\ &= \frac{g_\omega^2}{4\pi}\frac{\mathrm{e}^{-m_\omega r}}{r} + \frac{g_\rho^2}{4\pi}\frac{\mathrm{e}^{-m_\rho r}}{r} - \frac{g_\sigma^2}{4\pi}\frac{\mathrm{e}^{-m_\sigma r}}{r} \\ &\quad + \frac{g_2^2}{4\pi}\frac{\mathrm{e}^{-2m_\sigma r}}{r^2} + \frac{g_3^2}{4\pi}\frac{\mathrm{e}^{-3m_\sigma r}}{r^3} - \frac{g_\delta^2}{4\pi}\frac{\mathrm{e}^{-m_\delta r}}{r}. \end{aligned} \tag{9}$$

For a normal nuclear medium, the contribution $V_\delta$ of the $\delta$ meson can be neglected, compared to the magnitudes of both $V_\omega$ and $V_\sigma$. Hence, eq. (9), with the single-nucleon exchange effects [1], becomes

$$\begin{aligned} v_{\mathrm{eff}}(r) &= \frac{g_\omega^2}{4\pi}\frac{\mathrm{e}^{-m_\omega r}}{r} + \frac{g_\rho^2}{4\pi}\frac{\mathrm{e}^{-m_\rho r}}{r} - \frac{g_\sigma^2}{4\pi}\frac{\mathrm{e}^{-m_\sigma r}}{r} \\ &\quad + \frac{g_2^2}{4\pi}\frac{\mathrm{e}^{-2m_\sigma r}}{r^2} + \frac{g_3^2}{4\pi}\frac{\mathrm{e}^{-3m_\sigma r}}{r^3} + J_{00}(E)\delta(s), \end{aligned} \tag{10}$$

where $J_{00}(E)\delta(s)$ is the zero-range pseudopotential representing EX [1,25] and is given by

$$J_{00} = -276(1 - 0.005E/A_{\mathrm{c}(\alpha)})\ \mathrm{MeV\ fm}^3. \tag{11}$$

Here, $A_{\mathrm{c}(\alpha)}$ is the cluster (or $\alpha$-particle) mass and $E$ the energy measured in the centre of mass of the cluster- or $\alpha$-daughter nucleus system, is equal to the released $Q$-value.

As illustrative cases, using in eq. (10), the HS parameters [11], we get

$$v_{\rm eff}(r) = 11957\frac{{\rm e}^{-3.97r}}{4r} + 4099\frac{{\rm e}^{-3.90r}}{4r} - 6883\frac{{\rm e}^{-2.64r}}{4r} + J_{00}(E)\delta(s), \qquad (12)$$

and for NL3 parameters [26], eq. (10) becomes

$$\begin{aligned} v_{\rm eff}(r) &= 10395\frac{{\rm e}^{-3.97r}}{4r} + 1257\frac{{\rm e}^{-3.87r}}{4r} - 6554\frac{{\rm e}^{-2.58r}}{4r} \\ &\quad +6830\frac{{\rm e}^{-5.15r}}{4r^2} + 52384\frac{{\rm e}^{-7.73r}}{4r^3} + J_{00}(E)\delta(s), \end{aligned} \qquad (13)$$

with the corresponding effective *NN* interaction potentials, denoted LR3Y(HS), NR3Y(NL3), etc. as shown in figure 1, together with other effective *NN* interaction potentials, like M3Y without the one-pion exchange potential (OPEP) term, is given by

$$v_{\rm eff}(r) = 7999\frac{{\rm e}^{-4r}}{4r} - 2134\frac{{\rm e}^{-2.5r}}{2.5r} + J_{00}(E)\delta(s), \qquad (14)$$

where ranges are in fm and the strength in MeV. This M3Y-effective interaction, obtained from a fit of the *G*-matrix elements based on Reid–Elliott soft-core *NN* interaction [1], in an oscillator basis, is the sum of three Yukawa's with ranges 0.25 fm for a medium-range attractive part, 0.4 fm for a short-range repulsive part and 1.414 fm to ensure a long-range tail of the OPEP. Note that eq. (13) represents the spin- and isospin-independent parts of the central component of the effective *NN* interaction (eqs (1) and (2)), and that the OPEP contribution is absent here. Comparing eqs (12) and (13) with (14), we find a very similar behaviour of the *NN* interactions derived from RMF theory in figure 1,

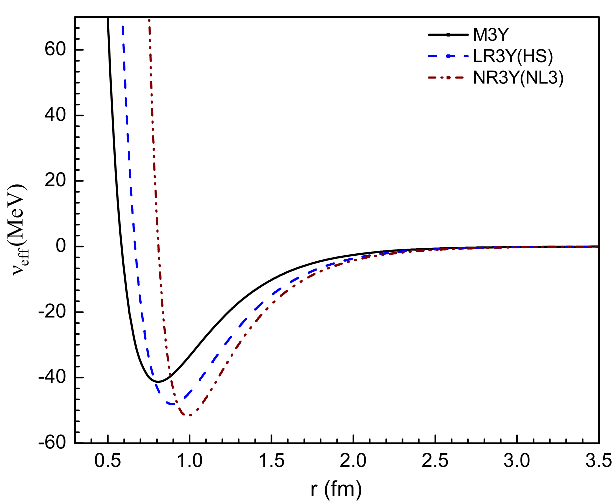


**Figure 1.** The NR3Y and the M3Y effective *NN* interaction potentials as a function of *r*.

which makes us believe that eq. (10) can be used to obtain the nucleus–nucleus optical potential. Using the optical potentials so obtained, we demonstrate in the next subsection, the applications of eqs (10) and (14) to various nuclear systems for evaluating some of the physical observables in the phenomenon of exotic proton and cluster radioactivity (CR).

### 2.2 *Optical potential and the half-lives study using the preformed cluster model (PCM)*

The nuclear interaction potential $V_{\mathrm{n}}(R)$ between the cluster (c) and daughter (d) nuclei, using the well-known double-folding procedure [1] and by single folding, with the respective RMF calculated nuclear matter densities $\rho_{\mathrm{c}}$ and $\rho_{\mathrm{d}}$ for M3Y forces is given as

$$V_{\mathrm{n}}(\vec{R}) = \int \rho_{\mathrm{c}}(\vec{r}_{\mathrm{c}})\rho_{\mathrm{d}}(\vec{r}_{\mathrm{d}})v_{\mathrm{eff}}(|\vec{r}_{\mathrm{c}} - \vec{r}_{\mathrm{d}} + \vec{R}| \equiv r)\mathrm{d}^3r_{\mathrm{c}}\mathrm{d}^3r_{\mathrm{d}} \tag{15}$$

and

$$V_{\mathrm{n}}(\vec{R}) = \int \rho_{\mathrm{d}}(\vec{r})v(|\vec{r} - \vec{R}|)\mathrm{d}^3r. \tag{16}$$

Adding Coulomb potential $V_{\mathrm{C}}(R)$ $(=Z_{\mathrm{d}}Z_{\mathrm{c}}\mathrm{e}^2/R)$ and centrifugal potential wherever necessary, the scattering potential is obtained as

$$V(R) = V_{\mathrm{N}}(R) + V_{\mathrm{C}}(R) + \frac{\hbar^2 L(L+1)}{2\mu R^2}, \tag{17}$$

where $R$ is the separation between the mass centre of the residual daughter nucleus and the emitted proton/cluster, $L$ is the angular momentum of the emitted proton in the case of proton radioactivity. The density distribution function $\rho$ has been calculated using RMFT formalism [11,18,27,28], in which an effective Lagrangian is taken to describe the interaction of nucleons through the effective meson and electromagnetic (E M) fields. The decay constant $\lambda$ or half-life time $T_{1/2}$ in the preformed cluster model (PCM) of Gupta and collaborators [29,30] is defined as

$$\lambda_{\mathrm{PCM}} = \frac{\ln 2}{T_{1/2}} = \nu_0 P_0 P, \tag{18}$$

with the 'assault frequency' $\nu_0$, i.e. the frequency with which the cluster hits the barrier, given by

$$\nu_0 = \frac{\text{velocity}}{R_0} = \frac{(2E_{\mathrm{c}}/\mu)^{1/2}}{R_0}. \tag{19}$$

Here $R_0$ is the radius of the parent nucleus and $E_{\mathrm{c}}$ is the kinetic energy of the emitted cluster. $P$ is the WKB penetration probability of the cluster tunnelling through the interaction potential $V(R)$ and is given by the WKB integral:

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_a}^{R_b} \{2\mu[V(R) - Q]\}^{1/2}\mathrm{d}R\right], \tag{20}$$

with $R_a$ and $R_b$ as the first and second turning points, satisfying $V(R_a) = V(R_b) = Q$. $\mu = A_d A_c/(A_d + A_c)$, the reduced mass, and $Q = BE_{\mathrm{p}} - (BE_{\mathrm{d}} + BE_{\mathrm{c}})$, where $BE_{\mathrm{p}}$,

$BE_c$ and $BE_d$ are the experimental ground state (g.s.) binding energies of the parent, cluster and daughter nuclei, taken from Audi and Wapstra [31]. We have also successfully demonstrated its application (with HS parameter set) to study the half-life of cluster decay [4], proton decay [32] and a recent study of the half-life of $\alpha$-decay [33] with the fusion cross-section of heavy-ion systems using the region-wise absorption method [34]. It is clearly seen from figure 4 of ref. [33] that the barrier (for $l = 0$) position and height play significant roles, not only in the study of fusion cross-sections of heavy nucleus but also in the study of half-life of proton decay [32] and $\alpha$-decay [33]. So to check the applicability of the present formalism we study the proton and cluster decay of heavy nuclei in the next section.

## 3. Results and discussions

The applicability of our new formalism is made for some highly unstable proton-rich trans-tin nuclei with the above-mentioned PCM of Gupta *et al*. Information on structure and nuclear mass of some exotic nuclei not yet experimentally accessible have also been obtained from the detailed analysis of the $p$, $\alpha$-cluster decay rates using the NL3 parameter set-based spherical, relativistic-mean-field densities. The region of nuclides just above $^{100}$Sn is of special interest for nuclear studies as it includes the heaviest $Z \cong N$ known nuclei, stable to the proton emission, and offers an unexpected richness and diversity of nuclear structures and new decay modes [37]. Nevertheless, our present formalism with the inclusion of non-linear terms in $\sigma$ meson show a good agreement with the experimental data and also compared with our earlier work with RMFT-HS densities given in table 1 and also shown in figure 2. We found that, although in many cases the NR3Y+EX is closer to the experimental value, in few of the cases the LR3Y+EX gives superier or comparable results. This implies the cluster decay property is less sensitive to the compressibility. Also, perhaps this value is indifferent to the detailed nuclear structure inheritated by the density while calculating the cluster decay property (mostly a surface phenomena). However, if one applies these folding potentials to some other nuclear phenomena where the structural property of the nuclei is important, the NR3Y+EX may work better. This is because of the high-quality predictive power of NL3 [26] over HS [11] throughout the periodic table. In addition to the shifting of barrier position and height, the effect of various model parameters cannot be neglected as one can observe from the fifth and seventh columns of table 1. We also study the sensitivity of half-lives to the orbital angular momentum L as we have clearly shown in figure 3 of ref. [32]. Here for $^{109}$I and $^{117}$La$^*$, we study the half-lives for different L and it is seen that NR3Y+EX *NN* interaction gives remarkably good result with the experiment; in fact, the $Q$ value is very compatible with the half-life.

Further, we apply the above formalism to study the $\alpha$-decay rates of some heavy nuclei and the summary is given in table 2. It is found that our NR3Y results are in good agreement with the widely used M3Y results. Whatever disagreement found with the experimental data may be due to excess number of valence nucleons. From the simple shell-model viewpoint, the $\alpha$-formation amplitude for these nuclei with many valence nucleons is extremely complicated [37], and good theoretical calculations are essentially impossible [38].

**Table 1.** The calculated half-lives of proton emitters are presented using M3Y+EX and NR3Y+EX *NN* interactions. The results of the present calculations have been compared with the experimental values and with the results of [35,36]. The experimental $Q$ values, half-lives and angular momentum (AM) $L$ values are taken from [35]. The asterisk symbol (∗) denotes the isomeric state.

| Parent nuclei | $Q$ (MeV) | AM $L$ | Expt. $\log_{10}T$(s) | (M3Y+EX) HS $\log_{10}T$(s) | (LR3Y+EX) HS $\log_{10}T$(s) | (M3Y+EX) NL3 $\log_{10}T$(s) | (NR3Y+EX) NL3 $\log_{10}T$(s) | [35] | [36] |
|---|---|---|---|---|---|---|---|---|---|
| $^{105}$Sb | 0.491 | 2 | 2.049 | 3.07 | 2.436 | 3.1 | 1.113 | 2.085 | 1.97 |
| $^{109}$I | 0.819 | 0 | −3.987 | −5.627 | −5.897 | −5.593 | −6.941 | – | – |
| | | 2 | | | | −5.522 | −3.666 | – | – |
| $^{112}$Cs | 0.814 | 2 | −3.301 | −2.857 | −3.555 | −2.835 | −4.705 | – | – |
| $^{113}$Cs | 0.973 | 2 | −4.777 | −5.236 | −5.803 | −5.204 | −7.017 | – | – |
| $^{117}$La | 0.803 | 2 | −1.628 | −1.943 | −2.504 | −1.922 | −3.878 | – | – |
| $^{117}$La* | 0.954 | 5 | −2.0 | 2.794 | 1.203 | – | −1.241 | – | – |
| | | 4 | | | | −0.226 | −3.266 | – | – |
| $^{131}$Eu | 0.940 | 2 | −1.749 | −2.097 | −2.764 | −2.085 | −4.256 | – | – |
| $^{140}$Ho | 1.094 | 3 | −2.221 | −1.374 | −2.132 | −1.376 | −4.007 | – | – |
| $^{141}$Ho | 1.177 | 3 | −2.387 | −2.487 | −3.298 | −2.468 | −5.038 | – | – |
| $^{141}$Ho* | 1.256 | 0 | −5.180 | −6.374 | −6.846 | −6.366 | −8.047 | – | – |
| $^{145}$Tm | 1.753 | 5 | −5.409 | −3.415 | −4.698 | −3.278 | −6.962 | −5.170 | −5.14 |
| $^{146}$Tm | 1.127 | 5 | −1.096 | 3.384 | 1.945 | 3.51 | −0.547 | – | – |
| $^{146}$Tm* | 1.307 | 5 | −0.698 | 0.919 | −0.484 | 1.043 | −2.870 | – | – |
| $^{147}$Tm | 1.071 | 5 | 0.591 | 4.191 | 2.775 | 4.369 | 0.315 | 1.095 | 0.98 |
| $^{147}$Tm* | 1.139 | 2 | −3.444 | −2.916 | −3.546 | −2.963 | −5.036 | −3.199 | −3.39 |

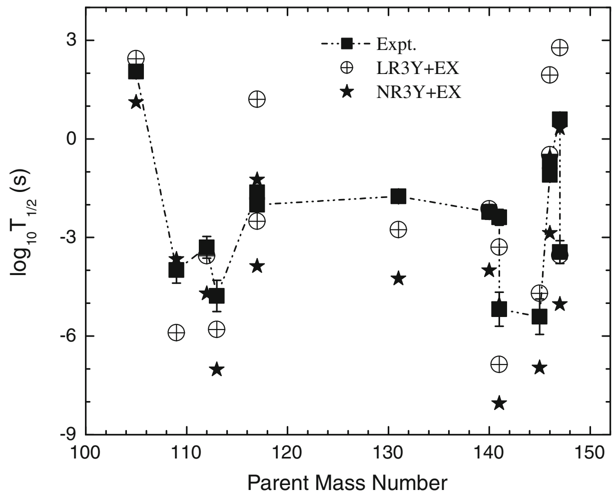


**Figure 2.** Half-lives for proton radioactivity of proton-rich parent nuclei. The present calculated results (NR3Y+EX, solid stars) agree quite well with the experimental data (solid squares) compared with the LR3Y+EX results.

**Table 2.** Summary of $\alpha$-half-lives calculations for the proton-rich trans-tin nuclei and the data taken from [37].

| Parent nuclei | $Q$ (MeV) | Expt. $\log_{10}T\,(s)$ | (M3Y+EX) $\log_{10}T\,(s)$ | (NR3Y+EX) $\log_{10}T\,(s)$ |
|---|---|---|---|---|
| $^{106}$Te | 4.320 | −4.30 | −5.779 | −9.858 |
| $^{108}$Te | 3.448 | 0.30 | −1.154 | −5.370 |
| $^{109}$Te | 3.214 | 2.06 | 0.401 | −3.846 |
| $^{110}$Te | 2.723 | 5.79 | 4.353 | 0.035 |
| $^{110}$I | 3.570 | 0.58 | −1.325 | −5.575 |
| $^{111}$I | 3.280 | 3.40 | 0.57 | −3.720 |
| $^{112}$I | 2.989 | 5.45 | 5.749 | −1.570 |
| $^{110}$Xe | 3.886 | −0.62 | −2.555 | −6.828 |
| $^{111}$Xe | 3.710 | −0.51 | −1.566 | −5.855 |
| $^{112}$Xe | 3.317 | 2.90 | 0.962 | −3.376 |
| $^{113}$Xe | 3.100 | 3.90 | 2.554 | −1.779 |
| $^{112}$Cs | 3.750 | | −1.209 | −5.554 |
| $^{113}$Cs | 3.474 | | 0.514 | −3.847 |
| $^{114}$Cs | 3.361 | 3.50 | 1.269 | −3.089 |
| $^{115}$Cs | 2.877 | | 5.104 | 0.683 |
| $^{113}$Ba | 3.902 | | −1.508 | −5.876 |
| $^{114}$Ba | 3.601 | 2.77 | 0.306 | −4.091 |
| $^{115}$Ba | 3.489 | | 1.023 | −3.370 |
| $^{116}$Ba | 3.004 | | 4.706 | 0.247 |

## 4. Summary and conclusions

In conclusion, the reported *NN* potential denoted here as NR3Y is presented eloquently in terms of the well-known inbuilt RMF parameters of $\sigma$, $\omega$ and $\rho$ meson fields, i.e. their masses ($m_\sigma$, $m_\omega$, $m_\rho$) and coupling constants ($g_\sigma$, $g_\omega$, $g_\rho$, $g_2$, $g_3$). Furthermore, in terms of the nucleus–nucleus folding optical potential, we have generated a bridge between the NR3Y and M3Y, which can be considered as a unification of the RMF model to predict the nuclear cluster decay properties. Hence we explain the proton and cluster decay properties of nuclei by using the RMF-derived NR3Y potential instead of the phenomenological M3Y interaction and found good agreement with the experimental data. In addition to this, to obtain the phenomenological incompresibility value of 200$\pm$30 MeV along with other basic reaction phenomena we have simply taken into account the non-linear terms in $\sigma$-meson coupling, which gives a new form of *NN* interaction alternate to the popular M3Y potential. While our method is not intended to provide descriptions of an *NN* data competitive with purely phenomenological models, the numerical results do encourage the use of this potential in the calculations of nuclear structure, nuclear matter, and few-nucleon systems at low energy.

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