PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 82, No. 2
February 2014
pp. 339–345

Impact of higher-order dispersion in the modulational instability spectrum of a relaxing coupled saturable media

K NITHYANANDAN[1,*], R VASANTHA JAYAKANTHA RAJA[2], T UTHAYAKUMAR[1] and K PORSEZIAN[1]

[1]Department of Physics, Pondicherry University, Puducherry 605 014, India
[2]School of Electrical and Electronics Engineering, SASTRA University, Thanjavur 613 401, India
*Corresponding author. E-mail: nithi.physics@gmail.com

DOI: 10.1007/s12043-014-0687-8; ***e*Publication:** 8 February 2014

Abstract. A theoretical analysis of modulational instability (MI) of optical pulses propagating near the zero dispersion wavelength of a lossless fibre with the effect of delayed saturable nonlinear response is presented. We calculate the exact dispersion relation with the effect of higher dispersion for the harmonic perturbation. We analysed the impact of fourth-order dispersion effects in the MI spectrum. We examine the possibility of MI in both dispersion regimes, regardless of the sign of the group velocity dispersion.

Keywords. Nonlinearity; modulational instability; optical fibre; four-wave mixing.

PACS Nos 42.65.−K; 42.79.Gn

1. Introduction

One of the most advanced frontiers of the nonlinear optics is modulation instability (MI). MI is a good old phenomenon featuring in most nonlinear wave systems in nature [1]. It is the process where weak perturbations, typically the noise imposed on a CW state, grow exponentially as a result of the interplay between nonlinear and dispersive effects. Although MI is known for more than a few decades, its applicability and versatility make it a subject of extensive research activity. The possibility of MI in optical fibre was introduced theoretically by Hasegawa and later experimentally realized by many groups. The perception of analysing MI takes different dimensions depending on the nonlinear contribution of the refractive index. The mechanism of MI is governed by the generalized nonlinear Schrödinger equation (GNLSE). Recently, much attention is paid to the influence of the relaxation of the nonlinear response and the saturable nonlinear (SNL) responses. The report by Liu *et al* introduced a new way of analysing the delay of the nonlinear response by a simple relaxational model [2]. Later, da Silva introduced the concept of saturable delayed nonlinear response [3,4]. It is evident from the literature that there exists another form of saturable nonlinear response called the coupled saturable nonlinear (CSN) response. In the context of MI, CSN in the presence of higher-order dispersion and the co-propagation of two beams with exponential nonlinearity has already been analysed by different groups. But there is no report till now about the combined action of CSN and delayed nonlinear response. Thus in this context, we would like to answer the question of what would happen to the MI spectrum for the combined action of delayed and saturable nonlinear response. Our analysis is two-fold: Phase I deals with MI dynamics for the pumping at the anomalous GVD regime and phase II features the more practicable case of optical fibre, where the propagation is around the zero dispersion wavelength (ZDW) in which higher-order dispersion plays a decisive role.

2. Theoretical model

In the regime of ultrashort pulses, the pulse propagation in nonlinear medium needs to include the finite response time of the nonlinearity. The delay in the saturable medium is given by the simple relaxation model as follows:

$$\frac{\partial E}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} + i\frac{\beta_3}{6}\frac{\partial^3 E}{\partial t^3} - \frac{\beta_4}{24}\frac{\partial^4 E}{\partial t^4} + \gamma N E, \qquad \frac{\partial N}{\partial t} = \frac{1}{\tau}\left(-N + f(\Gamma, |E|^2)\right), \tag{1}$$

where

$$f(\Gamma, |E|^2) = \frac{\Gamma|E|^2(2+\Gamma|E|^2/2)}{2(1+\Gamma|E|^2/2)^2}.$$

β_n and γ are the dispersion coefficient and nonlinear parameter, respectively, Γ is the saturation parameter ($\Gamma = 1/P_s$) and P_s is the saturation power of the nonlinear medium.

3. Linear stability analysis

The stability of the steady-state solution of the above dynamical equations against the presence of small harmonic perturbations can be studied using linear stability analysis [1]. The steady-state solution is given by

$$E_s = E_0 \exp[-if(\Gamma, |E_0|^2)], \qquad N_s = f(\Gamma, |E_0|^2).$$

The harmonic perturbation is of the form

$$E_p = (E_0 + a(z,t))\exp[-if(\Gamma, |E_0|^2)], \qquad N_p = n(z,t) + f(\Gamma, |E_0|^2),$$

where $a(z,t)$ and $n(z,t)$ are small deviations from the stationary solutions of the electric field and nonlinear index, respectively. After some mathematical manipulation, the dynamical equation for the perturbation can be written as

$$\frac{\partial a}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 a}{\partial t^2} + i\frac{\beta_3}{6}\frac{\partial^3 a}{\partial t^3} - \frac{\beta_4}{24}\frac{\partial^4 a}{\partial t^4} + \gamma n E_0$$

$$i\frac{\partial n}{\partial t} = \frac{1}{\tau}[-n + f(\Gamma, |E_0|^2), E_0(a + a^*)].$$

The dispersion relation (K) after some mathematical manipulation is given by

$$K = -\beta_3 \frac{\Omega^3}{6} \pm 2\sqrt{\left(\tilde{\gamma} E_0^2 + \frac{\beta_2}{2}\Omega^2 + \frac{\beta_4}{24}\Omega^4\right)^2 - \tilde{\gamma}^2 E_0^4},$$

$$\tilde{\gamma} = \frac{\gamma}{(1 + i\Omega\tau)(1 + \Gamma|E|^2/2)^3}. \qquad (2)$$

4. Modulational instability analysis

The above dispersion relation (eq. (2)) is a general case for the ultrashort pulse propagation near the zero dispersion wavelength. In principle, one can analyse MI in two distinct ways depending upon the wavelength of the propagation: (1) the pulse propagation at anomalous GVD regime, where the wavelength of the pump source is greater than the ZDW and (2) the pumping at or near the ZDW wavelength.

The dispersion relation for the pulse propagation at anomalous GVD regime is given by

$$K_{\mathrm{AD}} = \frac{1}{2}|\beta_2|\Omega\sqrt{\Omega + \mathrm{sgn}(\beta_2)\Omega_{\mathrm{AD}}^2}, \qquad \Omega_{\mathrm{AD}}^2 = \frac{4\tilde{\gamma} E_0^2}{|\beta_2|}. \qquad (3)$$

The dispersion relation for the pulse propagation at ZDW is given by

$$K_{\mathrm{ZDW}} = \beta_3 \frac{\Omega^3}{6} \pm \frac{|\beta_4|\Omega^2}{24}\sqrt{\Omega^4 + \mathrm{sgn}(\beta_2)\Omega_{\mathrm{ZD}}^4}, \qquad \Omega_{\mathrm{ZD}}^4 = \frac{48\tilde{\gamma} E_0^2}{|\beta_4|}, \qquad (4)$$

where Ω_{AD}^2 and Ω_{ZD}^4 are the critical modulation frequency (CMF) of SNL for the pulse centred at anomalous GVD and ZDW, respectively. The above dispersion relation leads to an exponential growth of the weak perturbation only for imaginary value of K. This can be achieved only when the frequency Ω registers value lower than the CMF, where Ω is the frequency shift from the central frequency ω_0.

5. MI in anomalous dispersion regime

For instance, we consider the anomalous dispersion regime whose dispersion relation is given by eq. (3). We plot the gain spectrum for some representative cases in figure 1a to have a quantitative picture of the MI in the presence of exponential saturation and relaxation. For instantaneous response, undoubtedly the CSN strongly reduces the MI gain and the OMF. Interestingly, due to the delay in the nonlinear response a second band of unstable frequencies emerges and this is attributed to the retarded Raman contribution to the nonlinear response of the medium. For a short response time, the delay in the nonlinear response does not have appreciable effect on the MI gain but extends the unstable regime. But for slow responses, both the MI gain and the MI instability are strongly reduced. It is

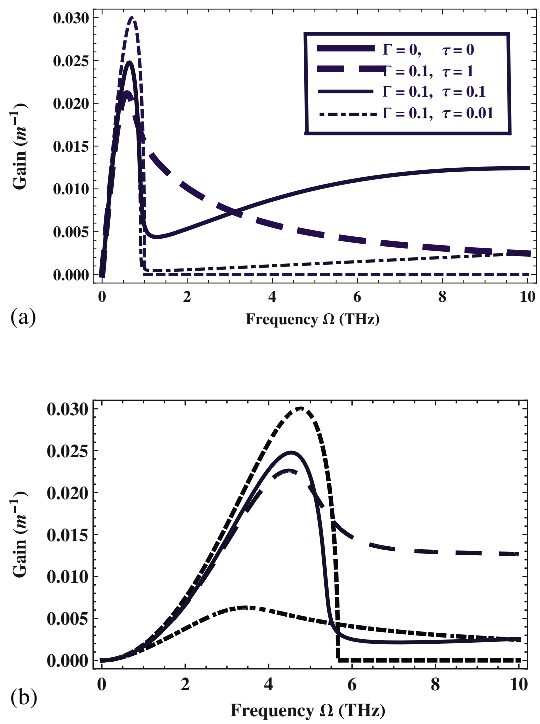

Figure 1. (**a**) MI spectrum in the anomalous GVD with $\beta_2 = -0.06$ ps^2/m, $\gamma = 0.015$ W^{-1}, $E_0 = 1$ W, (**b**) MI spectrum at ZDW for $\beta_2 = 0, \beta_4 = -0.0007$ ps^4/m, $\gamma = 0.015$ W^{-1}, $E_0 = 1$ W.

evident from figure 1a that two instability bands are resolved only at short response time of the medium.

6. MI in normal dispersion regime

Now we move on to the case of normal dispersion regime ($\beta_2 > 0$). It is a proven fact that MI is not possible in the single pump case for normal dispersion regime due to the lack of phase matching between the linear and nonlinear effects. Quite interestingly, the delay in the nonlinear response induces the emergence of an instability band even in this normal GVD. The emergence of instability in the normal dispersion regime is attributed to the fact that any finite relaxation time produces an imaginary part to the wave vector at any frequency, irrespective of the nature of the dispersion regime. The gain spectrum for a particular value of the saturation parameter and distinct response times is shown in figure 2a. Unlike the case of anomalous dispersion regime, here in the normal regime only single band is obtained which is the characteristic of the delayed Raman response. Figure 2a portrays the MI spectrum for the case of normal dispersion regime with varying saturation parameters and delay times.

7. MI at zero dispersion wavelength

We now extend the same for the case of pulse centred at or near ZDW, and for this case β_2 is very small or takes zero. In this context, the dispersion effect will be taken care of by the higher-order dispersion, especially the fourth-order dispersion (FOD). It is obvious that higher-order dispersion (HOD) more than four orders is literally insignificant. From the dispersion relation eq. (2), we can directly infer that the third-order dispersion is practically of no use in the MI dynamics and thus the entire dispersion effect is contributed by the FOD. Figures 1b and 2b show the MI gain spectra by virtue of FOD for the parameters quoted.

8. Results and discussion

This section features a comparative analysis on the influence of FOD in the delayed exponential saturable nonlinear system. For pulse propagation at ZDW wavelength, with $\beta_2 = 0$, FOD will be the dominant contender of the dispersion effect. From figures 1a and 1b corresponding to the anomalous dispersion regime one can see that MI can be achieved even when the GVD effects are ignored. It can also be observed like in the case of

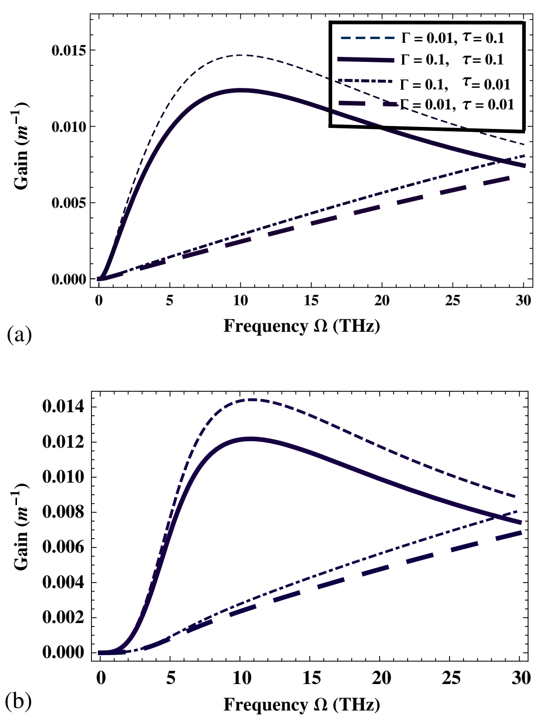

Figure 2. (**a**) MI spectrum in the normal GVD with $\beta_2 = 0.06$ ps^2/m, $\gamma = 0.015$ W^{-1}, $E_0 = 1$ W, (**b**) MI spectrum at ZDW for $\beta_2 = 0, \beta_4 = 0.0007$ ps^4/m, $\gamma = 0.015$ W^{-1}, $E_0 = 1$ W.

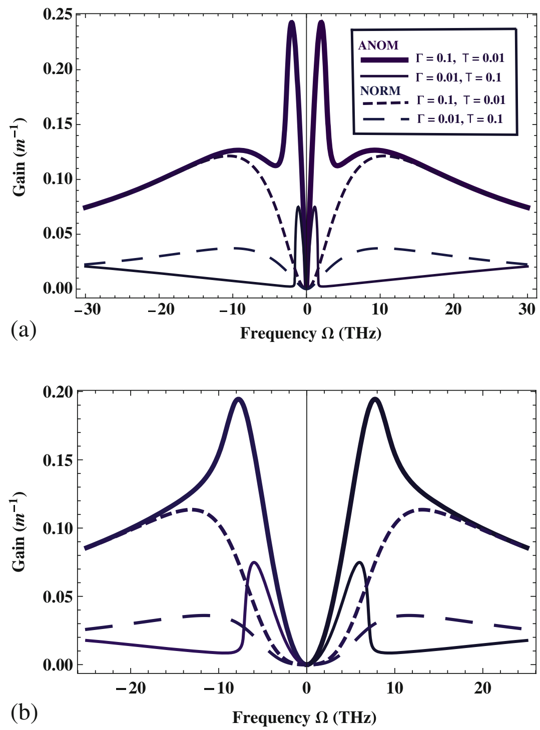

Figure 3. (**a**) MI spectrum with $\beta_2 = \pm 0.06$ ps^2/m, $\gamma = 0.015$ W^{-1}, $E_0 = 10$ W, (**b**) MI spectrum at ZDW for $\beta_2 = 0$, $\beta_4 = \pm 0.0007$ ps^4/m, $\gamma = 0.015$ W^{-1}, $E_0 = 10$ W.

anomalous GVD, FOD also results in two bands in the MI spectrum. Although MI gain is not of any appreciable change, FOD shifts the MI peak gain to the higher frequency side and also increases the instability region, as shown in figure 1b. As in the previous case, CSN suppress the MI gain and the instability region. To give a quantitative picture, we plot figures 3a and 3b to illustrate the role of FOD in minimum GVD regime. Figures 2a and 2b show the MI spectrum for the normal dispersion regime. Similar to our previous argument, FOD serves the necessary contribution to the phase matching condition here as well. One can observe that both the cases are identical, that a slight shift of the MI peak gain towards higher frequency side is observed in the case of FOD system (figure 2b).

9. Conclusion

We have presented the impact of higher-order dispersion in the MI spectrum of the saturable nonlinear system with delayed response. Using linear stability analysis, we have theoretically shown the role of FOD and CSN in the modulational instability spectrum with realistic fibre parameters. We have shown the possibility of MI in both the normal and anomalous dispersion regimes even in the absence of GVD and conclude that the incorporation of saturable nonlinear response certainly suppresses the MI irrespective of the nature of pumping regime.

Acknowledgements

KP wishes to thank the DST, DAE–BRNS and UGC, Government of India, for the financial support through major projects. K Nithyanandan thanks CSIR, Government of India for providing financial support by awarding Senior Research Fellowship (SRF). R V J Raja thanks DST, Government of India for financial support.

References

[1] G P Agrawal, *Applications of nonlinear fiber optics* (Academic Press, New York, 2006)
[2] X Liu, J W Haus and S M Shahriar, *Opt. Commun.* **281**, 2907 (2008)
[3] M L Lyra and A S Gouveia-Neto, *Opt. Commun.* **108**, 117 (1994)
[4] K Nithyanandan, R V J Raja and K Porsezian, *Phys. Rev. A* **87**, 043805 (2013)