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Electromagnetically-induced transparency in Doppler-broadened five-level systems

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Abstract. We study electromagnetically-induced transparency (EIT) of a probe field in a Doppler-broadened five-level K-type atomic system driven by three strong laser (coupling) fields. Effect of wave-vector mismatch occurring when the coupling field frequency is higher than that of the probe field frequency ($\lambda_c < \lambda_p$) are considered. Under the influence of the coherent coupling fields, the steady-state linear susceptibility of the probe laser shows that the system can have single, double or triple EIT windows depending on the amplitude and detuning of the coupling fields.

Keywords. Electromagnetically-induced transparency; atomic coherence and interference.

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1. Introduction

Electromagnetically-induced transparency (EIT) [1], which enables propagation of light through an otherwise opaque medium without significant attenuation, is one of the many unusual and interesting phenomena produced by atomic coherence and interference effects. In recent times, EIT has been of tremendous interest due to the possibility of its wide applications in optical switching via light velocity control [2,3], quantum information [4] and enhancing nonlinear optical processes [5]. Early studies on EIT focussed primarily on simple three-level systems, such as lambda, Vee and ladder (cascade) systems [1–5]. Nowadays there is considerable interest in the study of EIT in multilevel atomic systems formed by including extra, optical fields driven resonant transitions, to the three-level systems. Relevant to the present work is the five-level K-type system that in recent times has been utilized to investigate double electromagnetically-induced two-photon transparency [6] and the effect of spontaneously generated coherence on EIT [7] using homogenously broadened models.

2. Theory for the five-level K-system

2.1 Formulation

The interaction Hamiltonian in the interaction picture under resonant interaction condition and rotating wave approximation is obtained as

$$V^{\rm int} = -\hbar \Big[\Omega_{\rm p} {\rm e}^{{\rm i}(\vec{k}_{\rm p}\cdot\vec{r}+\Delta_{31}{\rm t})} \,|3\rangle\langle 1| + \Omega_{\rm c1} {\rm e}^{{\rm i}(\vec{k}_{\rm c1}\cdot\vec{r}+\Delta_{32}{\rm t})} \,|3\rangle\langle 2| + \Omega_{\rm c2} {\rm e}^{{\rm i}(\vec{k}_{\rm c2}\cdot\vec{r}+\Delta_{43}{\rm t})} \,|4\rangle\langle 3| + \Omega_{\rm c3} {\rm e}^{{\rm i}(\vec{k}_{\rm c3}\cdot\vec{r}+\Delta_{53}{\rm t})} \,|5\rangle\langle 3| + {\rm h.c.} \Big], \quad (1)$$

where $\Delta_{31} = \omega_{31} - \omega_{\rm p}$, $\Delta_{32} = \omega_{32} - \omega_{\rm c1}$, $\Delta_{43} = \omega_{43} - \omega_{\rm c2}$ and $\Delta_{53} = \omega_{53} - \omega_{\rm c3}$ denote the detuning of probe and coupling field frequencies from the atomic resonance frequencies $\omega_{31}, \omega_{32}, \omega_{43}$ and ω_{53} respectively, and $|i\rangle\langle j|\,(i, j = 1\text{–}5)$ are the atomic raising or lowering operators. Equation (1) is used to obtain the density matrix equations of motion in which the various relaxation processes such as spontaneous emissions are included phenomenologically. Doppler broadening of the vapour is also taken into account. As usual for the EIT problems, we make an expansion of the density matrix to all orders in the strong (coupling) field amplitudes and to the first order in probe amplitude to obtain the density matrix equations as

$$\dot{\tilde{\rho}}_{21}^{(1)} = -[{\rm i}(\Delta_{31} - \Delta_{32} + (\vec{k}_{\rm p} - \vec{k}_{\rm c1})\cdot\vec{v}) + \gamma_{21}]\tilde{\rho}_{21}^{(1)} + {\rm i}\Omega_{\rm c1}^{*}\tilde{\rho}_{31}^{(1)}, \quad (2a)$$

$$\dot{\tilde{\rho}}_{31}^{(1)} = -[{\rm i}(\Delta_{31} + \vec{k}_{\rm p}.\vec{v}) + (\gamma_{31} + \gamma_{32})]\tilde{\rho}_{31}^{(1)} + {\rm i}\Omega_{\rm p}\rho_{11}^{(0)} + {\rm i}\Omega_{\rm c1}\tilde{\rho}_{21}^{(1)} + {\rm i}\Omega_{\rm c2}^{*}\tilde{\rho}_{41}^{(1)} + {\rm i}\Omega_{\rm c3}^{*}\tilde{\rho}_{51}^{(1)}, \quad (2b)$$

$$\dot{\tilde{\rho}}_{41}^{(1)} = -[{\rm i}(\Delta_{31} + \Delta_{43} + (\vec{k}_{\rm p} + \vec{k}_{\rm c2})\cdot\vec{v}) + \gamma_{43}]\tilde{\rho}_{41}^{(1)} + {\rm i}\Omega_{\rm c2}\tilde{\rho}_{31}^{(1)}, \quad (2c)$$

$$\dot{\tilde{\rho}}_{51}^{(1)} = -[{\rm i}(\Delta_{31} + \Delta_{53} + (\vec{k}_{\rm p} + \vec{k}_{\rm c3})\cdot\vec{v}) + \gamma_{53}]\tilde{\rho}_{51}^{(1)} + {\rm i}\Omega_{\rm c3}\tilde{\rho}_{31}^{(1)}. \quad (2d)$$

In experimental situations, typically one considers a geometry in which the probe $\vec{E}_{\rm p}$ and coupler field $\vec{E}_{\rm c1}$ are copropagating, whereas the probe $\vec{E}_{\rm p}$ and other coupler fields $\vec{E}_{\rm c2}$ and $\vec{E}_{\rm c3}$ are counterpropagating along the z-axis. For this experimental configuration, we can henceforth set in eq. (2), the terms $\vec{k}_{\rm p}\cdot\vec{v} = k_{\rm p}v_{\rm z}$, $(\vec{k}_{\rm p} - \vec{k}_{\rm c1})\cdot\vec{v} = (k_{\rm p} - k_{\rm c1})v_{\rm z}$ and $(\vec{k}_{\rm p} + \vec{k}_{\rm cj})\cdot\vec{v} = (k_{\rm p} - k_{\rm cj})v_{\rm z}$, (where $j = 2, 3$). The steady-state solution obtained by setting the time derivates zero on the left-hand side of eq. (2) yields the one-dimensional velocity averaged one-photon coherence as

$$\frac{I_{31}^{(1)}}{\Omega_\mathrm{p}} = \int \tilde{\rho}_{31}^{(1)}(v_\mathrm{z})\mathrm{d}v_\mathrm{z}$$
$$= \int \mathrm{d}v_\mathrm{z} M(v_\mathrm{z})$$
$$\times \frac{\mathrm{i}}{(\gamma_{31}+\gamma_{32})+\mathrm{i}(\Delta_{31}+k_\mathrm{p}v_\mathrm{z})+\frac{|\Omega_{\mathrm{c}1}|^2}{\gamma_{21}+\mathrm{i}(\Delta_{31}-\Delta_{32})+\mathrm{i}(k_\mathrm{p}-k_{\mathrm{c}1})v_\mathrm{z}}+\frac{|\Omega_{\mathrm{c}2}|^2}{\gamma_{43}+\mathrm{i}(\Delta_{31}+\Delta_{43})+\mathrm{i}(k_\mathrm{p}-k_{\mathrm{c}2})v_\mathrm{z}}+\frac{|\Omega_{\mathrm{c}3}|^2}{\gamma_{53}+\mathrm{i}(\Delta_{31}+\Delta_{53})+\mathrm{i}(k_\mathrm{p}-k_{\mathrm{c}3})v_\mathrm{z}}}. \tag{3}$$

2.2 *Susceptibility*

The susceptibility of the medium is related to the velocity-averaged one-photon coherence as follows:

$$\chi = N\frac{|\mu_{31}|^2}{\hbar\gamma_\mathrm{D}}\left(\frac{I_{31}^{(1)}}{\Omega_\mathrm{p}/\gamma_\mathrm{D}}\right), \tag{4}$$

where N is the atomic density of the vapour and $\gamma_\mathrm{D}(= k_\mathrm{p}\overline{v})$ is the Doppler width in the system. As is well known, the imaginary (Imag(χ)) and real (Re(χ)) parts respectively of the susceptibility χ give the absorption and dispersion of the probe field.

3. Numerical results and discussion

It is seen from figure 1 that in a five-level atomic system of K-type configuration, levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ form a three-level Λ-configuration and level $|4\rangle$ ($|5\rangle$) together with

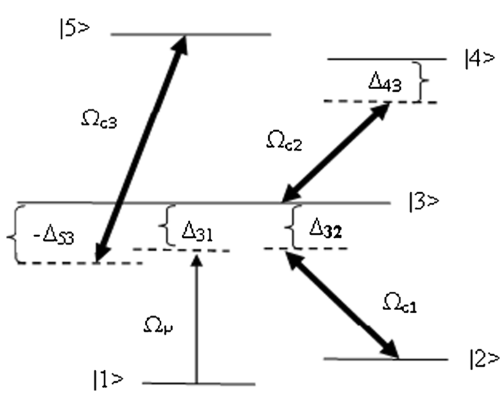

Figure 1. EIT scheme in five-level K-type atomic system. Here Ω_p is the Rabi frequency of the (weak) probe and $\Omega_{\mathrm{c}1}$, $\Omega_{\mathrm{c}2}$ and $\Omega_{\mathrm{c}3}$ respectively are the (strong) coupling field Rabi frequencies. The detunings of the probe and coupling fields from their respective atomic transitions are: $\Delta_{31} = \omega_{31}-\omega_\mathrm{p}$, $\Delta_{32} = \omega_{32}-\omega_{\mathrm{c}1}$, $\Delta_{43} = \omega_{43}-\omega_{\mathrm{c}2}$ and $\Delta_{53} = \omega_{53} - \omega_{\mathrm{c}3}$ respectively.

levels $|1\rangle$, $|3\rangle$ are in a usual three-level ladder-type configuration. We implement this scheme in atomic rubidium considering the transitions $5S_{1/2}(F = 1) \rightarrow 5P_{3/2} \rightarrow 5S_{1/2}(F = 2)$ which forms a three-level lambda system and $5S_{1/2}(F = 1) \rightarrow 5P_{3/2} \rightarrow 5D_{3/2}(5S_{1/2}(F = 1) \rightarrow 5P_{3/2} \rightarrow 5D_{5/2})$ which forms the other three-level ladder system of five-level K-type configuration. The level separation wavelength of the lower levels $5S_{1/2}(F = 1)(5S_{1/2}(F = 2))$ and intermediate level is $\lambda_p = 780$ nm ($\lambda_{c1} \simeq 780$ nm) and those of the intermediate and upper transitions are $\lambda_{c2} \approx \lambda_{c3} = 776$ nm. The wavelength mismatch between the counterpropagating probe ($\vec{E}_p$) and coupling ($\vec{E}_{c2}$ and $\vec{E}_{c3}$) fields in five-level K-type system introduce a residual Doppler width $(k_p - k_{cj})\overline{v}/k_p\overline{v} = -0.005$ (where $j = 2, 3$). For comparison the other case of exactly matched ($(k_p - k_{cj})\overline{v}/k_p\overline{v} = 0$) wave-vector is also considered hypothetically for this K-type atomic system. All parameters in the numerical calculations are expressed in units of the Doppler width $\gamma_D/2\pi\,(= 250$ MHz), i.e., $2\gamma_{21}/\gamma_D = 8 \times 10^{-6}$, $2\gamma_{31}/\gamma_D = 2\gamma_{32}/\gamma_D = 0.012$, $2\gamma_{43}/\gamma_D = 0.0003$ and $2\gamma_{53}/\gamma_D = 0.002$.

In figure 2, probe absorption ($\mathrm{Im}(I_{31}^{(1)}\gamma_D/\Omega_p)$) profiles are plotted as a function of the probe detuning for various cases of wave-vector mismatches of the coupling and probe fields. The coupling fields are on resonance $\Delta_{32} = \Delta_{43} = \Delta_{53} = 0$ and the Rabi frequencies are chosen equal ($\Omega_{c1} = \Omega_{c2} = \Omega_{c3} = 7.5$ MHz $(\times 2\pi)$. For the perfect wave-vector matching case $(k_p - k_{cj})\overline{v} = 0$, the two-photon resonances are Doppler-free and we observe the usual Doppler-free EIT profile (solid line), whereas in the case of negative wave-vector mismatch $(k_p - k_{cj})\overline{v}/k_p\overline{v} = -0.005$, we observe three EIT windows. The middle EIT resonance is very narrow compared to that in the previous case of perfect matching. Clearly, these unusual features in EIT profile arise from a more complicated interference effect caused due to overlap between the residual Doppler-broadened two-photon resonances of the ladder system with that of the Doppler free Λ system.

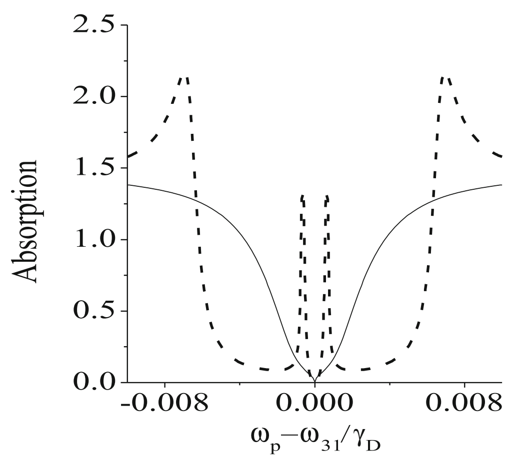

Figure 2. Probe absorption $[\mathrm{Im}(I_{31}^{(1)}\gamma_D/\Omega_p)]$ as a function of the probe field detuning $(\omega_p - \omega_{31})/\gamma_D$ for the coupling field Rabi frequencies $\Omega_{c1} = \Omega_{c2} = \Omega_{c3} = 7.5$ MHz $(\times 2\pi)$ and various wave-vector mismatch cases, $(k_p - k_{cj})\overline{v}/k_p\overline{v} = -0.005$ (dashed line) and $k_p = k_{cj}$ (solid line) (where $j = 2, 3$). The coupling fields are on resonance $\Delta_{32} = \Delta_{43} = \Delta_{53} = 0$.

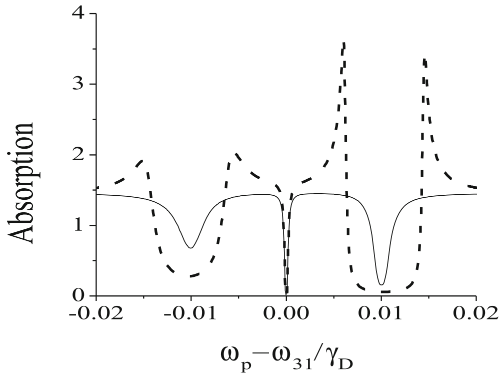

Figure 3. Probe absorption $[\mathrm{Im}(I_{31}^{(1)}\gamma_D/\Omega_p)]$ as a function of the probe field detuning $(\omega_p - \omega_{31})/\gamma_D$ for the coupling field Rabi frequencies $\Omega_{c1} = 3.75$ MHz $(\times 2\pi)$, $\Omega_{c2} = \Omega_{c3} = 7.5$ MHz $(\times 2\pi)$ and various wave-vector mismatch cases, $k_p - k_{cj} = -0.005k_p$ (dashed line) and $k_p = k_{cj}$ (solid line) (where $j = 2, 3$). The coupling fields detuning chosen are $\Delta_{32} = 0$, $\Delta_{43} = -\Delta_{53} = 2.5$ MHz $(\times 2\pi)$.

We now consider the finite detuning case when the coupling fields (Ω_{c2}, Ω_{c3}) are detuned on either side of the intermediate level. Figure 3 shows the probe absorption $(\mathrm{Im}(I_{31}^{(1)}\gamma_D/\Omega_p))$ variation as the probe frequency is tuned through the coupling field detuning. The probe absorption profile for this case splits into three distinct transparency windows corresponding to lambda and two distinct cascade subsystems. The transparency window occurring at $\Delta_{31} - \Delta_{32} = 0$ corresponds to $5S_{1/2}(F = 1) \rightarrow 5P_{3/2} \rightarrow 5S_{1/2}(F = 2)$ and two-photon resonance $\Delta_{31} + \Delta_{43} = 0$ ($\Delta_{31} + \Delta_{53} = 0$) corresponds to $5S_{1/2}(F = 1) \rightarrow 5P_{3/2} \rightarrow 5D_{3/2}$ ($5S_{1/2}(F = 1) \rightarrow 5P_{3/2} \rightarrow 5D_{5/2}$) transitions. Since these three subsystems are decoupled, the depth and width of each transparency window is now governed by the two-photon dephasing rate parameters and the coupling field Rabi frequency in that particular subsystem. The asymmetry in the depth of the transparency on either side of the middle transparency window occurs as the two-photon decay rates in the two cascade subsystems are dissimilar in the present K system.

4. Conclusion

In conclusion, we have studied the electromagnetically-induced transparency (EIT) of a probe field in a Doppler-broadened five-level K-type atomic system that can be considered to be composed of *three* three-level sub-systems; two of ladder-type and the third one of Λ-type configurations. Yet we find that the EIT response of the composite system can be dramatically distinct and more complicated than those of the constituent systems. For large field frequency detuning, we get the three well-separated usual EIT profiles of the Λ and the two-ladder system. This suggests that this unusual behaviour at exact resonance is due to further interference between the regimes of overlap of the very narrow absorption profile of the Λ-type system with the very wide transparency window of the two-ladder-type systems. Such systems might be useful in any EIT-based optical switching device as control and variation of EIT are the main features of this system.

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