PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 81, No. 6
December 2013
pp. 1045–1049

Shape transition of state density for bosonic systems

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MS received 25 June 2013; accepted 29 August 2013
DOI: 10.1007/s12043-013-0622-4; ***e*Publication:** 26 November 2013

Abstract. For a finite m boson system, the ensemble-averaged state density has been computed with respect to the body interaction rank k. The shape of such a state density changes from Gaussian to semicircle as the body rank of the interaction increases. This state density is expressed as a linear superposition of Gaussian and semicircular states. The nearest-neighbour spacing distribution (NNSD), which is one of the most important spectral properties of a system, is studied. The NNSDs are rather independent of body rank k and show a Wigner distribution throughout.

Keywords. Random matrix ensembles; interacting bosons; Gaussian orthogonal ensemble; bosonic embedded Gaussian orthogonal ensemble; state density; spacing distribution.

PACS Nos 05.40.−a; 05.30.Jp; 05.45.Mt

Random matrix theory (RMT) was introduced and studied around 1960 by Wigner, Dyson, Mehta and others [1,2]. Recently, the interest in RMT is renewed largely due to its applicability to the spectra of chaotic systems. The ensemble of symmetric random matrices, with each matrix element being a Gaussian random variate with zero mean and unit variance (for diagonal matrix elements variance = 2), is called Gaussian orthogonal ensemble (GOE). The GOE for m-particles in the space defined by N single particle states, in fact describes simultaneous interactions between all the particles because of the statistical independence of the matrix elements. Realistic interactions are however 2-body in nature. Interest in 3-body interactions has also been revived recently [3]. In general, we can choose an ensemble of k-body interactions by generating a GOE in k-particle space and then propagating it to generate an ensemble in m-particle space. Such an ensemble is termed EGOE(k).

The ensemble-averaged state density for GOE is semicircle, while the shape of the state density for EGOE(2) with $m \gg 2$, is close to Gaussian. The change in shape of the state density, from semicircle to Gaussian for EGOE(k), as m increases from k to $m \gg k$ for fermions, has been explained mathematically by Mon and French [4] and also by Benet *et al* [5]. Further, Benet *et al* have shown that the semicircle to Gaussian transition point is $m = 2k$. This shape transition has also been numerically demonstrated [2,6]. All these works refer to the system of fermions.

Similar behaviour is expected from EGOE(k) for the system of bosons also; the corresponding ensemble is termed as BEGOE(k) (B stands for bosons). It has been shown [7,8] that the state density for BEGOE(2) in the dense limit (m-particles in N-single particle states with $m \to \infty$, $N \to \infty$ and $m/N \to \infty$) approaches a Gaussian. The state density for m-bosons with $m \gg 2$, is Gaussian for BEGOE(2) which has been already demonstrated [9]. However, the transition of the state density from Gaussian to semicircle as the rank of the interaction k increases from 2 to m has not been explored. We demonstrate such a transition in this note.

For m bosons to be distributed in N orbits, the dimensionality of the spaces is given by $d(N, m) = \binom{N+m-1}{m}$. For BEGOE($m$), the number of independent matrix elements (IME) is given by $\frac{d(N,m)\,(d(N,m)+1)}{2}$. Table 1 gives the dimensionality and IME for k-body interaction with $N = 4$ and $m = 10$.

We have considered a system of 50 members with $m = 10$ bosons distributed in $N = 4$ single-particle states and the body rank of the interaction going from $k = 2, 3, 4, \ldots, 10$.

It is important to remember during these calculations that the eigenvalue spectrum for each member of the ensemble is first zero centred and scaled to unit width. Figure 1 clearly shows a Gaussian to semicircular transition for the state density as we move the body rank from $k = 2$ to $k = m$ (here 10).

The ensemble-averaged state density can be expressed as a superposition of the two limiting forms by introducing a parameter μ as

$$\rho(E, \mu) = \mu\rho_{\mathrm{G}}(E) + (1 - \mu)\rho_{\mathrm{SC}}(E), \tag{1}$$

where $\rho_{\mathrm{G}}(E)$ and $\rho_{\mathrm{SC}}(E)$ are Gaussian and semicircle distribution respectively. μ is the interpolating parameter between Gaussian and semicircular distribution. Its value is obtained by standard least square fitting procedure. For low values of k the ensemble-averaged state density is close to Gaussian form, while for k close to m it is close to semicircular form.

Table 1. Dimensionality and number of independent matrix elements.

m	$d(N, m)$	No. of IME
2	10	55
3	20	210
4	35	630
5	56	1596
6	84	3570
7	120	7260
8	165	13695
9	220	24310
10	286	41041

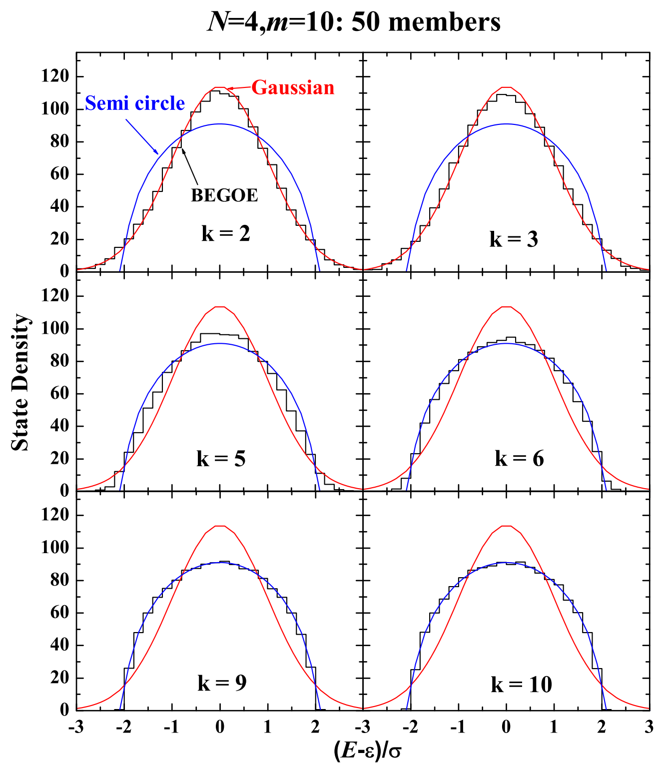

Figure 1. Ensemble-averaged state density as a function of normalized energy $(E - \varepsilon)/\sigma$ (ε is the centroid and σ is the width of the spectrum) for different k for a 50-member BEGOE(k) ensemble with $N = 4$ and $m = 10$.

Figure 2 shows a plot between μ and body rank k for $N = 4$ and $N = 5$ also which depicts that the transition ($\mu = 0.5$) is between $k = 4$ and 5, which is close to the theoretically predicted transition [6] point $k = m/2$.

For boson spaces (N, m), we have obtained the nearest-neighbour spacing distribution (NNSD) $P(s)$ which concerns with the character of the ensemble. It is known that for embedded systems in chaotic domain, the nature of NNSD is Wigner, whereas in the integrable domain, NNSD is in the form of Poisson distribution.

$$P(s) = \exp(-s),$$
$$P(s) = \left(\frac{\pi s}{2}\right) \exp\left(\frac{-\pi s^2}{4}\right). \tag{2}$$

Figure 3 shows ensemble-averaged NNSD $P(s)$ computed for each body rank $k = 2$ varying up to m with 50-member BEGOE(k) for $N = 4$. It is well understood that, whether for 2-body interaction, i.e. $k = 2$, or as we move ahead up to the maximum body interaction (here $k = m = 10$), the distribution $P(s)$ shows a Wigner form consistently.

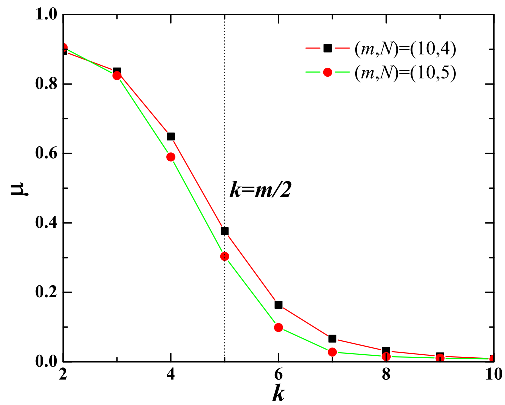

Figure 2. The interpolating parameter μ is plotted against the body rank k for $(m, N) = (10, 4)$ and also for $(10, 5)$.

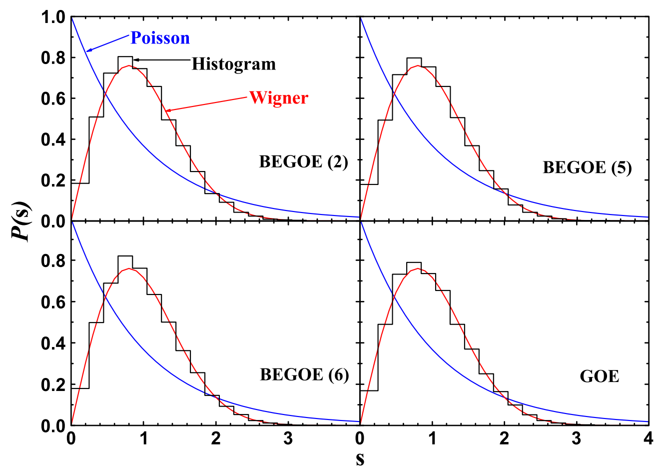

Figure 3. NNSD for a 50-member BEGOE(k) ensemble with $N = 4$ for different body ranks. The numerical results are compared with Poisson and Wigner (GOE) forms.

For a BEGOE(k) system, the state density form changes from Gaussian (for $k = 2$) to semicircle as k increases. The transition can be said to occur near $k = m/2$. Moreover, the NNSD shows a consistent Wigner (GOE) behaviour independent of the body rank of interaction.

Acknowledgement

The authors acknowledge grants F.No:40- 425/2011(SR) and No.F.6-17/10(SA-II) from University Grants Commission, New Delhi.

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