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The classification of the single travelling wave solutions to the generalized Pochhammer–Chree equation

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Abstract. By the complete discrimination system for the polynomial, we invest the classifications of single travelling wave solutions to the generalized Pochhammer–Chree (PC) equation with $p = 1/2$ and $p = 3/2$.

Keywords. Exact solution; single travelling wave solution; complete discrimination system for the polynomial; the generalized PC equation.

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1. Introduction

Nonlinear differential equations have been widely used in the field of Mathematics, Physics and Engineering. The exact solutions to the nonlinear differential equations are closely related to our lives. In the past several years, the issue of the exact solutions of high-order nonlinear differential equations has been thoroughly researched and a large number of methods for finding exact solutions have been established and developed, such as inverse scattering method, Backlund transformation, Lie group method, Painleve method [1] and transformed rational function method for finding exact solutions to integrable cases of nonlinear wave equations [2], Frobenius decomposition method that provides a general idea for solving ODEs by using integrable ODEs [3]. Of course, some other powerful methods have also been proposed to solve nonlinear differential equations [3–10]. For example, the generalized bilinear equations and their solutions are also characterized recently by Bell polynomials [10]. These methods have been widely applied to many nonlinear differential equations for obtaining the exact solutions. Recently, a method named as the complete discrimination system for polynomial method has been proposed by Liu [11–16]. By Liu's method, we can obtain the classification of single travelling wave solutions to some nonlinear differential equations [11–19]. If we take the travelling wave transformation and integrating it, the nonlinear differential equation can be directly reduced to the integral form as follows:

$$\pm(\xi - \xi_0) = \int \frac{\mathrm{d}u}{\sqrt{p_n(u)}}, \tag{1}$$

where $p_n(u)$ is the nth-order polynomial. We can obtain the classification of all solutions to the right side integral in eq. (1). For example, Wang and Li [17] used Liu's method to invest the single and multisolitary solutions to a class of nonlinear evolution equations; Yang [18] studied the envelop solutions to DS equation; Fan [19] gave the classification of the single travelling wave solutions to the generalized equal width equation and so on. In the present paper, we shall use Liu's method to the generalized Pochhammer–Chree equation and give the classification of all its single travelling wave solutions with $p = 1/2$ and $p = 3/2$. The generalized Pochhammer–Chree equation

$$u_{tt} - u_{ttxx} - u_{xx} - \frac{1}{p}(u^p)_{xx} = 0 \tag{2}$$

is used to describe the propagation of longitudinal deformation waves in an elastic rod, where $p = 3$ and 5 reflect two possible constitutive choices for the material [20,21]. Bogolubsky [20] and Clarkson [21] gave some solitary-wave solutions of eq. (2) with $p = 2$, 3 and 5. Clarkson *et al* [21] considered the Painleve property for the generalized PC equation

$$u_{tt} - u_{ttxx} - (\sigma(u))_{xx} = 0 \tag{3}$$

and pointed out that for eq. (3) to be of Painleve type, $\sigma(u)$ must be of the form $\sigma(u) = a_0 + a_1u + a_2u^2 + a_3u^3$, then we gain the following generalized PC equation:

$$u_{tt} - u_{ttxx} - (a_1u + a_2u^2 + a_3u^3)_{xx} = 0, \tag{4}$$

where $a_2 \neq 0$ or $a_3 \neq 0$. Zhang and Ma [22] gave explicit solitary wave solutions of eq. (4).

2. Exact solutions of the PC equation with $p = 1/2$

When $p = 1/2$, eq. (2) becomes

$$u_{tt} - u_{ttxx} - u_{xx} - 2(u^{1/2})_{xx} = 0. \tag{5}$$

Taking the travelling wave transformation $u = u(\xi)$ and $\xi = x - ct$, we gain the corresponding reduced ODE:

$$c^2u'' - c^2u^{(4)} - u'' - \frac{1}{2}u^{-3/2}(u')^2 + u^{-1/2}u'' = 0. \tag{6}$$

Integrating it, we obtain the following equation:

$$(u')^2 = -\frac{8}{3c^2}u^{3/2} + \frac{c^2-1}{c^2}u^2 + \frac{2g_1}{c^2}u + \frac{2g_2}{c^2}, \tag{7}$$

where g_1 and g_2 are two arbitrary constants.

Taking the transformation $w^2 = u$, eq. (7) becomes

$$(w')^2 = \frac{1}{w^2}(a_4 w^4 + a_3 w^3 + a_2 w^2 + a_0), \tag{8}$$

where

$$a_4 = \frac{c^2-1}{4c^2}, \quad a_3 = \frac{-2}{3c^2}, \quad a_2 = \frac{g_1}{2c^2}, \quad a_0 = \frac{g_2}{2c^2}.$$

The integral form of eq. (8) is

$$\pm(\xi - \xi_0) = \int \frac{w\,\mathrm{d}w}{\sqrt{a_4 w^4 + a_3 w^3 + a_2 w^2 + a_0}}. \tag{9}$$

According to eq. (9), we shall give single travelling wave solutions to the generalized PC equation (eq. (5)). There exist two cases to be discussed.

Case 2.1. $a_0 = 0$. Then, eq. (9) becomes

$$\pm(\xi - \xi_0) = \int \frac{\mathrm{d}w}{\sqrt{a_4 w^2 + a_3 w + a_2}}. \tag{10}$$

We denote

$$F(w) = a_4 w^2 + a_3 w + a_2, \tag{11}$$

and $\Delta = a_3^2 - 4a_4 a_2$.

Case 2.1.1. $\Delta = 0$,

$$F(w) = a_4 \left(w + \frac{a_3}{2a_4}\right)^2. \tag{12}$$

The corresponding solutions are

$$u(x,t) = \exp\left[\pm\sqrt{\frac{c^2-1}{4c^2}}(x - ct - \xi_0)\right] + \frac{16}{9(c^2-1)^2} \pm \frac{8}{3(c^2-1)} \times \exp\left[\pm\sqrt{\frac{c^2-1}{4c^2}}(x - ct - \xi_0)\right], \quad c^2 > 1. \tag{13}$$

Case 2.1.2. $\Delta > 0$,

$$F(w) = a_4 \left(w + \frac{a_3}{2a_4}\right)^2 - \frac{a_3^2 - 4a_4 a_2}{4a_4}. \tag{14}$$

The corresponding solutions are

$$u(x,t) = \left\{\pm\frac{1}{2}\exp\left[\pm\sqrt{\frac{c^2-1}{4c^2}}(x - ct - \xi_0)\right] + \left(\frac{8}{9(c^2-1)^2} - g_1\right) \times \exp\left[\mp\sqrt{\frac{c^2-1}{4c^2}}(x - ct - \xi_0)\right] \pm \frac{4}{3(c^2-1)}\right\}^2, \quad c^2 > 1, \tag{15}$$

$$u(x,t) = \left(\frac{16}{9(c^2-1)^2} - \frac{2g_1}{c^2-1}\right)\sin^2\left[\pm\sqrt{\frac{1-c^2}{4c^2}}(x-ct-\xi_0)\right] + \frac{16}{9(c^2-1)^2} \mp \frac{16}{9(c^2-1)^2}\sqrt{\frac{8-9g_1(c^2-1)}{2}} \times \sin\left[\pm\sqrt{\frac{1-c^2}{4c^2}}(x-ct-\xi_0)\right], \qquad c^2<1. \tag{16}$$

Case 2.1.3. $\Delta < 0$, $c^2 > 1$, the corresponding solutions are the same as Case 2.1.2.

Case 2.2. $a_0 \neq 0$. When $c^2 > 1$, we take the transformation

$$v = a_4^{1/4}\left(w + \frac{a_3}{4a_4}\right), \quad \xi_1 = a_4^{1/4}\xi,$$

then, eq. (9) becomes

$$\pm a_4^{1/4}(\xi_1 - \xi_0) = \int \frac{(v-a)\mathrm{d}v}{\sqrt{v^4 + pv^2 + qv + r}}, \tag{17}$$

where

$$a = -\left(\frac{256}{81c^2(c^2-1)^3}\right)^{1/4}, \quad p = \frac{a_2}{\sqrt{a_4}},$$
$$q = a_4^{-1/4}\left(\frac{a_3^3}{8a_4^2} - \frac{a_2a_3}{2a_4}\right), \quad r = -\frac{3a_3^4}{256a_4^3} + \frac{a_2a_3^2}{16a_4^2} + a_0. \tag{18}$$

When $c^2 < 1$, we take the transformation

$$v = (-a_4)^{1/4}\left(w + \frac{a_3}{4a_4}\right), \quad \xi_1 = (-a_4)^{1/4}\xi,$$

then, eq. (9) becomes

$$\pm(-a_4)^{1/4}(\xi_1 - \xi_0) = \int \frac{(v-a)\mathrm{d}v}{\sqrt{-(v^4 + pv^2 + qv + r)}}, \tag{19}$$

where

$$a = -\left(\frac{256}{81c^2(1-c^2)^3}\right)^{1/4}, \quad p = -\frac{a_2}{\sqrt{-a_4}},$$
$$q = (-a_4)^{-1/4}\left(-\frac{a_3^3}{8a_4^2} + \frac{a_2a_3}{2a_4}\right), \quad r = \frac{3a_3^4}{256a_4^3} - \frac{a_2a_3^2}{16a_4^2} - a_0. \tag{20}$$

Equations (17) and (19) become

$$\pm|a_4|^{1/4}(\xi_1 - \xi_0) = \int \frac{(v-a)\mathrm{d}v}{\sqrt{\epsilon(v^4 + pv^2 + qv + r)}}, \tag{21}$$

where $\epsilon = \pm 1$.

We denote

$$F(v) = v^4 + pv^2 + qv + r, \tag{22}$$

and write its complete discrimination system as follows:

$$\begin{aligned} &D_1 = 4, \quad D_2 = -p, \quad D_3 = 8rp - 2p^3 - 9q^2, \\ &D_4 = 4p^4 r - p^3 q^2 + 36prq^2 - 32r^2 p^2 - \frac{27}{4} q^4 + 64r^3, \\ &E_2 = 9q^2 - 32pr. \end{aligned} \tag{23}$$

We must notice that in Case 2.2, the parameter a in eq. (21) is not equal to the roots of $F(v) = 0$.

Case 2.2.1. $D_4 = 0$, $D_3 = 0$, $D_2 < 0$. We have

$$F(v) = ((v - l)^2 + s^2)^2, \tag{24}$$

where l, s are real numbers, $s > 0$. When $\epsilon = +1$, the corresponding solutions are

$$\begin{aligned} \pm\sqrt{\frac{c^2-1}{c^2}}(x - ct - \xi_0) = &\ \frac{1}{2}\ln\left[\left(\frac{(\frac{c^2-1}{4c^2})^{1/4}(\sqrt{u} - \frac{2}{3(c^2-1)}) - l}{s}\right)^2 + 1\right] \\ &+ \frac{\left(\frac{256}{81c^2(c^2-1)^3}\right)^{1/4} - l}{s} \\ &\times \arctan\frac{(\frac{c^2-1}{4c^2})^{1/4}(\sqrt{u} - \frac{2}{3(c^2-1)}) - l}{s}. \end{aligned} \tag{25}$$

Case 2.2.2. $D_4 = 0$, $D_3 = 0$, $D_2 = 0$. We have

$$F(v) = v^4. \tag{26}$$

When $\epsilon = +1$, the corresponding solutions are

$$\begin{aligned} \pm\sqrt{\frac{c^2-1}{c^2}}(x - ct - \xi_0) = &\ \ln\left|\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u} - \frac{2}{3(c^2-1)}\right)\right| \\ &- \frac{2}{\pm 3(c^2-1)\sqrt{u} - 2}. \end{aligned} \tag{27}$$

Case 2.2.3. $D_4 = 0$, $D_3 = 0$, $D_2 > 0$, $E_2 > 0$. We have

$$F(v) = (v - \alpha)^2 (v - \beta)^2, \tag{28}$$

where α, β are real numbers, $\alpha > \beta$. When $\epsilon = +1$, the corresponding solutions are

$$\begin{aligned}\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0) = {} & \frac{-(\frac{256}{81c^2(c^2-1)^3})^{1/4}-\alpha}{\alpha-\beta} \\ & \times \ln\left|\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3c^2-3}\right)-\alpha\right| \\ & + \frac{(\frac{256}{81c^2(c^2-1)^3})^{1/4}+\beta}{\alpha-\beta} \\ & \times \ln\left|\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3c^2-3}\right)-\beta\right|. \end{aligned} \tag{29}$$

Case 2.2.4. $D_4 = 0$, $D_3 > 0$, $D_2 > 0$. We have

$$F(v) = (v-\alpha)^2(v-\beta)(v-\gamma), \tag{30}$$

where α, β, γ are real numbers and $\beta > \gamma$. When $\epsilon = +1$, the corresponding solutions are

$$\begin{aligned} & \pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0) = 2\ln\left[\sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\beta}\right. \\ & \left. + \sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\gamma}\,\right] + \frac{\alpha+\left(\frac{256}{81c^2(c^2-1)^3}\right)^{1/4}}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}} \\ & \times \ln\frac{\left[\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\gamma-\alpha)}-\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\gamma)(\alpha-\beta)}\,\right]^2}{|(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha|}, \\ & \alpha > \beta, \quad \left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right) > \beta, \end{aligned} \tag{31}$$

$$\begin{aligned} & \pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0) = 2\ln\left[\sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\beta}\right. \\ & \left. + \sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\gamma}\,\right] + \frac{\alpha+\left(\frac{256}{81c^2(c^2-1)^3}\right)^{1/4}}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}} \\ & \times \ln\frac{\left[\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\alpha-\gamma)}-\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\gamma)(\beta-\alpha)}\,\right]^2}{(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha}, \\ & \alpha < \gamma, \quad \left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right) > \beta, \end{aligned} \tag{32}$$

$$\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0)=2\ln\left[\sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\beta}\right.$$
$$\left.+\sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\gamma}\ \right]+\frac{\alpha+\left(\frac{256}{81c^2(c^2-1)^3}\right)^{1/4}}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}$$
$$\times\arcsin\frac{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\alpha-\gamma)+((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\gamma)(\alpha-\beta)}{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha)(\beta-\gamma)},$$
$$\beta>\alpha>\gamma,\quad\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)>\beta, \tag{33}$$

$$\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0)=2\ln\left[\sqrt{\beta-\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)}\right.$$
$$\left.+\sqrt{\gamma-\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)}\ \right]+\frac{\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4}}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}$$
$$\times\ln\frac{\left[\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\alpha-\gamma)}-\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\gamma)(\beta-\alpha)}\ \right]^2}{\alpha-(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})},$$
$$\alpha>\beta,\quad\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)<\gamma, \tag{34}$$

$$\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0)=2\ln\left[\sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\beta}\right.$$
$$\left.+\sqrt{\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\gamma}\ \right]+\frac{\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4}}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}$$
$$\times\ln\frac{\left[\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\alpha-\gamma)}-\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\gamma)(\alpha-\beta)}\ \right]^2}{|\alpha-(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})|},$$
$$\beta>\gamma>\alpha,\quad\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)<\gamma, \tag{35}$$

$$\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0)=2\ln\left[\sqrt{\beta-\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)}\right.$$

$$\left.+\sqrt{\gamma-\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)}\,\right]+\frac{\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4}}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}$$

$$\times\arcsin\frac{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\alpha-\gamma)+((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\gamma)(\alpha-\beta)}{|((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha)(\beta-\gamma)|},$$

$$\gamma<\alpha<\beta,\quad\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)<\gamma. \tag{36}$$

When $\epsilon=-1$, the corresponding solutions are

$$\pm\sqrt{\frac{1-c^2}{c^2}}(x-ct-\xi_0)=-\arcsin\frac{\beta+\gamma-(\frac{c^2-1}{4c^2})^{1/4}\left(\pm2\sqrt{u}-\frac{4}{3(c^2-1)}\right)}{\beta-\gamma}$$

$$+\frac{\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4}}{\sqrt{(\alpha-\beta)(\gamma-\alpha)}}$$

$$\times\ln\frac{\left[\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)(\gamma-\alpha)}-\sqrt{(\gamma-(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)}))(\alpha-\beta)}\right]^2}{|(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha|},$$

$$\gamma<\alpha<\beta,\quad\gamma<\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)<\beta, \tag{37}$$

$$\pm\sqrt{\frac{1-c^2}{c^2}}(x-ct-\xi_0)=-\arcsin\frac{\beta+\gamma-(\frac{c^2-1}{4c^2})^{1/4}(\pm2\sqrt{u}-\frac{4}{3(c^2-1)})}{\beta-\gamma}+\frac{\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4}}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}$$

$$\times\arcsin\frac{(\gamma-\alpha)((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\beta)+(\alpha-\beta)(\gamma-(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)}))}{(\beta-\gamma)|(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha|},$$

$$\alpha>\beta\text{ or }\alpha<\gamma. \tag{38}$$

Case 2.2.5. $D_4=0$, $D_3=0$, $D_2>0$, $E_2=0$. We have

$$F(v)=(v-\alpha)^3(v-\beta), \tag{39}$$

where α, β are real numbers. When $\epsilon = +1$, $v > \alpha, v > \beta$, or $v < \alpha, v < \beta$. The corresponding solutions are

$$\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0)$$
$$= \ln\left|2\sqrt{\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\beta\right)\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\alpha\right)}\right.$$
$$\left.+2\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)\right|$$
$$-\frac{2\left(\alpha+\left(\frac{256}{81c^2(c^2-1)^3}\right)^{1/4}\right)\sqrt{\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\beta\right)\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\alpha\right)}}{(\alpha-\beta)\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-\alpha\right)}, \quad (40)$$

When $\epsilon = -1$, $v > \alpha, v < \beta$, or $v < \alpha, v > \beta$. The corresponding solutions are

$$\pm\sqrt{\frac{1-c^2}{c^2}}(x-ct-\xi_0) = -\arcsin\frac{\beta+\alpha-(\frac{c^2-1}{4c^2})^{1/4}(\pm2\sqrt{u}-\frac{4}{3(c^2-1)})}{|\beta-\alpha|}$$
$$+\frac{2(\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4})\sqrt{\beta-(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})}}{(\beta-\alpha)\sqrt{(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha}}. \quad (41)$$

Case 2.2.6. $D_4 = 0$, $D_2D_3 < 0$. Then we have

$$F(v) = (v-\alpha)^2[(v-l)^2+s^2], \quad (42)$$

where α, l and s are real numbers. When $\epsilon = +1$, the corresponding solutions are

$$\pm\sqrt{\frac{c^2-1}{c^2}}(x-ct-\xi_0) = \ln\left|\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-l\right)\right.$$
$$\left.+\sqrt{\left(\left(\frac{c^2-1}{4c^2}\right)^{1/4}\left(\pm\sqrt{u}-\frac{2}{3(c^2-1)}\right)-l\right)^2+s^2}\right|+\frac{\alpha+(\frac{256}{81c^2(c^2-1)^3})^{1/4}}{\sqrt{(\alpha-l)^2+s^2}}$$
$$\times\ln\left|\frac{\gamma(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})+\delta-\sqrt{((\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-l)^2+s^2}}{(\frac{c^2-1}{4c^2})^{1/4}(\pm\sqrt{u}-\frac{2}{3(c^2-1)})-\alpha}\right|, \quad (43)$$

where

$$\gamma = \frac{\alpha-2l}{\sqrt{(\alpha-l)^2+s^2}}, \quad \delta = \sqrt{(\alpha-l)^2+s^2}-\frac{\alpha(\alpha-2l)}{\sqrt{(\alpha-l)^2+s^2}}.$$

3. Exact solutions of the PC equation with $p = 3/2$

When $p = 3/2$, eq. (2) reads as

$$u_{tt} - u_{ttxx} - u_{xx} - \frac{2}{3}(u^{3/2})_{xx} = 0. \tag{44}$$

Taking the travelling wave transformation $u = u(\xi)$ and $\xi = x - ct$, we can obtain the corresponding reduced ODE as

$$c^2u'' - c^2u^{(4)} - u'' - \frac{1}{2}u^{-1/2}(u')^2 + u^{1/2}u'' = 0. \tag{45}$$

Integrating it, we can obtain the following equation:

$$(u')^2 = -\frac{8}{15c^2}u^{5/2} + \frac{c^2-1}{c^2}u^2 + \frac{2g_1}{c^2}u + \frac{2g_2}{c^2}, \tag{46}$$

where g_1 and g_2 are two arbitrary constants.

Taking the transformation $w^2 = u$, eq. (46) becomes

$$(w')^2 = \frac{1}{w^2}(b_5w^5 + b_4w^4 + b_2w^2 + b_0), \tag{47}$$

where

$$b_5 = \frac{-2}{15c^2}, \quad b_4 = \frac{c^2-1}{4c^2}, \quad b_2 = \frac{g_1}{2c^2}, \quad b_0 = \frac{g_2}{2c^2}.$$

The integral form of eq. (47) is

$$\pm(\xi - \xi_0) = \int \frac{w\mathrm{d}w}{\sqrt{b_5w^5 + b_4w^4 + b_2w^2 + b_0}}. \tag{48}$$

According to eq. (48), we shall give the single travelling wave solutions to the generalized PC equation (44). There exist two cases to be discussed.

Case 3.1. $b_0 = 0$. Equation (48) becomes

$$\pm(\xi - \xi_0) = \int \frac{\mathrm{d}w}{\sqrt{b_5w^3 + b_4w^2 + b_2}}. \tag{49}$$

Letting $w_1 = b_5^{1/3}w$, $d_2 = b_4b_5^{-2/3}$, $d_0 = b_2$, eq. (49) becomes

$$\pm b_5^{1/3}(\xi - \xi_0) = \int \frac{\mathrm{d}w_1}{\sqrt{w_1^3 + d_2w_1^2 + d_0}}. \tag{50}$$

We denote

$$F(w_1) = w_1^3 + d_2w_1^2 + d_0, \tag{51}$$

and write its complete discrimination system as follows:

$$\Delta = -27\left(\frac{2d_2^3}{27} + d_0\right)^2 + \frac{4d_2^6}{27}, \quad D_1 = -\frac{d_2^2}{3}. \tag{52}$$

Case 3.1.1. $\Delta = 0$, $D_1 < 0$. Then we have

$$F(w_1) = (w_1 + 2\beta)^2 (w_1 - \beta). \tag{53}$$

When $w_1 > \beta$, the corresponding solutions are

$$u(x,t) = \left(\frac{-2}{15c^2}\right)^{-2/3} \times \left\{3\beta \tan^2\left[\sqrt{3\beta}\left(\frac{-1}{60c^2}\right)^{1/3}(x - ct - \xi_0)\right] + \beta\right\}^2, \quad \beta > 0, \tag{54}$$

$$u(x,t) = \left(\frac{-2}{15c^2}\right)^{-2/3} \times \left\{-3\beta \tanh^2\left[\sqrt{-3\beta}\left(\frac{-1}{60c^2}\right)^{1/3}(x - ct - \xi_0)\right] + \beta\right\}^2, \quad \beta < 0, \tag{55}$$

$$u(x,t) = \left(\frac{-2}{15c^2}\right)^{-2/3} \times \left\{-3\beta \coth^2\left[\sqrt{-3\beta}\left(\frac{-1}{60c^2}\right)^{1/3}(x - ct - \xi_0)\right] + \beta\right\}^2, \quad \beta < 0. \tag{56}$$

Case 3.1.2. $\Delta > 0$, $D_1 < 0$. Then we have

$$F(w_1) = (w_1 - \alpha)(w_1 - \beta)(w_1 - \gamma), \tag{57}$$

where $\alpha < \beta < \gamma, \alpha\gamma + \alpha\beta + \beta\gamma = 0$. When $\alpha < w_1 < \gamma$, the corresponding solutions are

$$u(x,t) = \left(-\frac{1}{15c^2}\right)^{-2/3} \times \left\{\alpha + (\beta - \alpha)\,\mathrm{sn}^2\left(\frac{\sqrt{\gamma - \alpha}}{2}\left(-\frac{1}{15c^2}\right)^{1/3}(x - ct - \xi_0), m\right)\right\}^2, \tag{58}$$

where

$$m^2 = \frac{\beta - \alpha}{\gamma - \alpha}.$$

When $w_1 > \gamma$, the corresponding solutions are

$$u(x,t) = \left(-\frac{1}{15c^2}\right)^{-2/3} \left\{\frac{\gamma - \beta\,\mathrm{sn}^2(\frac{\sqrt{\gamma-\alpha}}{2}(-\frac{1}{15c^2})^{1/3}(x - ct - \xi_0), m)}{\mathrm{cn}^2(\frac{\sqrt{\gamma-\alpha}}{2}(-\frac{1}{15c^2})^{1/3}(x - ct - \xi_0), m)}\right\}^2, \tag{59}$$

where

$$m^2 = \frac{\beta - \alpha}{\gamma - \alpha}.$$

Case 3.1.3. $\Delta < 0$. Then we have

$$F(w_1) = (w_1 - \alpha)(w_1^2 + pw_1 + \alpha p), \tag{60}$$

where $p^2 - 4\alpha p < 0$. When $w_1 > \alpha$, the corresponding solutions are

$$u(x,t) = \left(-\frac{1}{15c^2}\right)^{-2/3} \times \left\{\alpha - \sqrt{\alpha^2 + 2p\alpha} + \frac{2\sqrt{\alpha^2 + 2p\alpha}}{1 + \mathrm{cn}((\alpha^2 + 2p\alpha)^{1/4}(-\frac{1}{15c^2})^{1/3}(x - ct - \xi_0), m)}\right\}^2, \tag{61}$$

where

$$m^2 = \frac{1}{2}\left(1 - \frac{\alpha + \frac{p}{2}}{\sqrt{\alpha^2 + 2p\alpha}}\right).$$

Case 3.2. $b_0 \neq 0$. We take the transformation $w_1 = b_5^{1/5}(w + (b_4/5b_5)), \xi_1 = b_5^{1/5}\xi$, eq. (48) becomes

$$\pm b_5^{1/5}(\xi_1 - \xi_0) = \int \frac{(w_1 - a)\mathrm{d}w_1}{\sqrt{w_1^5 + pw_1^3 + qw_1^2 + rw_1 + s}}, \tag{62}$$

where

$$a = \frac{1}{5}b_4 b_5^{-4/5}, \quad p = -\frac{2}{5}b_4^2 b_5^{-8/5}, \quad q = \frac{4}{25}b_4^3 b_5^{-12/5} + b_2 b_5^{-2/5},$$
$$r = -\frac{3}{125}b_4^4 b_5^{-16/5} + \frac{2}{5}b_4 b_2 b_5^{-6/5}, \quad s = \frac{4}{3125}b_4^5 b_5^{-4} + \frac{1}{25}b_2 b_4^2 b_5^{-2} + b_0. \tag{63}$$

Denote

$$F(w_1) = w_1^5 + pw_1^3 + qw_1^2 + rw_1 + s, \tag{64}$$

and write its complete discrimination system as follows:

$$D_2 = -p, \quad D_3 = 40rp - 12p^3 - 45q^2,$$
$$D_4 = -4p^3q^2 + 12p^4r + 117pq^2r - 88p^2r^2 - 40qsp^2 - 27q^4 - 300qrs + 160r^3,$$

$$\begin{aligned} D_5 &= -1600qsr^3 - 3750pqs^3 + 2000ps^2r^2 - 4p^3q^2r^2 + 16p^3q^3s \\ &\quad -900rs^2p^3 + 825p^2q^2s^2 + 144pq^2r^3 + 2250rq^2s^2 \\ &\quad +16p^4r^3 + 108p^5s^2 - 128r^4p^2 - 27r^2q^4 \\ &\quad +108sq^5 + 256r^5 + 3125s^4 - 72rsqp^4 \\ &\quad +560sqr^2p^2 - 630prsq^3, \\ E_2 &= 160r^2p^3 + 900q^2r^2 - 48rp^5 + 60rp^2q^2 + 1500pqrs \\ &\quad +16q^2p^4 - 1100qsp^3 + 625s^2p^2 - 3375sq^3, \\ F_2 &= 3q^2 - 8rp. \end{aligned} \tag{65}$$

We must notice that in Case 3.2, the parameter a in eq. (62) is not equal to the roots of $F(w_1) = 0$.

Case 3.2.1. $D_5 = 0$, $D_4 = 0$, $D_3 > 0$, $E_2 \neq 0$. Then we have

$$F(w_1) = (w_1 - \alpha)^2(w_1 - \beta)^2(w_1 - \gamma), \tag{66}$$

where $\alpha \neq \beta \neq \gamma$. When $w_1 > \gamma$, the corresponding solutions are (here ξ_0 has been rescaled, the same as follows)

$$\begin{aligned} &\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0) \\ &= \frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\alpha}{\sqrt{\gamma-\alpha}} \arctan\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}}{\sqrt{\gamma-\alpha}} \\ &\quad -\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\beta}{\sqrt{\gamma-\beta}} \arctan\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}}{\sqrt{\gamma-\beta}}, \quad \gamma>\alpha, \gamma>\beta, \end{aligned} \tag{67}$$

$$\begin{aligned} &\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0) = \frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\alpha}{\sqrt{\gamma-\alpha}} \\ &\quad\times \arctan\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}}{\sqrt{\gamma-\alpha}} \\ &\quad -\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\beta}{2\sqrt{\beta-\gamma}} \\ &\quad\times \ln\left|\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}-\sqrt{\beta-\gamma}}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}+\sqrt{\beta-\gamma}}\right|, \quad \gamma>\alpha, \gamma<\beta, \end{aligned} \tag{68}$$

$$\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0)=\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\alpha}{2\sqrt{\alpha-\gamma}}$$
$$\times\ln\left|\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}-\sqrt{\alpha-\gamma}}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}+\sqrt{\alpha-\gamma}}\right|$$
$$-\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\beta}{\sqrt{\gamma-\beta}}\arctan\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}}{\sqrt{\gamma-\beta}},$$
$$\gamma<\alpha,\gamma>\beta, \tag{69}$$

$$\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0)=\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\alpha}{2\sqrt{\alpha-\gamma}}$$
$$\times\ln\left|\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}-\sqrt{\alpha-\gamma}}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}+\sqrt{\alpha-\gamma}}\right|$$
$$-\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\beta}{2\sqrt{\beta-\gamma}}\ln\left|\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}-\sqrt{\beta-\gamma}}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\gamma}+\sqrt{\beta-\gamma}}\right|,$$
$$\gamma<\alpha,\gamma<\beta. \tag{70}$$

Case 3.2.2. $D_5=0$, $D_4=0$, $D_3=0$, $D_2\neq 0$, $F_2\neq 0$. Then we have

$$F(w_1)=(w_1-\alpha)^3(w_1-\beta)^2, \tag{71}$$

where $\alpha\neq\beta$. When $w_1>\alpha$, the corresponding solutions are

$$\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0)=-\frac{\left(\frac{81(c^2-1)^5}{81920c^2}\right)^{1/5}-\alpha}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}$$
$$-\frac{\left(\frac{81(c^2-1)^5}{81920c^2}\right)^{1/5}-\beta}{\sqrt{\alpha-\beta}}\arctan\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}{\sqrt{\alpha-\beta}},\quad \alpha>\beta, \tag{72}$$

$$\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0)=-\frac{\left(\frac{81(c^2-1)^5}{81920c^2}\right)^{1/5}-\alpha}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}$$
$$-\frac{\left(\frac{81(c^2-1)^5}{81920c^2}\right)^{1/5}-\beta}{2\sqrt{\beta-\alpha}}\ln\left|\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}-\sqrt{\beta-\alpha}}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}+\sqrt{\beta-\alpha}}\right|,$$
$$\alpha<\beta. \tag{73}$$

Case 3.2.3. $D_5 = 0$, $D_4 = 0$, $D_3 = 0$, $D_2 \neq 0$, $F_2 = 0$. Then we have

$$F(w_1) = (w_1 - \alpha)^4 (w_1 - \beta), \tag{74}$$

where $\alpha \neq \beta$. When $w_1 > \beta$, the corresponding solutions are

$$\begin{aligned} &\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0) \\ &= -\frac{((\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\alpha)\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\beta}}{2((-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha)} \\ &-\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}+\alpha-2\beta}{2\sqrt{\beta-\alpha}} \\ &\times \arctan\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\beta}}{\sqrt{\beta-\alpha}}, \quad \alpha<\beta, \end{aligned} \tag{75}$$

$$\begin{aligned} &\pm(\beta-\alpha)\left(\frac{1}{1800c^2}\right)^{1/5}(x-ct-\xi_0) \\ &= -\frac{((\frac{81(c^2-1)^5}{81920c^2})^{1/5}-\alpha)\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\beta}}{2((-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha)} \\ &-\frac{(\frac{81(c^2-1)^5}{81920c^2})^{1/5}+\alpha-2\beta}{4\sqrt{\alpha-\beta}} \\ &\times \ln\left|\frac{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\beta}-\sqrt{\alpha-\beta}}{\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\beta}+\sqrt{\alpha-\beta}}\right|, \quad \alpha>\beta. \end{aligned} \tag{76}$$

Case 3.2.4. $D_5 = 0$, $D_4 = 0$, $D_3 = 0$, $D_2 = 0$. Then we have

$$F(w_1) = (w_1 - \alpha)^5. \tag{77}$$

$p = q = r = s = 0$. So $b_4 = b_2 = b_0 = 0$. This case is included in Case 1.

Case 3.2.5. $D_5 = 0$, $D_4 = 0$, $D_3 < 0$, $E_2 \neq 0$. Then we have

$$F(w_1) = (w_1 - \alpha)(w_1^2 + \beta w_1 + \gamma)^2, \tag{78}$$

where $\beta^2 - 4\gamma < 0$. When $w_1 > \beta$, the corresponding solutions are

$$\pm\left(-\frac{2}{15c^2}\right)^{1/5}(x-ct-\xi_0) = \frac{2}{\rho\sqrt{4\gamma-\beta^2}}\left\{\frac{\sin\varphi}{2}\right.$$
$$\times\left[\rho^2\ln\frac{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha+\rho^2-2\rho\cos\varphi\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha+\rho^2+2\rho\cos\varphi\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}\right.$$
$$-\left(\alpha-\left(\frac{81(c^2-1)^5}{81920c^2}\right)^{1/5}\right)$$
$$\left.\times\ln\frac{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha-\rho^2-2\rho\cos\varphi\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha-\rho^2+2\rho\cos\varphi\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}\right]$$
$$-\cos\varphi\left[\rho^2+\alpha-\left(\frac{81(c^2-1)^5}{81920c^2}\right)^{1/5}\right]$$
$$\left.\times\arctan\frac{2\rho\sin\varphi\sqrt{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha}}{(-\frac{2}{15c^2})^{1/5}(\sqrt{u}-\frac{3(c^2-1)}{8})-\alpha-\rho^2}\right\}, \tag{79}$$

where

$$\rho = (\alpha^2+\alpha\beta+\gamma)^{1/4}, \quad \varphi = \frac{1}{2}\arctan\frac{\sqrt{4\gamma-\beta^2}}{-2\alpha-\beta}.$$

4. Conclusion

In this paper, we take the travelling wave transformation and get the corresponding reduced ODE to the generalized PC equation. By integrating and taking some transformation, we gain the corresponding integral in terms of the polynomial. We apply the complete discrimination system for polynomial to obtain the classification of single travelling wave solutions to the generalized PC equation with $p = 1/2$ and $p = 3/2$. If we take the concrete parameters, the solutions in the classification can be realized.

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