PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 81, No. 5
November 2013
pp. 791–798

Next-to-next-to-leading order calculation of the strong coupling constant α_s by using moments of event-shape variables

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MS received 22 April 2013; revised 4 August 2013; accepted 13 August 2013
DOI: 10.1007/s12043-013-0615-3; ***e*Publication:** 18 October 2013

Abstract. The next-to-next-to-leading order (NNLO) quantum chromodynamics (QCD) correction to the first three moments of the four event-shape variables in electron–positron annihilation, the thrust, heavy jet mass, wide, and total jet broadening, is computed. It is observed that the NNLO correction gives a better agreement between the theory and the experimental data. Also, by using the above observables, the strong coupling constant (α_s) is determined and how much its value is affected by the NNLO correction is demonstrated. By combining the results for all variables at different centre-of-mass energies $\alpha_s(M_{Z^0}) = 0.1248 \pm 0.0009\,(\text{exp.})^{+0.0283}_{-0.0144}\,(\text{theo.})$ is obtained.

Keywords. Event shape; quantum chromodynamics; next-to-next-to-leading order correction.

PACS No. **13.66.Bc**

1. Introduction

Quantum chromodynamics (QCD) is generally accepted to be the correct theory for describing the strong interaction between quarks and gluons [1]. If the quark masses and their mixing angles are fixed, then the only free parameter of this theory is the strong coupling constant (α_s). Therefore, it is of paramount importance to measure this parameter to the best possible precision. One way to determine α_s is by calculating the moment of event-shape variables. The nth moment of an event-shape observable y is defined as

$$\langle y^n \rangle = \frac{1}{\sigma_{\text{had}}} \int_0^{y_{\max}} y^n \frac{\mathrm{d}\sigma}{\mathrm{d}y} \mathrm{d}y, \tag{1}$$

where $y_{\max}$ is the kinematically allowed upper limit of the observable [2]. The most common observables are: thrust T [3,4] (where moments of $y = (1 - T)$ is taken), the heavy jet mass $\rho = M_{\text{H}}^2/S$ [5], the wide and total jet broadenings, B_{w} and B_{T} [6,7].

While the NNLO correction should offer a better value for α_s (to the inclusion of additional radiation at parton level), it is worthwhile to study this correction not only to the first but also to higher moments of these observables. Besides the experimental uncertainty, one need to take into account the theoretical uncertainty on such a fundamental quantity.

2. NNLO correction to event-shape moments

The perturbative contribution to $\langle y^n \rangle$ upto NNLO can be given in terms of three dimensionless coefficients $\bar{A}_{y,n}$, $\bar{B}_{y,n}$ and $\bar{C}_{y,n}$ as

$$\begin{aligned}\langle y^n \rangle (s, \mu^2) &= \left(\frac{\alpha_s(\mu)}{2\pi}\right) \bar{A}_{y,n} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \left(\bar{B}_{y,n} + \bar{A}_{y,n}\beta_0 \log \frac{\mu^2}{s}\right) \\ &\quad + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 \left(\bar{C}_{y,n} + 2\bar{B}_{y,n}\beta_0 \log \frac{\mu^2}{s} \right. \\ &\quad \left. + \bar{A}_{y,n}\left(\beta_0^2 \log^2 \frac{\mu^2}{2} + \beta_1 \log \frac{\mu^2}{s}\right)\right) + \mathcal{O}(\alpha_s^4), \qquad (2)\end{aligned}$$

where s denotes the centre-of-mass energy squared and μ is the QCD renormalization scale [8]. The NLO expression is obtained by suppressing all terms at order α_s^a. The first two coefficients of the QCD β-function are

$$\beta_0 = \frac{11C_A - 4T_R N_F}{6}$$

and

$$\beta_1 = \frac{17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F}{6},$$

with $C_A = N$, $C_F = ((N^2 - 1)/N)$, $T_R = \frac{1}{2}$ for $N = 3$ colours and N_F quark flavours.

The perturbative coefficients in eq. (2) are independent of the centre-of-mass energy. They are obtained by integrating parton-level distributions, which were calculated recently to the NNLO level [9–12].

In figure 1, we show the energy dependence of the first moments of all event-shape variables at NLO and NNLO. The theory predictions are compared to PYTHIA Monte Carlo. Comparing NLO and NNLO predictions, we observe that the NNLO contribution comes closer to the Monte Carlo data for $1 - T$ and B_T, while negligible improvement over NLO is observed for other observables.

The higher moments of the four event shapes are displayed in figures 2 ($1 - T$), 3 (ρ), 4 (B_T) and 5 (B_w). Compared again to the Monte Carlo data, the qualitative behaviour of the higher moments of the different shape variable is similar to what was observed for the first moments. Although, for $1 - T$ and B_T, higher moments cause an increase in NNLO correction (almost 10% and 30% respectively), for ρ it is the reverse and for B_w, it is negligible. All figures and data are consistent with the data from OPAL and JADE experiments [13] and also with NLO and NNLO corrections in ref. [12].

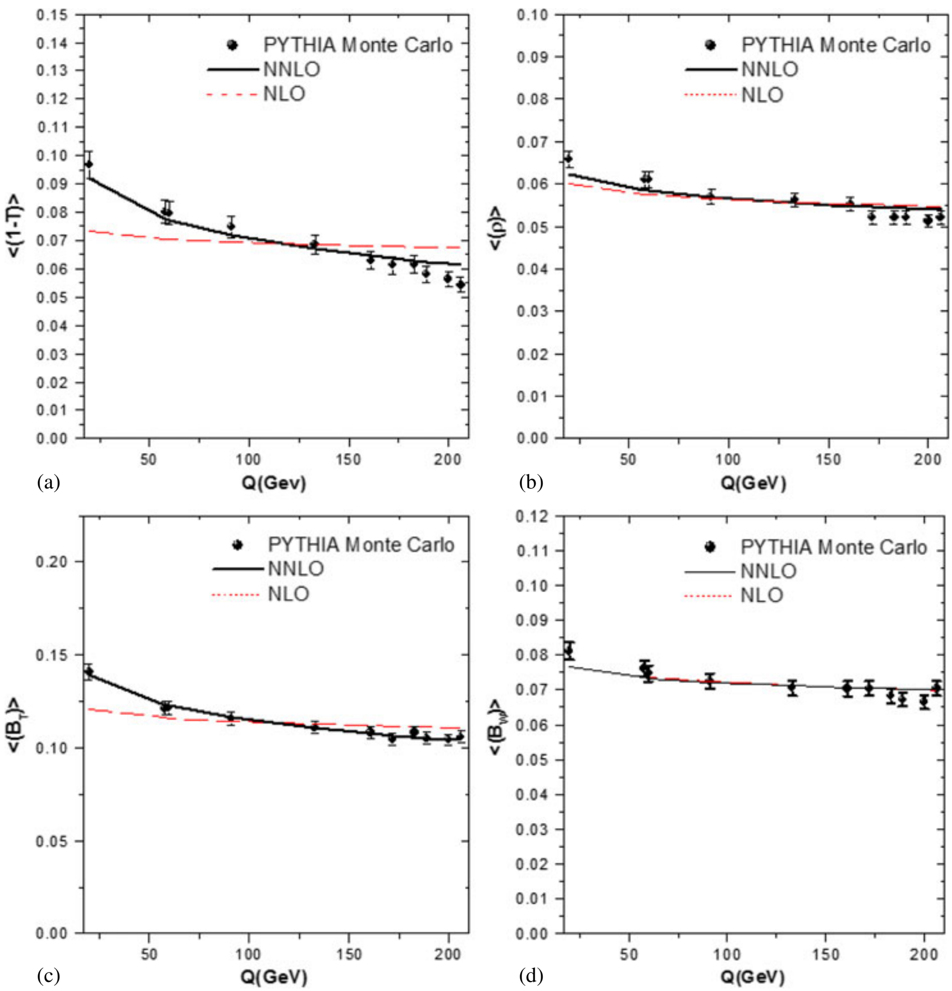

Figure 1. First moment of four event-shape variables: (**a**) $1 - T$, (**b**) ρ, (**c**) B_{T}, (**d**) B_{w}.

3. Determination of the strong coupling constant

3.1 *Theoretical uncertainties*

Comparing the different sources of error in the extraction of α_s from hadronic data, one finds that the purely experimental error is negligible compared to the theoretical uncertainty. There are two sources of theoretical uncertainty: the theoretical description of the parton-to-hadron transition (hadronization uncertainty) and the uncertainty stemming from the truncation of the perturbative series at a certain order, as estimated by the scale variation (perturbative or scale uncertainty). Although the accuracy of the hadronization uncertainty is debatable and perhaps often underestimated, it is conventional to consider

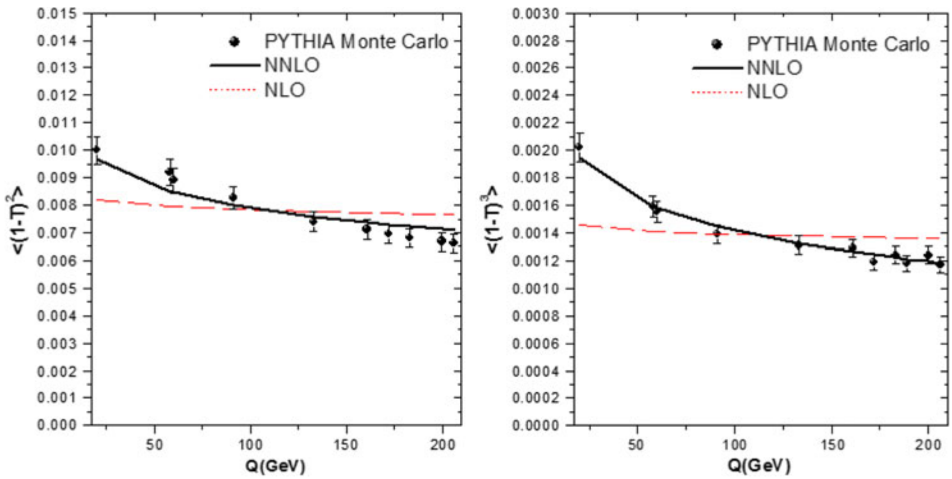

Figure 2. Perturbative corrections to the higher moments of $1 - T$.

the scale uncertainty as the dominant theoretical error on the precise determination of α_s [8]. The scale uncertainty is obtained by repeating the fit for different values of the renormalization scale in the interval $0.5\sqrt{s} \leq \mu \leq 2\sqrt{s}$. In tables 1–4 theoretical errors are also added to make the value of α_s more precise.

3.2 *Comparison of NNLO correction with NLO*

To further clarify the situation, we compare the NNLO correction to NLO correction. The reason for doing this is to see if the value of α_s is affected by increasing the order of our

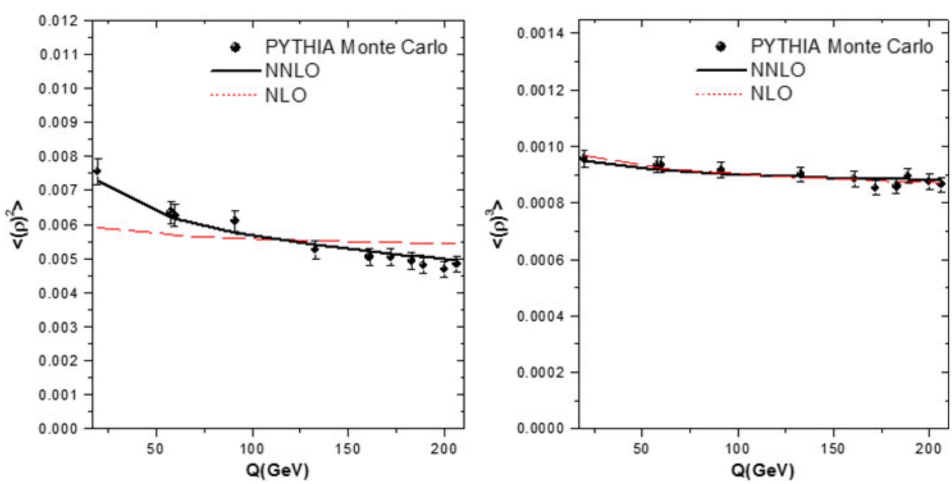

Figure 3. Perturbative corrections to the higher moments of the normalized heavy jet mass ρ.

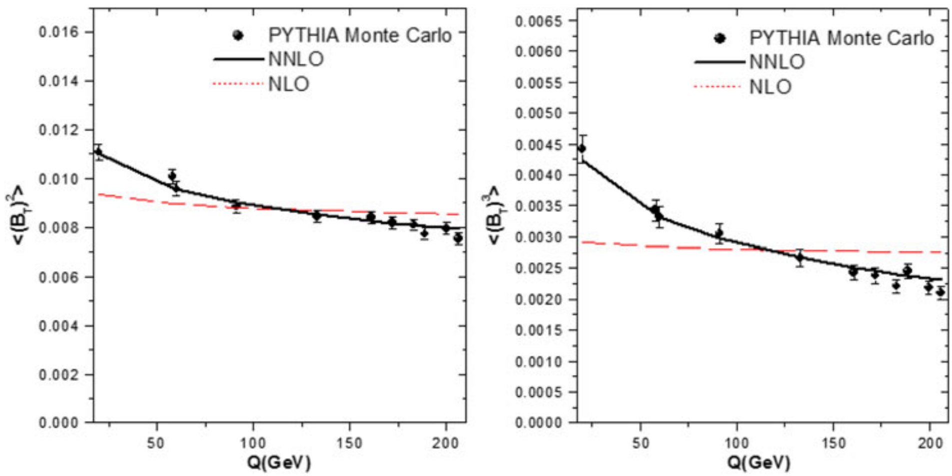

Figure 4. Perturbative corrections to the higher moments of the total jet broadening B_T.

calculation. In the case of pure fixed-order predictions, the main source of arbitrariness in the prediction is the choice of the renormalization scale χ_μ. The residual dependence of the fitted value of $\alpha_s(M_Z^2)$ on the renormalization scale is shown in figure 6. A dramatic reduction of the scale dependence is observed when going from NLO to NNLO.

Next we fit the QCD predictions (NLO and NLO corrections) for hadronization to the data for a given observable and moment $n = 1, 2, 3$ individually with $\alpha_s(M_{Z^0})$ as the only free parameter. The results for $\alpha_s(M_{Z^0})$ are summarized in figure 7. For higher moments,

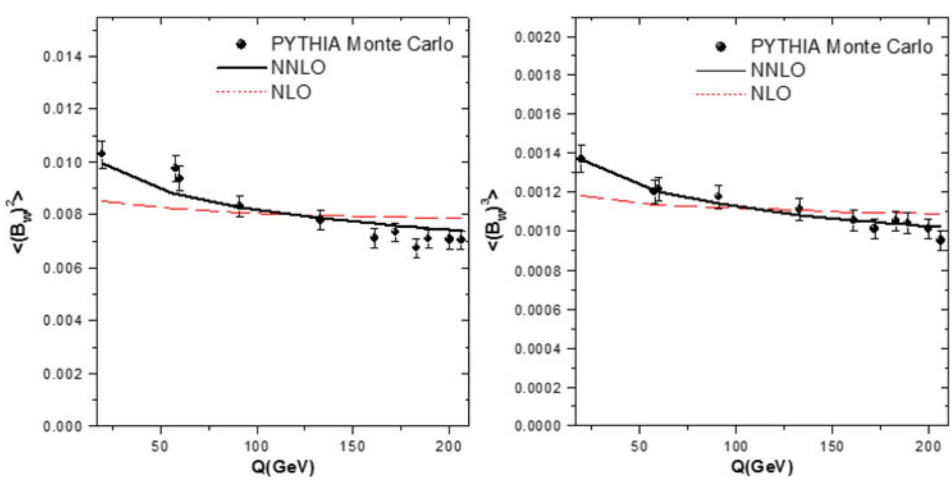

Figure 5. Perturbative corrections to the higher moments of the wide jet broadening B_w.

Table 1. Values of α_s measured from fit to the first and higher moments of thrust.

	$\langle(1-T)\rangle$	$\langle(1-T)^2\rangle$	$\langle(1-T)^3\rangle$
α_s	$0.1324 \pm 0.0015\ ^{+0.0289}_{-0.0145}$	$0.1305 \pm 0.0008\ ^{+0.0239}_{-0.0132}$	$0.1339 \pm 0.0006\ ^{+0.0245}_{-0.0148}$

Table 2. Values of α_s measured from fit to the first and higher moments of wide jet broadening.

	$\langle(B_w)\rangle$	$\langle(B_w)^2\rangle$	$\langle(B_w)^3\rangle$
α_s	$0.1116 \pm 0.0011\ ^{+0.0143}_{-0.0115}$	$0.1218 \pm 0.0019\ ^{+0.0287}_{-0.0149}$	$0.1189 \pm 0.0007\ ^{+0.0302}_{-0.0142}$

Table 3. Values of α_s measured from fit to the first and higher moments of total jet broadening.

	$\langle(B_T)\rangle$	$\langle(B_T)^2\rangle$	$\langle(B_T)^3\rangle$
α_s	$0.1221 \pm 0.0003\ ^{+0.0229}_{-0.0147}$	$0.1300 \pm 0.0008\ ^{+0.0145}_{-0.0085}$	$0.1366 \pm 0.0009\ ^{+0.0230}_{-0.0134}$

Table 4. Values of α_s measured from fit to the first and higher moments of heavy jet mass.

	$\langle\rho\rangle$	$\langle(\rho)^2\rangle$	$\langle(\rho)^3\rangle$
α_s	$0.1018 \pm 0.0007\ ^{+0.0394}_{-0.0170}$	$0.1364 \pm 0.0011\ ^{+0.0344}_{-0.0154}$	$0.1214 \pm 0.0007\ ^{+0.0545}_{-0.0205}$

we observe that the values of $\alpha_s(M_{Z^0})$ increase with n for the observables $\langle(1-T)^n\rangle$ and $\langle(B_T)^n\rangle$, while for $\langle(B_w)^n\rangle$ and $\langle(\rho)^n\rangle$, its value decrease. What is more, due to an increase in the order of correction, experimental errors also drop significantly. ($\langle(B_w)^n\rangle$ is less sensitive to NNLO correction.)

To obtain a combined value for the strong coupling constant we take an unweighted average of the 12 values, presented in tables 1–4. Similarly, we estimate the overall theoretical and experimental errors from their simple average. The result of the combination is

$$\alpha_s(M_{Z^0}) = 0.1248 \pm 0.0009\,(\text{exp.})\ ^{+0.0283}_{-0.0144}\ (\text{theo.})\,,$$

which is above but still consistent with the world average (0.1184 ± 0.0027 [14]). Combining only the fit results from $\langle(1-T)\rangle$, $\langle(B_w)\rangle$, $\langle(B_T)\rangle$ and $\langle\rho\rangle$ yields a value of

$$\alpha_s(M_{Z^0}) = 0.1170 \pm 0.0009\,(\text{exp.})\ ^{+0.0263}_{-0.0144}\ (\text{theo.})\,.$$

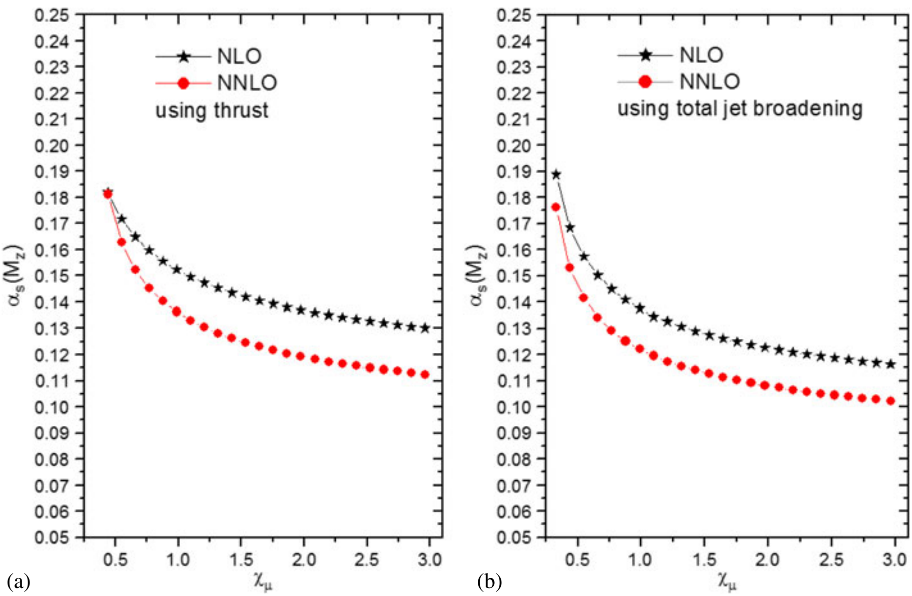

Figure 6. Dependence of the extracted α_s on the renormalization scale when fitting (**a**) the thrust and (**b**) the total jet broadening moments with predictions at different orders of pertubative theory.

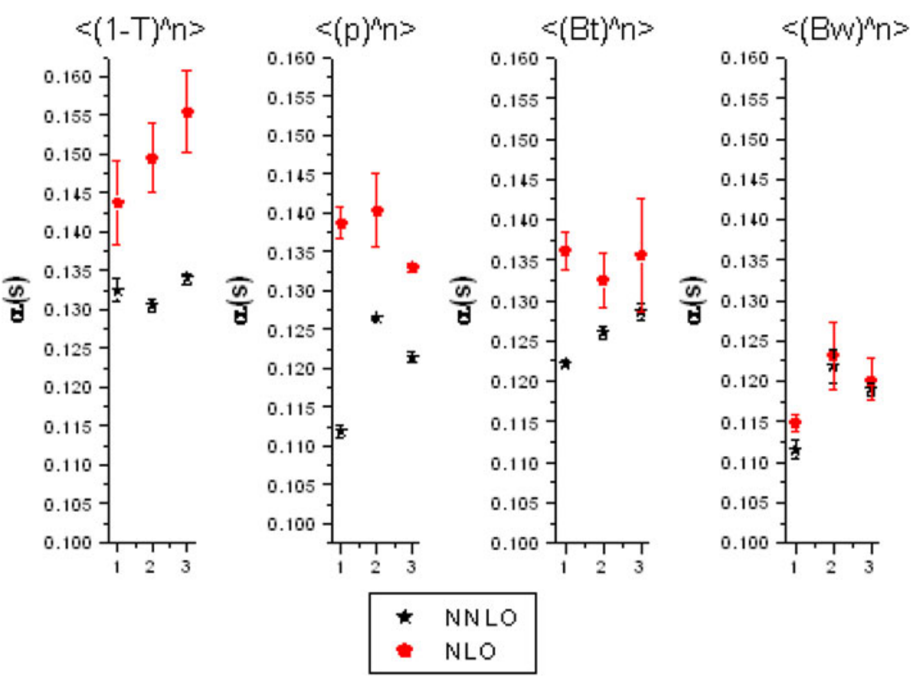

Figure 7. Variation of $\alpha_s(M_Z)$ in terms of first and higher moments for NLO and NNLO corrections. Error is experimental.

The slightly smaller error of α_s reflects the fact that the lower-order moments were less sensitive to the multijet region of the event-shape distributions.

4. Conclusions

In this paper we used NNLO QCD corrections to event-shapes moments $\langle(1-T)^n\rangle$, $\langle(B_W)^n\rangle$, $\langle(B_T)^n\rangle$ and $\langle(\rho)^n\rangle$ in e^-e^+ annihilation to determine the strong coupling constant α_s. The uncertainties of these measurements are dominated by the theoretical uncertainty arising from the unknown higher-order contribution in the calculation. The NNLO corrections to different event shapes are sizeable and the value of α_s obtained in NNLO is closer to the world average value.

References

[1] R K Ellis, W J Stirling and B R Webber, *QCD and collider physics* (Cambridge University Press, Cambridge, UK, 1996)
G Dissertori, I G Knowles and M Schmelling, *Quantum chromodynamics: High energy experiments and theory* (Oxford University Press, Oxford, UK, 2003)

[2] T Gehrmann, M Jaquier, G Luisoni, *Eur. Phys. J. C* **67**, 57 (2010)

[3] S Brandt, C Peyrou, R Sosnowski and A Wroblewski, *Phys. Lett.* **12**, 57 (1964)

[4] E Farhi, *Phys. Rev. Lett.* **39**, 1587 (1977)

[5] L Clavelli and D Wyler, *Phys. Lett. B* **103**, 383 (1981)

[6] P E L Rakow and B R Webber, *Nucl. Phys. B* **191**, 63 (1981)

[7] S Catani, G Turnock and B R Webber, *Phys. Lett. B* **295**, 269 (1992)

[8] A Gehrmann-De Ridder, T Gehrmann, E W N Glover and G Heinrich, *J. High Energy Phys.* **0905**, 106 (2009), arXiv:0903.4658

[9] A Gehrmann-De Ridder, T Gehrmann, EWN Glover and G Heinrich, *Phys. Rev. Lett.* **99**, 132002 (2007), arXiv:0707.1285

[10] A Gehrmann-De Ridder, T Gehrmann, E W N Glover and G Heinrich, *Phys. Rev. Lett.* **100**, 172001 (2008), arXiv:0802.0813

[11] S Weinzierl, *J. High Energy Phys.* **0906**, 041 (2009), arXiv:0904.1077

[12] A Gehrmann-De Ridder, T Gehrmann, E W N Glover and G Heinrich, *J. High Energy Phys.* **0712**, 094 (2007), arXiv:0711.4711J

[13] C J Pahl, *Untersuchungperturbativer und nichtperturbativer Struktur der Momente hadronischer Ereignisformvariablen mit den Experimenten JADE und OPAL*, Ph.D. Thesis (TU Munich, Munich, Germany, 2007)

[14] S Bethke, *Nucl. Phys. Proc. Suppl.* **13**, 345 (2004)