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Low-lying ($K^\pi = 0^+$) states of gadolinium isotopes

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Abstract. The sd-interacting boson approximation (sd-IBA) and the df-interacting boson approximation (df-IBA) can be related to each other and the states of the interacting boson approximation model can be identified with the fully symmetric states in the sdf interacting boson approximation model. A systematic study of the sdf-IBA model showed that the constructed Hamiltonian can successfully describe the strong octupole correlations in the deformed nuclei. We showed that the interacting boson approximation may account for many of these $K^\pi = 0^+$ states. It was found that the calculated energy spectra of the gadolinium isotopes agree quite well with the experimental data. The observed $B(E2)$ values were also calculated and compared with the experimental data.

Keywords. Interacting boson approximation; octupole character; excited $K^\pi = 0^+$ states.

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1. Introduction

The rare-earth region, with many well-deformed and transitional nuclei, is an ideal venue for studying the origin of deformation and collective motion. Many of the collective modes and quadrupole excitations can form $K^\pi = 0^+$ states. Probably because of this, such states are often complex and remain poorly understood. The nature of collective $K^\pi = 0^+$ states in even–even deformed nuclei is the most controversial subject and observation of a number of $K^\pi = 0^+$ states opened a new window and emphasized the importance of microscopic approach to atomic nuclei [1–3]. Especially, the structure of the 0^+ excitation on energy levels as possible as determined has become a research field by itself. For that reason, it is important to understand the origin of such a large number of 0^+ modes.

Recently, 0^+ excitations have attracted much attention and the 0^+ states in deformed nuclei have been studied by many workers. Gerceklioglu [4] has studied the 0^+ excitations in ^{158}Dy, ^{164}Er and ^{172}Hf using the quasiparticle random phase approximation (QRPA) method. Iudice *et al* [5] used QPM including monopole and quadrupole pairing with a quadrupole–quadrupole force term. Making a detailed analysis, they presented the calculations on the microscopic properties including energies, $E2$, $E0$ transitions and two-nucleon spectroscopic factors with the shell and multiphonon structure of the 0^+ states. Another study has been made in [6] by using the pairing-plus-quadrupole model (PPQ), including only monopole pairing; a good description has been given for the distribution and the nature of the 0^+ states. Garrett [7] has studied the experimental properties of the first excited 0^+ states in deformed rare-earth nuclei. Octupole vibrational states were studied and the positive- and negative-parity states of ^{150}Nd isotope were revealed by Elver *et al* [8] and compared with interacting boson approximation (IBA) calculations. Meyer *et al* [9] have obtained 0^+ states of nuclei such as ^{152}Gd, ^{154}Gd, ^{162}Dy, ^{168}Er, ^{176}Hf, ^{180}W, ^{184}W and ^{190}Os.

The studies mentioned above imply that new microscopic models and interactions that can give new contributions are necessary. Certainly, various collective 0^+ modes can exist and it is clearly interesting to determine how many such excitations appear below $\sim$3 MeV. The aim of the present work is to systematically study the gadolinium isotopes within the sdf-IBA model to give a comprehensive view of these isotopes in a rather simple way. The model we are using has been extensively described in [10] and so we shall present here only the results of the calculation and refer the reader to [10] for details of the model. We restrict the discussion to the $^{154-156}$Gd isotopes because the discussion on ^{158}Gd isotope was already presented elsewhere [11].

2. The energy levels

The interacting boson approximation is widely accepted as a tractable theoretical scheme for correlating, describing and predicting low-energy collective properties of complex nuclei. In this approximation, it is assumed that low-lying collective states of even–even nuclei can be described as states of a given (fixed) number N of bosons. Each boson can occupy energy level 1 with angular momentum $L = 0$ (s-boson), usually with lower energy, with $L = 2$ (d-boson) and with $L = 3$ (f-boson). In the original form of the model known as IBA-1, proton- and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure associated with it. In terms of s- and d-boson operators, the most general sd-IBA Hamiltonian can be expressed as [10]

$$H_{\mathrm{sd}} = \varepsilon' n_{\mathrm{d}} + \frac{1}{2}\eta(LL) + \frac{1}{2}\kappa(QQ) - 5\sqrt{7}\omega\big[(d^+\tilde{d})^{(3)} \times (d^+\tilde{d})^{(3)}\big]_0^{(0)} + 15\xi\big[(d^+\tilde{d})^{(4)} \times (d^+ + \tilde{d})^{(4)}\big]_0^{(0)}, \quad (1)$$

where

$$LL = -10\sqrt{3}\big[(d^+\tilde{d})^{(1)} \times (d^+\tilde{d})^{(1)}\big]_0^{(0)} \quad (2)$$

and

$$Q_{\mathrm{s}}Q_{\mathrm{d}} = \sqrt{5}\big[\{(s^+\tilde{d}+\tilde{d}s)^{(2)}+\chi(d^+\tilde{d})^{(2)}\}\times\{(s^+\tilde{d}+d^+s)^{(2)}+\chi(d^+\tilde{d})^{(2)}\}\big]_0^{(0)}. \quad (3)$$

Again the parameters were fit by reproducing the deformed nature of this nucleus (e.g., the ground-state rotational band) and the properties of the γ-vibration resulting in the following parameters: $\varepsilon' = 0.308$ MeV, $\omega = 0.001$ MeV, $\eta = 0.0155$ MeV,

$\kappa = -0.02$ MeV, $\chi = -0.910$ and $\xi = 0.0001$ MeV. These parameters are similar to those found for other nuclei in this region. Again, the model is capable only of accounting for relatively few of the now known 0^+ states. For the simple df-boson Hamiltonian space is as follows [10]:

$$H_{\rm df} = \eta'(L_{\rm d}L_{\rm f}) + \kappa'(Q_{\rm d}Q_{\rm f}) - \chi'(O^{(3)}O^{(3)}), \tag{4}$$

where

$$L_{\rm d}L_{\rm f} = -2\sqrt{210}\big[(d^+\tilde{d})^{(1)} \times (f^+\tilde{f})^{(1)}\big]_0^{(0)}, \tag{5}$$

$$Q_{\rm d}Q_{\rm f} = -2\sqrt{35}\big[\big\{(s^+\tilde{d} + d^+s)^{(2)} - \chi(d^+\tilde{d})^{(2)}\big\} \times (f^+\tilde{f})^{(2)}\big]_0^{(0)} \tag{6}$$

and

$$O^{(3)} = (s^+\tilde{f} + f^+s)^{(3)} + \chi(d^+\tilde{f} + f^+\tilde{d})^{(3)}. \tag{7}$$

Here $\eta' = 0.0155$ MeV, $\kappa' = -0.02$ MeV and $\chi' = -0.97$ are the strength of LL-force, QQ-force and octupole force, respectively. Therefore, we carried out further IBA calculations with the help of sd-boson Hamiltonian and df-boson Hamiltonian spaces. The simple Hamiltonian (in terms of sd-df bosons) is,

$$H = H_{\rm sd} + H_{\rm df} \tag{8}$$

and the quadrupole operator is,

$$Q_{\rm sdf} = Q_{\rm sd} + Q_{\rm df}. \tag{9}$$

The parameters ε', ω, η and χ were the same as in the sd-IBA calculation. The calculations with the boson energy $\varepsilon' = 0.308$ MeV reproduce the experimental level states rather well, as seen in figure 1, which shows the predicted 0^+ states below ~3 MeV. We made no attempt to fine-tune the calculations to the empirical 0^+ states (there is insufficient data on the detailed structure of these states to accurately fix the parameter of the full Hamiltonian), and there is no point in invoking a precise energy cut-off. Therefore, it is also appropriate to look slightly above 2.9 MeV where there is a continuing spectrum of 0^+ states, amounting to 15 excited 0^+ states below 3 MeV.

The calculations were done in a simple and straightforward way. No attempt was made to fit individual 0^+ states and no claim was made that specific predicted 0^+ states have a correspondence with specific empirical states. The point was to rather see the large number of 0^+ excitations in the energy range up to ~2–3 MeV.

The empirical 0^+ states and theoretical works of ^{154,156,158}Gd are shown in figure 1. Given that gadolinium is typical of many rare earth and actinide deformed nuclei, one can expect that similar numbers of 0^+ excitations appear throughout the deformed regions of nuclei. It is therefore important to understand the origin of such a large number of $K^\pi = 0^+$ modes. It is known that the sd-IBA can account only for about six or seven excited 0^+ states below ~2.9 MeV but that inclusion of the octupole degree of freedom (f-boson) allows one to predict, perhaps unexpectedly, nearly 15 excited 0^+ states below 3 MeV.

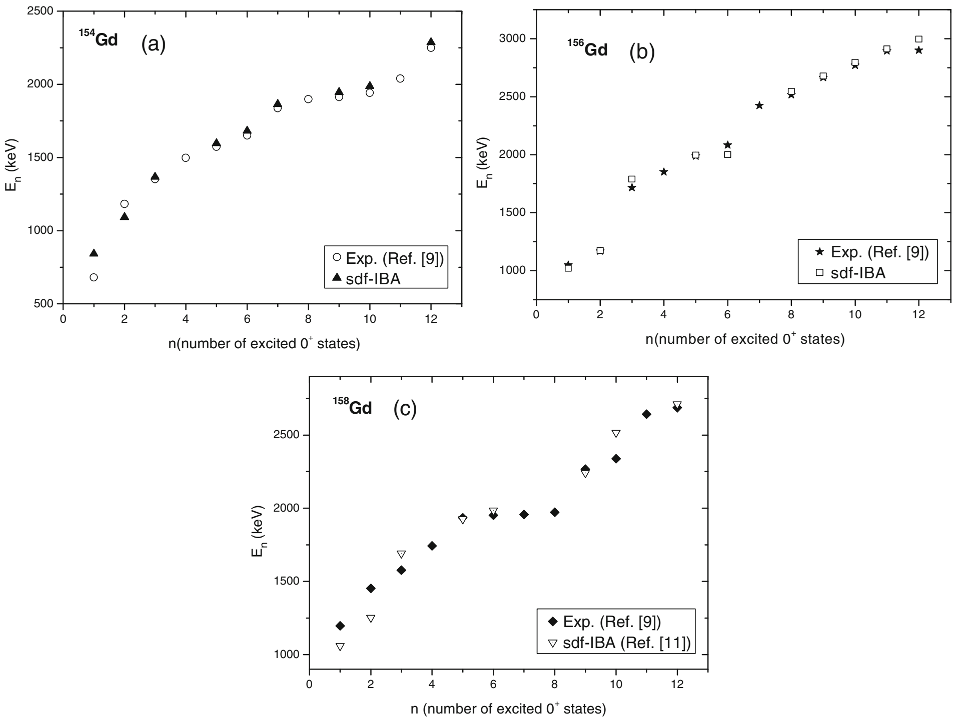

Figure 1. Distribution of the calculated and experimental [9] ($K^{\pi} = 0^{+}$) excitation energies of gadolinium isotopes.

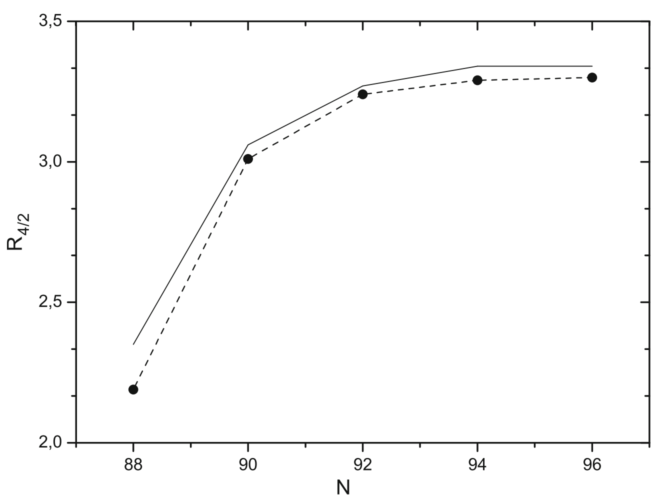

Figure 2. Evolution of experimental [9] (symbols with dashed lines) and calculated (solid lines) $R_{4/2}$ values for gadolinium isotopes as a function of increasing neutron number.

A useful measure of collectivity is the $R_{4/2} = E4_1^+/E2_1^+$ energy ratio and it is the fundamental observable to describe the structure of a nucleus. Usually the region $2 < R_{4/2} < 2.4$ is called the vibrational, $3 < R_{4/2} < 3.33$ is called the rotational and $2.4 < R_{4/2} < 3$ is called transition regions. For the nuclei included in this study, all chains begin as vibrational with $R_{4/2}$ near 2.2 and move towards rotational ($R_{4/2} \to 3.33$) as neutron number is increased. This behaviour is illustrated in figure 2 using the gadolinium chains as examples. For the Gd chain, the change in $R_{4/2}$ is quite sharp and the calculations reproduce these behaviours.

3. Electromagnetic transition probabilities

The Gd nuclei are at the beginning of the deformation region $150 < A < 190$. The nucleus is a rotor which shows a developed vibrational band. To explain the form of a nucleus, the binding energy of the nucleus, the transition probabilities between different energy levels, electric and magnetic multipole moments, the quadrupole moments and the rest of the observable quantities must be known. The pairing and the quadrupole forces are important in deformed nuclei. These forces especially influence the particles in the unfilled states. The pairing force keeps the nucleus in spherical symmetry. The quadrupole charge distribution causes what is known as the quadrupole force. This force takes the nucleus to the deformed state.

A successful nuclear model must give a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties. The most important electromagnetic features are the $E2$ transitions. The $B(E2)$ values were calculated by using the $E2$ operator,

$$E2 = e_\pi Q_\pi + e_v Q_v \tag{10}$$

$$Q_\rho = (d^+ \times s + s^+ \times \tilde{d})^{(2)}_\rho + \chi_\rho (d^+ \times \tilde{d})^{(2)}_\rho, \quad \rho = \pi, v, \tag{11}$$

where e_π and e_v are the 'effective charges' of the proton bosons and the neutron bosons.

In order to find the value of the effective charge we have fitted the calculated absolute strengths $B(E2)$ of the transitions within the ground state band to the experimental ones. The best agreement is obtained with the value $e_\pi = e_v = e = 0.14$ eb, as shown in table 1. The $B(E2)$ values depend quite sensitively on the wave functions suggesting that the wave functions obtained in this work are reliable.

Table 1. $B(E2; I \to I-2)$ values for ground-state bands of ^{154,156,158}Gd isotopes.

	$B(E2)$ (in W.u.)				$B(E2)$ ratios	
	$4_1 \to 2_1$		$2_1 \to 0_1$		$(4_1 \to 2_1)/(2_1 \to 0_1)$	
N	Theory	Exp. [12]	Theory	Exp. [12]	Theory	Exp. [12]
90	211	245 (9)	109	157 (1)	1.93	1.56
92	251	263 (5)	156	187 (5)	1.60	1.40
94	283	289 (5)	191	198 (6)	1.48	1.45

4. Results and discussions

The present work demonstrates that the sd-df IBA Hamiltonian parameters based on the IBA-1 model gave good results for the excitation energies and the electric quadrupole transition probability $B(E2; 0_i^+ \rightarrow 2_1^+)$ of the ^{154,156,158}Gd isotopes. For the states which are not completely symmetric, we renormalized the parameters (ε and κ) and obtained good results. In the present calculations, we have shown the ability of the projection in correlating different properties of gadolinium isotopes in terms of a few parameters.

Gadolinium is typical of many rare-earth and actinide-deformed nuclei and one can expect that similar numbers of 0^+ excitations appear throughout the deformed regions of the nuclei. It is therefore important to understand the origin of such a large number of $K^\pi = 0^+$ modes. The number of 0^+ states in heavy deformed regions opened a new window and emphasized the importance of the microscopic approach to the atomic nucleus. In fact, studies in this field imply the need for new approximations based on the microscopic approach.

In figure 1, clearly there are many 0^+ states, at relatively low energy, with dominant predicted two-phonon octupole character. As stressed, there is no assurance that these correspond to specific empirical 0^+ states. Nevertheless, the results highlight the importance of including the octupole degree of freedom if one is studying deformed nuclei above the pairing gap. We interpret these results as a proof that models including the octupole degree of freedom are able to predict a large number of relatively low-lying 0^+ states.

The observed γ-decay 0^+ states [13] support the concept of widespread octupole character. Recent measurements of the lifetimes of almost all of the 0^+ states show that the $B(E2; 0_i^+ \rightarrow 2_1^+)$ values are at most a few Weisskopt unit (W.u.). Combined with the decay to octupole modes, this is consistent with the double octupole phonon character of some of them. These results, along with further measurements of γ-ray branches from higher-lying 0^+ states, will allow one to define clearly the parameters of the Hamiltonian and the electromagnetic transition operators.

Experimental $B(E2)$ values for transitions between positive-parity states are compared in table 1 with our results, which were obtained from the (modified) program PHINT [14] by using the wave functions to fit the energy levels as described in §3.

The $B(E2)$ values depend quite sensitively on the wave functions, suggesting that the wave functions obtained in this work are reliable. This method of approximation may be applied to many other even–even nuclei and its many other nuclear properties [15,16].

In conclusion, we have shown that it is possible to predict nearly as many 0^+ states below ~3.2 MeV in ^{154,156,158}Gd isotopes if the octupole degree of freedom is taken into account. Every 0^+ states, whose γ-decay is known, de-excites by $E1$ transitions to lower lying negative-parity states. This is consistent with a strong two-phonon octupole character in many of these 0^+ states. If this association of structure is correct, it also implies that only a few of the 0^+ states are largely two quasiparticle in character and that double octupole 0^+ states may be common near and above the pairing gap in deformed nuclei.

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