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The steady state of a particle in a vibrating box and possible application in short pulse generation of charged particles

NANDAN JHA[1,*] and SUDHIR R JAIN[2]

[1]High Pressure & Synchrotron Radiation Physics Division, Bhabha Atomic Research Centre, Mumbai 400 085, India
[2]Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai 400 085, India
*Corresponding author. E-mail: nandanj@barc.gov.in

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Abstract. In this paper the classical evolution of a particle is studied which bounces back and forth in a 1D vibrating cavity such that the reflection from the wall does not change the speed of the particle. A peculiar behaviour of the particle motion can be seen where the time evolution of the motion shows superposition of linear and oscillatory behaviour. In particular, the parameter range is found in which the particle oscillates between the walls in steady state as if the wall was static and it is showed that for these parameter ranges the particle settles to this steady state for all initial conditions. It is proposed that this phenomenon can be used to bunch charged particles in short pulses where the synchronization proposed in our model should work against the space charge effect in the charged particle bunch.

Keywords. Nonlinear dynamics; synchronization; accelerators.

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1. Introduction

Ultrashort electron pulses in the range of hundreds of femtosecond can be a great tool for time-resolved electron microscopy. Electron pulses below 100 fs duration are produced by irradiating sharp metallic needles by ultrashort laser pulses [1]. Similarly, other charged particle ultrashort pulses have many applications in material science and FEL development [2]. But it is not possible to get such short-duration pulses with large number of particles in one bunch due to their Coulomb repulsion. In this paper, we present a model which can be used to produce short-duration pulses of charged particles.

We consider a charged particle, e.g. a relativistic electron, in a Racetrack Microtron. The only change from the usual Microtron design is that in this case the energy of the electron is maintained constant by the Linac. We want to draw a parallel between the relativistic electron in this storage ring and a particle bouncing between two parallel walls (like particle in a 1D box). The motion of an electron in one half of the round trip in microtron can be mapped to the motion of particle from one wall to the other and the motion of electron in the other half of the round trip can be mapped to the return motion of the particle in the 1D box.

2. Model of particle in an oscillating box

Now, we shall study the particle in this 1D box. The special property of this 1D box system is that the speed of the particle remains the same even after a collision with the walls, even if the walls are vibrating. This property distinguishes this model from Fermi acceleration model [3] where the speed of the particle changes after collision with the oscillating wall. This can happen in two cases: (a) if the particle considered is moving at (or very near) the speed of light and the walls are suitable reflectors that only reverse the direction of the particle. This case is closely related to an electron in the storage ring since there also the electron is relativistic and the speed remains the same throughout the ring. Small perturbations can only change their paths but not their speeds. (b) If the particle has a finite velocity and the walls are made such that they reflect the particles back with the same speed they came with.

We consider the case when one of the walls is oscillating sinusoidaly (but without changing the speed of the particle on collision). The speed of the particle will not change on collision with the wall if the particle is highly relativistic and small perturbation in energy by colliding with the moving wall will not change the velocity of the particle. Therefore, the particle in each round trip will travel more or less distance depending on the phase of the oscillating wall it encounters.

Let the time taken by the particle for one full round trip when both walls are static is T_0. If one wall starts oscillating at time zero and time taken for the particle to reach the static wall after N round trips is denoted by T_N, then after $(N+1)$th round trip,

$$T_{N+1} = T_N + (T_0 + t_0 \cos\phi_{N+1}), \tag{1}$$

where t_0 is amplitude of the time delay due to the extra distance travelled by the particle because of the vibration of the wall and ϕ_{N+1} is the phase of oscillation of the wall. We are recording the time T_N when the particle returns back to the static wall after N round trips. The phase ϕ_N is the phase of the oscillating wall when the particle reaches the oscillating wall in Nth round trip. For the phase ϕ_N we have

$$\phi_{N+1} - \phi_N = \omega t, \tag{2}$$

where ω is the angular velocity of oscillations of the wall and t is the time taken by the particle to reach the vibrating wall after one round trip from the previous interaction with the oscillating wall. Therefore,

$$\phi_{N+1} - \phi_N = \omega\left(\frac{T_0 + t_0\cos\phi_N}{2} + \frac{T_0 + t_0\cos\phi_{N+1}}{2}\right)$$

$$\phi_{N+1} - \frac{\omega t_0(\cos\phi_N + \cos\phi_{N+1})}{2} = \phi_N + \omega T_0. \tag{3}$$

To see the change in the total time after $(N+1)$ round trips due to the oscillating wall, we subtract $(N+1)T_0$ from eq. (1) and get

$$T_{N+1} - (N+1)T_0 = T_N + (T_0 + t_0 \cos\phi_{N+1}) - (N+1)T_0$$

$$T_{N+1} - (N+1)T_0 = T_N - NT_0 + (t_0 \cos\phi_{N+1}).$$

If we define the change in round trip time due to vibration of the wall as our new variable, i.e. $(T_m - mT_0) = R_m$, then we have a map:

$$R_{N+1} = R_N + t_0 \cos\phi_{N+1}. \quad (4)$$

Equations (3) and (4) are to be solved simultaneously.

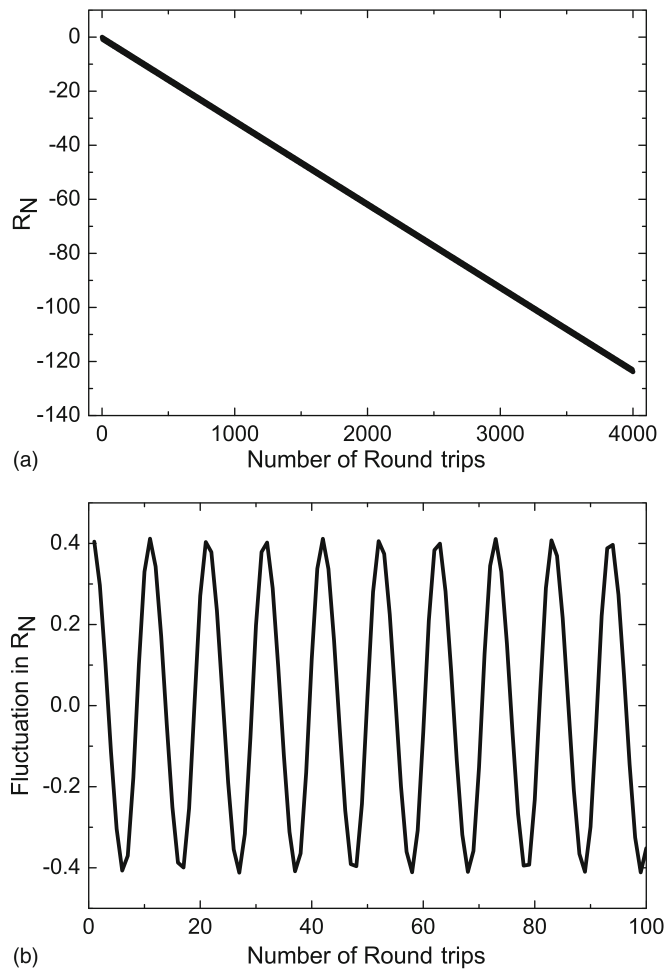

Figure 1. (**a**) R_N vs. number of round trips and (**b**) fluctuation of R_N over the linear variation.

During the motion of particle in the cavity, it will interact with the vibrating wall at all possible phases of the vibration of wall. But when the following condition is satisfied, ϕ_N will settle down to a steady state ϕ_* given by eq. (6)

$$\omega\,(T_0 + t_0 \cos\phi_*) = N\pi, \tag{5}$$

where N is an integer. From this we get

$$\phi_* = \cos^{-1}\left[\frac{N\pi}{\omega t_0} - \frac{T_0}{t_0}\right]. \tag{6}$$

Therefore, if parameters T_0, t_0, ω are chosen such that eq. (5) is satisfied, then the particle will eventually settle down to this steady state after a few round trips.

3. Numerical simulation results

To simulate this system, we fix $T_0 = 1$, $t_0 = 0.25$ and see the behaviour of R_N and ϕ_N numerically as a function of N for different values of ω. For all values of ω that do not satisfy eq. (5), the behaviour of R_N is generically the same. They all are linear functions of the round trips N with some fluctuations superposed on this linear behaviour. In figure 1, we have plotted the behaviour of R_N for the case when $\omega = 0.2\pi$.

The time error R_N increases or decreases linearly with time depending on the system parameters. But after subtracting the straight line behaviour from the plot we see the oscillatory pattern shown in figure 1b.

In the Fourier spectrum of these oscillatory fluctuations in figure 2, we see sharp peaks in the Fourier spectrum. Sharp peaks in Fourier spectrum are indicative of sinusoidal behaviour. From the theory of Fourier transforms, we know that the location of these peaks in the Fourier spectrum give the frequencies of the periodic motions involved. Therefore, as we have two sharp peaks in the Fourier spectrum of fluctuations, we can

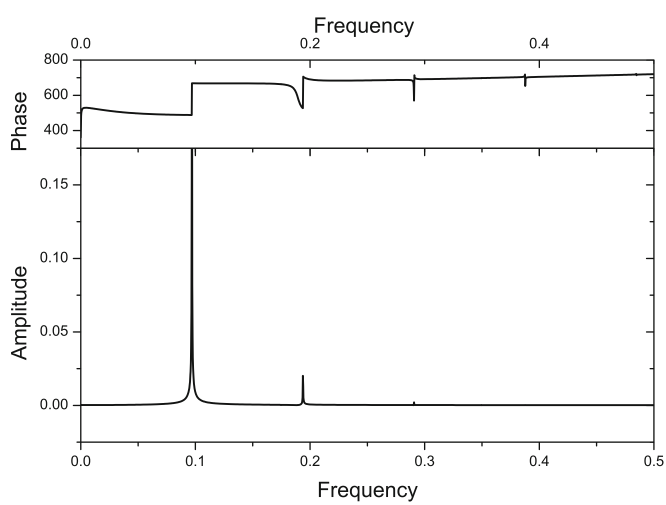

Figure 2. FFT spectrum of the fluctuations in R_N.

say that these fluctuations are due to the overlap of two different periodic motions. In our system two periodic motions are taking place, one is the periodic oscillations of the wall and the other is the periodic motions of the particle between the two walls. Therefore, these periodic motions can be seen in the Fourier spectrum of fluctuations. The fact that these fluctuations are superposed on a linearly varying function is not trivial and can only be seen from the numerical simulation.

When we choose ω that satisfies eq. (5) (e.g. $\omega = 2\pi$ for our parameters), the phase ϕ_N settles to a steady state after a few round trips (in our case it settles to $\pi/2$). R_N has no fluctuation. It either varies linearly with the number of round trips or it settles to a constant value if ϕ_N settles to $\pi/2$.

We can see in figure 3 that for angular frequency of oscillation $\omega = 2\pi$ of the wall, the particle synchronizes itself with the oscillation of the wall. Therefore, if we have more

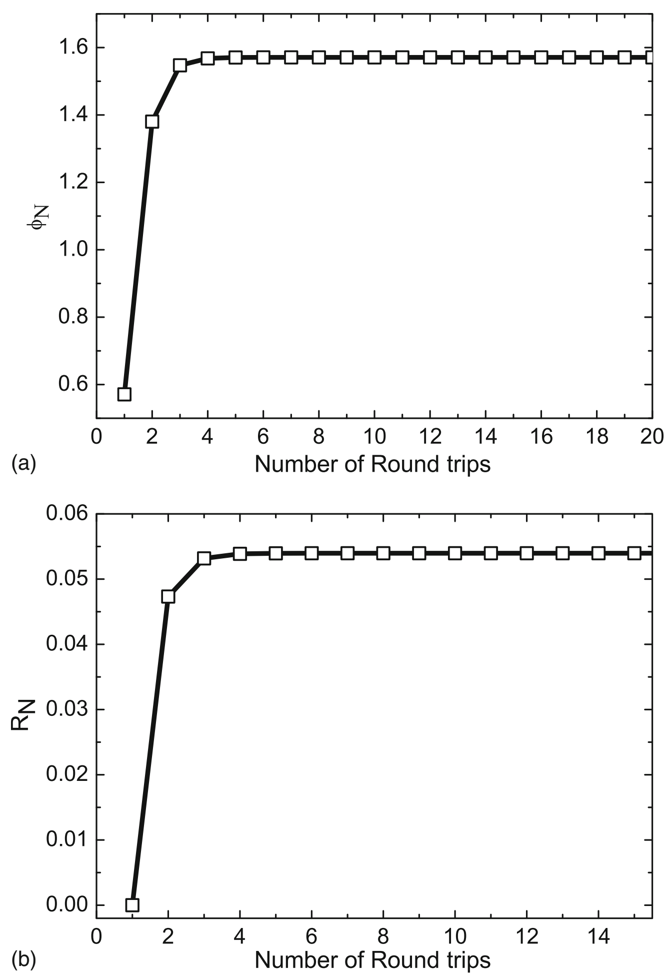

Figure 3. (**a**) ϕ_N vs. number of round trips and (**b**) R_N vs. number of round trips for $\omega = 2\pi$.

than one particle in the cavity, the motion of these particles will tend to synchronize with the oscillating wall and since the phase of the oscillating wall at which they all synchronize to is independent of initial conditions, after a few round trips they will all bunch together at the same point and move together in the cavity. If this result can be applied to the storage ring made from Racetrack Microtron, then this process of synchronization will act towards bunching the electrons and will act as a force opposing the mutual Coulomb repulsion of the electrons. Racetrack Microtron has two semicircular D's where a constant magnetic field is applied to bend the electron by 180° and in between these D's, the electron travels in straight path and is accelerated by a linear accelerator [4]. For our case that accelerator will only supply enough energy to compensate for the radiation losses when the electrons bend in the D's.

To apply the model proposed in this paper in Racetrack Microtron, it is required that the magnetic field in one of the D's is time-dependent to mimic the vibrations of the wall in our model. The requirement from the time dependence is to increase or decrease the path taken by the electron in one round trip periodically. This can be achieved if the region where electron gets reflected by the magnetic field is made to oscillate sinusoidaly. However, to do this mechanically by making one of the D's vibrate is impossible. But, we can achieve this by switching on and off the magnetic field periodically in the boundary region where the electron enters the D. This will only lead to the electron getting reflected a little later or earlier causing an increase or decrease in the path. The vibration of the magnetic field in the region where the electron enters the D and where it leaves the D must be out of phase by time $m\pi/Bq$ such that the electron spends the same time in the magnetic field regardless of the phase of vibration of the magnetic field.

4. Conclusion

We have shown that for a particle in a vibrating cavity, we can set the parameters of the vibration and length of the cavity to make sure that the particle settles to a steady state condition for all initial conditions. We have also proposed a possible way to implement this result in producing charged particle bunches. To actually show that this proposal can work for charged particle bunching, more sophisticated numerical simulations are to be done.

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