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Higgs interchange and bound states of superheavy fermions

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Abstract. Hypothetical superheavy fourth-generation fermions with a very small coupling with the rest of the Standard Model can give rise to long enough lived bound states. The production and the detection of these bound states would be experimentally feasible at the LHC. Extending, in the present study, the analysis of other authors, a semirelativistic wave equation is solved using an accurate numerical method to determine the binding energies of these possible superheavy fermion-bound states. The interaction given by the Yukawa potential of the Higgs boson exchange is considered; the corresponding relativistic corrections are calculated by means of a model based on the covariance properties of the Hamiltonian. We study the effects given by the Coulomb force. Moreover, we calculate the contributions given by the Coulombic and confining terms of the strong interaction in the case of superheavy quark bound states. The results of the model are critically analysed.

Keywords. Superheavy fermions; heavy quarks; relativistic bound states.

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1. Introduction

The Standard Model (SM), particularly after the recent experimental observation of a new neutral boson with a mass of $m_{\mathrm{H}} = 126$ GeV, which is compatible with the Higgs boson [1], is considered capable of reproducing most of the available data of particle physics with high accuracy.

However, some mysteries remain and investigations about physics beyond the Standard Model are very active. In this context, we consider the possible existence of a fourth generation (4G) of quarks and leptons. Up to now only three generations, accommodated in the SM, have been experimentally observed but 4G fermions, whose masses are beyond the present experimental search, can also exist.

If these 4G fermions couple to the Higgs field with the standard Yukawa coupling [2], they can feel a strong attractive force because of the exchange of the scalar Higgs boson. The adimensional coupling constant for that interaction has the form:

$$\alpha_{\mathrm{H}} = \frac{m_{\mathrm{f}}^2}{v^2 4\pi}, \tag{1}$$

where m_{f} represents the fermion mass and $v = 246$ GeV is the Higgs vacuum expectation value.

Note that α_{H} is proportional to m_{f}^2, the square of the fermion mass. This Higgs exchange interaction between a pair of superheavy fermions can give rise to bound states with new physical properties, whose study represents the hypothetical but exciting subject of the present work.

On the one hand, free and bound 4G leptons can be found. On the other hand, the properties of the confined 4G quark systems should be significantly determined by the interaction due to the Higgs exchange (together with the colour QCD interaction) giving rise to bound states with different properties with respect to the standard, known, hadronic particles. The hypothesis of superheavy quark open-flavour bound states also has been explored.

In the present work we consider fermion–antifermion pairs, extending a study performed by other authors [3] of superheavy quark–antiquark systems.

The remaining part of this paper is organized as follows. In §2, analysing some works on this subject, we discuss, in general terms, the existence of the 4G and superheavy fermion bound states. In more detail, in §2.1 the proposed experimental and theoretical constraints are examined. In §2.2 some experimental searches of 4G free and bound states are revised. Their dynamical production mechanisms are also considered. In §2.3 the existing theoretical models for 4G bound states are discussed. In §2.4 the general aspects of the present model are introduced.

In §3 the dynamical model of the present work is discussed in detail. In particular, in §3.1 the relativistic kinetic energy term and its nonrelativistic expansion are displayed. The screened Yukawa potential is discussed in §3.2. The relativistic and retardation correction terms (collectively denoted as relativistic corrections) are analysed in §3.3 also recalling the basic principles that have been used to determine their form. Finally, the long-range Coulomb and effective QCD interactions are introduced in §3.4.

In §4 the procedure of the numerical calculations is briefly described. In §5 the numerical results are shown and discussed. Finally, in §6 some comments and conclusions are drawn. Throughout the article we set $\hbar = c = 1$.

2. The 4G fermions and their possible bound states

In this section we discuss the fundamental problems related to a 4G and the possibility of studying the 4G bound states.

2.1 *Experimental and theoretical constraints on the 4G*

The hypothetical existence of 4G represents the necessary condition for the development of the present work.

We point out that this issue is extremely complicated and highly controversial. An interesting, but not complete, collection of studies on this subject can be found, for example, in http://www.nu.to.infn.it/Fourth_generation/ by C Giunti and M Laveder.

A complete and detailed analysis of this problem goes beyond the scope of the present paper. We shall only cite some articles that can be potentially relevant for the study of 4G fermion bound states. Furthermore, we shall in general rely on the conclusions given by their authors without criticizing their works.

We begin by considering some physical reasons for introducing a 4G, as given, for example, in [4,5] where the theoretical virtues of a 4G are discussed. In these works, the 4G was related, on the one hand, to the possible solution of internal consistency of the SM: namely, the Higgs naturalness problem and the fermion mass hierarchy problem; on the other hand, it can provide a new source of CP violation, in particular for explaining the baryon asymmetry of the Universe, and can also introduce new stable particles as Dark Matter candidates.

In [6] it is pointed out that heavy 4G Majorana neutrinos can be stable and contribute to a small fraction of the relic Dark Matter density. The mass of the 4G neutrino is constrained by means of some Dark Matter experimental data.

In [7], the authors make the hypothesis of a TeV-scale 4G lepton family. In this case, the heavy, nearly stable, Majorana neutrino can mix with the SM charged leptons. Via its semileptonic decays, this neutrino can be a source of electron and positron cosmic rays. Their spectra are studied and favourably compared with the rise of positron fraction above 10 GeV measured by PAMELA [8] and with the electron and positron fluxes measured by Fermi-LAT [9].

The possibility of the existence of a 4G is usually studied in terms of the oblique parameters that parametrize the effects of new physics (here, we consider in particular a 4G) on the available electroweak experimental data. We recall that, in the original nomenclature, the propagator corrections due to vacuum polarization effects were defined oblique as they participate in the computation in an indirect manner [10]. In a theory beyond the SM with new fermion doublets, the S parameter estimates the size of the new fermion sector and the T parameter measures the isospin violation in that sector.

A detailed numerical analysis [10] suggested that enough space was still available for a 4G. However, this study was performed before the Higgs observation [1] and the Higgs mass range $120 < m_{\mathrm{H}} < 600$ GeV was excluded from the calculations.

Murayama *et al* [11] considered (criticizing another approach [12]) the case of a long-lived 4G whose longevity is due to very small mixings with the lighter three generations. This hypothesis would open the possibility of exact B–L symmentry of nature, with important consequences on the origin of neutrino mass and baryon asymmetry.

The experimental observation of a new neutral boson with a mass $m_{\mathrm{H}} = 126$ GeV, assuming that it is the true SM Higgs boson, represents, in any case, a very strong constraint on the properties of the 4G. Moreover, according to some works that will be discussed in the following, if no further modifications are made on the SM, the Higgs observation excludes the possibility of a 4G. For this reason, we shall mainly focus our attention on the (recent) works that take explicitly into account that measurement.

In [13], the authors analyse the viability of a sequential 4G by using a large set of data, including the Higgs decays and the electroweak precision observables. In this work, the value $m_{\rm H} = 125$ GeV is taken and a 4G Dirac neutrino is assumed. The authors affirm that a 4G seems to be disfavoured with respect to SM, but not excluded by the data. They also find the best fit values for the masses of the hypothetical 4G fermions. In a subsequent work [14] (in which they used the following two values: $m_{\rm H} = 126.5$ GeV and $m_{\rm H} = 147$ GeV, that respectively are the preferred Higgs mass of the SM with 3G and preferred Higgs mass of SM with a 4G) a new global fit of the SM with a 4G is performed. The results are even more pessimistic but the authors admit that a sequential 4G can be allowed in the presence of an extended Higgs sector.

In [15] it has been clearly pointed out that, if the recently discovered particle at 126 GeV is really the Higgs boson, the experimental data and the internal consistency of the theory (i.e. the stability and the little hierarchy fine-tuning problems) strongly disfavour a straightforward 4G extension of the SM. However, if new quanta, non-SM Higgs sectors or a non-elementary Higgs-like particle are introduced, then a 4G can still be considered viable. The author has studied, for 4G models, the effects of the two-loop corrections on the electroweak data, finding a big and beneficial effect on the electroweak fits, in terms of S and T parameters. It has been estimated that perturbation theory can be correctly applied for 4G quark mass of mass $m_{\rm Q} \simeq 600$ GeV while it becomes marginal for $m_{\rm Q} \geq 900$ GeV.

The CKM mixing parameters of the 4G are extremely relevant for determining the interactions of the 4G family. Their values have been constrained in previous works [12,16] by the same author.

As for the internal consistency of the theory in the presence of a 4G, it has been observed [17,18] that, by using partial-wave analysis at high energy, a critical mass value (for the superheavy fermions) can be found, beyond which the strength of the coupling invalidates the perturbation expansion. Also, the superheavy fermions would produce, at low energies, large one-loop radiative corrections. The authors find, in their model, that one of these corrections determines a specific upper value for the mass of the superheavy lepton around 700 GeV.

Moreover, in another work [19], the problems related to the ultraviolet singularity of the running coupling of a scalar field theory, defined as Landau pole, have been examined, considering the presence of new physics, represented by superheavy fermions. The presence of the Landau pole indicates a mass scale at which new physics must intervene. But the new physics affects the running coupling constant also at low energies. The author explored these low-energy effects using the renormalization group equation technique.

Recently, non-perturbative lattice calculations [20,21] have been performed to determine upper and lower Higgs boson mass bounds in the presence of a heavy 4G. The authors find that for a standard Higgs with mass around 125 GeV the maximum 4G quark mass is of the order of 300 GeV, which is already not compatible with direct searches.

We point out that, however, in such a theoretical framework, there is a possibility of the existence of vector fermions interacting by means of a singlet scalar. In this case, the oblique parameter bounds would be evaded avoiding a discrepancy with the existing

Standard Model experimental data. Furthermore, the running of the Yukawa coupling would also be avoided (at one loop). These vector fermions would not couple to the SM Higgs, so that the existence of new singlet or triplet scalars should also be assumed. For the sake of clarity, we anticipate that, however, in the present work, the existing experimental value of the Higgs mass will be tentatively taken to parametrize the fermion pair potential.

2.2 *Experimental searches of free and bound 4G particles*

Assuming that, due to its specific properties [11] and/or due to the physical theory beyond SM [15], the 4G can really exist, we now consider the works that study the possibility of detecting these particles at the LHC.

Ozcan *et al* [22] proposed a possible channel for the discovery of new charged leptons in which the final state contains three leptons of the same sign. The method has been illustrated for different physical cases, in particular for a 4G family. The authors show that some events can be expected with pp collisions at 14 TeV of CM energy.

Extensive studies [23,24] have been devoted to the specific signatures of superheavy quarks and of their bound states at the LHC. The authors, accepting the standard point of view, note that the Higgs search results strongly disfavour a new chiral family of fermions. But, on the other hand, they affirm that the existence of vector-like quarks remains a viable and interesting hypothesis. In particular, analogously to [11], this kind of particles would decouple in the limit of vanishing mixing and, being long-lived could also form bound states. From the experimental point of view, long-lived particles would evade the current searches as they propagate over sizeable distances in the detectors. With respect to the objective of the present study, a very interesting analysis is made [23] about the possibility that a superheavy quark–antiquark pair ($Q\bar{Q}$) can form a bound state, defined there as a heavy quarkonium (η_{Q}). The production process is assumed to proceed at the LHC through gluon fusion, that is, $pp \rightarrow gg \rightarrow \eta_{\mathrm{Q}}$. The authors give an estimate of the cross-section of this process pointing out that it depends on the square of the $Q\bar{Q}$ radial wave function at the origin. Assuming that the $Q\bar{Q}$ interaction is only given by QCD and that it can be represented by a standard Coulombic potential, the authors conclude that the η_{Q} production rate typically amounts to a few percent of the pair production cross-section. However, the authors also admit that the Yukawa Higgs exchange interaction should be considered to obtain a more accurate description of the process. This point can represent an important motivation for developing the present study in which an accurate variational study of the superheavy bound state wave functions is performed.

The authors also study the effects of the heavy quarkonium binding energies and give a qualitative description of the expected final-state signatures of the η_{Q} decay, depending on the chiral or, more probably, vector-like character of the superheavy quark Q. Furthermore, they also analyse the signatures for open-flavour mesons such as $Q\bar{q}$, $\bar{Q}q$, etc. opening the possibility of observing a rich spectroscopy. Dedicated searches of these new hypothetical bound states are proposed as promising strategies of investigation.

For completeness, we recall that production and decay processes of other new (very heavy) bound states at the LHC have also been extensively studied [25].

2.3 *Theoretical studies on 4G bound states*

Assuming that 4G superheavy particles can exist and form experimentally detectable bound states, we now move on to examine some of the proposed models for the possible bound states.

Incidentally, we recall that the Higgs-induced bags of superheavy fermions and bosons have been studied in various works [26–29]. However, apart from a possible role in baryogenesis, multifermion bags are extremely difficult to detect using particle accelerators.

In the work [30] a nonrelativistic Schrödinger equation is solved variationally by means of a one-component wave function in order to determine the ground-state energy of two superheavy particle-bound systems. The Yukawa Higgs exchange potential is considered as the main interaction. The effects of long-range Coulomb and strong interactions are also studied. The authors calculate the relativistic and radiative corrections. In order to avoid unphysically high contributions from the relativistic terms, a cut-off radius is phenomenologically introduced in the Higgs exchange potential. Radiative corrections are shown to be small.

In another study [3], the authors analyse in detail the possibility that a 4G can exist. They point out that a 4G is not in contrast with the electroweak precision data, justifying in this way their investigation on superheavy $Q\bar{Q}$ bound states. In particular, the 4G quarks are considered to be long-lived. They use a Schrödinger equation variationally solved by means of a one-component exponential wave function. The interaction is given by the usual Yukawa Higgs exchange term, the QCD Coulombic term, relativistic and all the perturbative corrections to the potential given by the underlying field theory. A very interesting point of this work is that they study not only the standard colour singlet states but also the colour octet ones, in which the QCD Coulombic interaction term is repulsive. Also in this case a bound state is possible, due to the strength of the Higgs Yukawa coupling for ultraheavy quarks.

The colour octet states have been studied in other works also [31–33].

In [31] a relativistic study of the superheavy quark bound states was previously performed. Then in [31,32] the authors affirm that at the LHC one interesting possibility of producing 4G quarks is represented by the colour octet, isosinglet vector state. Furthermore, the decay modes of this state are carefully studied considering their potential interest for the LHC phenomenology.

In [33], the authors find as it can be expected, that the colour octet state has a greater energy than the singlet one. As a consequence, the octet state can decay into the standard singlet state with the emission of a gluon, also contributing to the total production cross-section of the singlet state. The authors give an estimate of the decay width of this process by using a chromomagnetic dipole ($M1$) transition amplitude.

2.4 *The model of the present article for 4G bound states*

Taking into account the complex phenomenological situation and the results of the works quoted in the previous subsections, we study in the present work the bound (ground) states of superheavy fermion–antifermion pairs, assuming that the interaction given by the Higgs interchange plays a dominant role.

To this aim we introduce a semirelativistic potential model in which the Hamiltonian is given by the sum of a relativistic kinetic operator and an interaction operator. The latter is used to represent all the interactions of the fermion–antifermion pairs.

With respect to other investigations [30], we try to analyse the different possible physical situations in an orderly way. To this aim three dynamical cases will be examined separately:

(1) a pair of electrically neutral leptons, only interacting by means of the Higgs-induced potential;
(2) a pair of electrically charged leptons, for which the Coulomb potential is added to the Higgs one;
(3) finally, the case of superheavy quarks, for which an effective QCD interaction is also considered. In this case we limit our attention to the colour singlet case.

The potential model of the present work, as in the case of other conventional physical systems, can be consistently applied only if the relativistic and field effects are not too strong. In particular, in this work the relativistic form of the kinetic energy is always taken, assuming that the semirelativistic wave equation is able to sum up (nonperturbatively) the infinite series of Higgs interchange diagrams in the ladder approximation, as usually done in hadronic physics [34].

As for the Higgs interaction, that represents the main object of this study, a screened Yukawa potential is adopted, also briefly analysing the unscreened case. The relativistic corrections for the interaction operator, given, in this case, by the screened Yukawa potential, are calculated by means of an expression based on the relativistic covariance properties of the Hamiltonian. In this way, the form of these relativistic corrections is completely determined. Their general expression, related to the exchange of a scalar particle, was developed and applied to study the charmonium spectrum [35].

The use of a screened potential (and its specific form) represents one of the most relevant uncertainties of this kind of investigations. We point out that the use of the standard (unscreened) Yukawa potential, when the two fermion distance r goes to zero, gives rise to a very relativistic bound system with huge values of the relativistic corrections. Such a pathological situation is always found, also assuming different theoretical schemes for the calculation of the relativistic corrections. As a consequence, the introduction of a screening effect has been shown to be strictly necessary [30].

In the present work, a phenomenological screening factor (that we do not try to relate to specific physical effects) is consistently introduced from the beginning of the calculation, modifying the form of Yukawa potential so that the singularity for $r \to 0$ is removed. As a consequence, the form of the relativistic corrections is also modified and their numerical values always remain sufficiently small, allowing for a consistent perturbative treatment.

The numerical calculations are performed using three different values for the screening mass (that is introduced to parametrize the screening radius) in order to investigate the dependence of the results on that quantity. The specific, purely phenomenological, form of the screening factor and the choice of the numerical values of the screening radius will be discussed in the following.

As for the long-range interactions, we consider the standard electromagnetic Coulomb potential and, for the (effective) strong interaction, the Coulombic and Cornell potentials. Taking into account the hypothetical character of the present study and the uncertainties of the results, no relativistic effects are introduced for these interactions.

The semirelativistic wave equation for the kinetic and the leading potential terms is solved numerically with an accurate variational-diagonalization (V-D) technique, successfully developed for studying conventional hadronic systems [35], while the relativistic corrections and the non-leading potential terms are added perturbatively.

In the present work, the V-D technique of solution of the semirelativistic wave equation allows to study in detail the effects of the confining Cornell potential.

For the specific calculation, the values of the fermion mass $m_{\rm f}$ are taken in the hypothetical range of $500 \le m_{\rm f} \le 1000$ GeV. This choice, analogous to that of [3], is essentially made for pedagogical reasons, without ignoring the limitations imposed by perturbativity, as discussed in §2.1.

As for the Higgs mass, we use the value $m_{\rm H} = 126$ GeV that corresponds to the recent experimental finding [1].

3. The dynamical model

The two superheavy fermion system is described, in its rest frame, by means of Hamiltonian operator H given by a standard relativistic kinetic term T plus an interaction term V:

$$H = T + V. \tag{2}$$

Negative energy and antiparticle virtual states are not considered here explicitly. As a consequence, the interaction is represented by a standard potential to which relativistic corrections are added. The present scheme, that has been successfully adopted for the constituent-quark description of hadronic particles (with a strong effective potential), is expected to work if the relativistic and multiparticle field effects are not too strong and, as a consequence, can be treated perturbatively.

3.1 *The kinetic energy*

The kinetic energy term, for two equal mass fermions, has the form

$$T = 2\sqrt{\vec{p}^2 + m_{\rm f}^2} - 2m_{\rm f}, \tag{3}$$

where $m_{\rm f}$ is the fermion mass and $\vec{p}$ represents the relative fermion momentum, conjugate to the distance of the two fermions $\vec{r} = \vec{r}_1 - \vec{r}_2$. In the definition of the kinetic energy T, the masses of the two fermions have been subtracted, in order to obtain directly the binding energy of the system. The previous relativistic expression has always been used in the calculations.

In the following subsections, we turn to consider the interaction terms.

3.2 *The Yukawa potential*

The standard Yukawa potential given by the Higgs boson exchange has the form [30]

$$V_{\mathrm{Y}}^{\mathrm{st}}(r) = -\frac{\alpha_{\mathrm{H}}}{r}\exp(-m_{\mathrm{H}}r), \tag{4}$$

with α_H defined in eq. (1) and $r = |\vec{r}|$.

The singularity of the Yukawa potential when $r \to 0$, due to the small radius of the bound system, can give unphysical results especially when calculating the relativistic corrections. This difficulty was first noticed in ref. [30] and was also found when developing the present work, where a slightly different form of the relativistic corrections is considered (see §3.3). To avoid this problem, a screening factor must be introduced. We take the following relatively simple form, proposed for hadronic composite systems [36]:

$$f_{\mathrm{scr}}(r) = 1 - \exp\left(\frac{-r}{r_{\mathrm{scr}}}\right), \tag{5}$$

that, by means of the screening radius r_{scr}, can take into account hypothetical short distance physical effects whose study is, in any case, beyond the scope of the present work.

For convenience, we parametrize the screening radius by means of a screening mass m_{scr}, in the form

$$r_{\mathrm{scr}} = \frac{1}{m_{\mathrm{scr}}}. \tag{6}$$

Phenomenologically, we consider that m_{scr} can be of the order of 1 TeV. In more detail, for the numerical calculations, the following three values will be used: $m_{\mathrm{scr}} = 1000$, 1200 and 1400 GeV.

Summarizing, the screened, nonsingular, Yukawa potential has the form:

$$V_{\mathrm{Y}}^{\mathrm{scr}}(r) = V_{\mathrm{Y}}^{\mathrm{st}}(r) \cdot f_{\mathrm{scr}}(r), \tag{7}$$

that will be used in the following calculations. In order to highlight the critical aspect of this part of the model, the results of the calculation with no screening and no relativistic corrections will also be shown.

Finally, we point out that the treatment of the screening in the present work is different from that of ref. [30]. In that work a screening function of the following form is used:

$$g_{\mathrm{scr}}(r) = \frac{r}{(r_{\mathrm{scr}} - r)}. \tag{8}$$

Then, the screened potential $g_{\mathrm{scr}}(r)$ is used in the calculations, in particular when solving the relativistic wave equation, but in the final part of the calculation the contribution

$$V_{\mathrm{Y}}^{\mathrm{st}}(r) - V_{\mathrm{Y}}^{\mathrm{scr}}(r)$$

is added perturbatively to the bound state energy in order to avoid a strong dependence of the physical results on r_{scr}. We consider that such cut-off procedure should be justified in the context of a specific physical field theory (not yet known) at TeV scale. For this reason we prefer to take the screening as a phenomenological parametrization that reflects our ignorance above 1 TeV and is strictly necessary to obtain reasonable numerical results.

3.3 *The relativistic corrections*

As for the relativistic and retardation corrections for the interaction operator, it is necessary to point out that the nonrelativistic reduction (up to order c^{-2}) of a scalar effective theory is not completely straightforward.

On the other hand, in the vector case of quantum electrodynamics, the corresponding reduction, i.e. the Fermi–Breit equation, is derived by means of a consistent theoretical procedure and also experimentally tested with the positronium energy spectrum.

Note that the Fermi–Breit equation is obtained by fixing the Coulomb gauge for the propagator, while the use of the Feynman gauge propagator and a direct expansion of the fermion energy transfer give the wrong sign for the retardation term [37]. One can infer that this latter procedure would give a wrong result also for a scalar theory.

For this reason, we use an approach developed for studying charmonium spectrum is based on the relativistic covariance of the Hamiltonian. The interested reader can refer to ref. [35], for all the details of the derivation. In the following we simply summarize the results.

For a given scalar interaction $V_{\rm int}(r)$ of the order c^0, the relativistic correction terms take the following form:

$$V_{\rm rc} = V_{\rm rc}^{mi} + V_{\rm rc}^{md}, \tag{9}$$

with

$$V_{\rm rc}^{mi} = -\frac{1}{4m_{\rm f}^2}\nabla^2 V_{\rm int}(r) - \frac{1}{2m_{\rm f}^2} V'_{\rm int}(r)\frac{1}{r}\vec{s}\cdot\vec{l} \tag{10}$$

and

$$\begin{aligned} V_{\rm rc}^{md} = & -\frac{1}{8m_{\rm f}^2}\left[\{p^2, V_{\rm int}(r)\} + 2p^\alpha V_{\rm int}(r) p^\alpha\right] \\ & + \frac{1}{4m_{\rm f}^2}\left[\{p^\alpha p^\beta, \hat{r}^\alpha \hat{r}^\beta W_{\rm int}(r)\} + 2p^\alpha \hat{r}^\alpha \hat{r}^\beta W_{\rm int}(r) p^\beta\right], \end{aligned} \tag{11}$$

$\hat{r}^\alpha$ being the Cartesian components of the unit vector $\vec{r}/r$.

The two contributions displayed in eqs (10) and (11) represent the momentum-independent and the momentum-dependent contributions respectively. The latter has a different form with respect to other works [30]. In eq. (10) the standard Darwin and spin-orbit terms are displayed. The spin-orbit term, with the two-fermion spin defined as $\vec{s} = \vec{s}_1 + \vec{s}_2$, will give no contribution to the present calculation due to the s-wave character of the ground bound state.

All the previous relativistic corrections represent the terms of order c^{-2} of the work [35] (for two fermions of equal mass $m_{\rm f}$) given there in eqs (5) and (10) with the definition of eq. (9). For convenience, that definition is rewritten here as

$$W_{\rm int}(r) = -\frac{1}{2}V'_{\rm int}(r)r. \tag{12}$$

This last term appears in eq. (11) of the present work.

In the present calculation the scalar interaction of the order c^0 is always taken in the form of the screened Yukawa potential of eq. (7), that is

$$V_{\rm int}(r) = V_{\rm Y}^{\rm scr}(r) \tag{13}$$

that determines, by means of the relativistic covariance constraint, the form of the relativistic correction operator $V_{\rm rc}$, of the order c^{-2}.

3.4 *The long-range interactions*

The Higgs exchange interaction, introduced in §3.2, has a very short range, that is $1/m_{\rm H}$. In the present calculation we also consider the long-range Coulomb and effective QCD interactions.

The standard Coulomb term

$$V_{\rm C}(r) = q_1 q_2 \frac{\alpha_{\rm em}}{r}, \tag{14}$$

where $\alpha_{\rm em}$ is the fine structure constant and q_i, with $i = 1, 2$, represents the fermion charge in units of elementary charge. We shall consider a charged lepton–antilepton pair with $q_1 q_2 = -1$ and quark–antiquark pairs with the two possibilities: $q_1 q_2 = -1/9$ and $q_1 q_2 = -4/9$.

As for the quark–antiquark case, we also take an effective QCD term in the Cornell [38] form:

$$V_{\rm QCD}(r) = -\frac{4}{3}\frac{\alpha_{\rm s}}{r} + \sigma r. \tag{15}$$

In the present calculations, we take for the parameters of the previous equation the same numerical values of other works. In particular, our value of the effective strong coupling constant is $\alpha_{\rm s} = 0.12$ as in refs [3,30]; our value of the parameter of the confining term is $\sigma = 0.2$ GeV^2 as in refs [30,39]. This last value, in any case, is not far from the original Cornell [38] value of $\sigma_{\rm Corn} = 0.26$ GeV^2.

3.5 *The three specific dynamical cases*

As anticipated in the Introduction, we study three different dynamical cases. We display here, in each case, the total potential, explaining which terms will be treated perturbatively.

(1) For a pair of electrically neutral leptons we only take the screened Yukawa Higgs potential of eq. (7) and the relativistic corrections of eq. (9), that is,

$$V = V_{\rm Y}^{\rm scr}(r) + V_{\rm rc}, \tag{16}$$

where, for the first term the V-D technique is used while the contribution of $V_{\rm rc}$ is calculated perturbatively.

(2) For a pair of charged leptons, we take

$$V = V_{\rm Y}^{\rm scr}(r) + V_{\rm rc} + V_{\rm C}(r), \tag{17}$$

where the first and the third terms are calculated using the V-D technique and, again, only $V_{\rm rc}$ is calculated perturbatively.

(3) Finally, for a pair of quarks, the interaction is

$$V = V_{\mathrm{Y}}^{\mathrm{scr}}(r) + V_{\mathrm{rc}} + V_{\mathrm{C}}(r) + V_{\mathrm{QCD}}(r). \tag{18}$$

In this case the first and the fourth terms are calculated using the V-D technique and V_{rc} and $V_{\mathrm{C}}(r)$ are calculated perturbatively. Contrary to Case 2, here the contributions of $V_{\mathrm{C}}(r)$ are surely smaller than those given by $V_{\mathrm{QCD}}(r)$.

Furthermore, considering the hypothetical character of the object of this study and the uncertainties of the numerical results, many physical effects are not considered in the present work:

- the radiative corrections related to the Higgs exchange. Their contributions have been shown to be relatively small [30],
- the Z exchange interaction, also usually considered small [30],
- the relativistic and radiative corrections for the electromagnetic interaction,
- the effects related to a possible fermion decay,
- finally, the relativistic effects for the QCD effective potential, in particular the spin–spin interaction that would be relevant to reproduce the ground-state splitting. As for the QCD effective potential, a more accurate analysis would also require a different parametrization, instead of using the standard Cornell potential. However, given that the main interest of this study is to reproduce, in a first approximation, the ground-state energy, the adopted effective potential should be sufficient to reach that objective.

4. The numerical calculations

In order to find approximate, numerical, solutions of the eigenvalue wave equation for the Hamiltonian H of eq. (2), a V-D procedure is performed and, subsequently, the perturbative contributions are added.

Schematically, the V-D procedure consists of the following three steps:

- diagonalizing the Hamiltonian matrix of the model in the selected basis of trial wave functions that will be introduced in eq. (19),
- varying, in the trial wave functions, the dimensional parameter $\bar{r}$ that will be defined immediately after eq. (19),
- repeating the two previous steps until the minimum value of the ground-state energy is found.

On variational basis, we adopt, for the radial wave functions, an orthonormal set of functions with a decreasing exponential asymptotic spatial behaviour [40]. They have the form:

$$R_{\mathrm{nl}}(r) = \frac{1}{\bar{r}^{3/2}}\left[\frac{n!}{\Gamma(2l+3+n)}\right]^{1/2} s^l L_n^{2l+2}(s)\exp(-s/2), \tag{19}$$

where $\bar{r}$ is the variational dimensional parameter, $s = r/\bar{r}$ is introduced as an adimensional argument, and, finally, $L_n^{\alpha}(s)$ represent the standard Laguerre polynomials.

Due to their asymptotic behaviour, these wave functions are particularly suitable for studying the dynamical Cases 1 and 2 in which the potential is not confining. They have also been used successfully for studying the spectrum of confined quark systems [41]. No difficulty is found when studying the ground state, in the dynamical Case 3. Some numerical tests have also been performed by using a standard harmonic oscillator basis.

In the present calculation only s-wave ($l = 0$) wave functions are considered. The matrix elements of the relativistic kinetic energy of eq. (3) are calculated in the momentum representation. To this aim, the Fourier transformation of the wave functions given in eq. (19) is performed analytically.

We use 10 orthonormal wave functions, so that a 10×10 Hamiltonian matrix is constructed and diagonalized.

For the V-D procedure, we take in the Hamiltonian the operators indicated in §3.5 according to the three dynamical cases. By means of the wave function obtained we calculate the contributions of other operators at the first order of the perturbation theory.

5. The results of the model

The results of the calculations are shown in tables 1–4 where the following notations have been adopted: In the first column, the values of the fermion mass m_{f}, in GeV, are given. In the columns labelled as E_1, E_2 and E_3 the binding energies, in GeV, for the three dynamical cases are displayed, respectively.

For Case 3, as discussed in §3.4, the two possible values of the product of the electric charges of the quark–antiquark pair, give two slightly different results for the binding energies. In order to avoid displaying these results with useless accuracy, in the column labelled as E_3 we show the mean value of the two energies, that is,

$$E_3 = [E(q_1 q_2 = -1/9) + E(q_1 q_2 = -4/9)]/2. \tag{20}$$

Table 1. Binding energies obtained with $V_{\mathrm{Y}}^{\mathrm{st}}(r)$ and no relativistic corrections.

m_{f}	E_1	E_2	E_3 (δ)
500	−0.235	−0.248	−9.88 (0.25)
550	−0.612	−0.702	−16.4 (0.36)
600	−1.514	−1.981	−27.2 (0.48)
650	−9.727	−10.94	−45.3 (0.67)
700	−23.87	−25.64	−73.0 (0.90)
750	−48.02	−50.49	−115.0 (1.21)
800	−86.54	−89.98	−179.0 (1.66)
850	−146.4	−151.1	−274.0 (2.29)
900	−237.6	−244.2	−415.0 (3.18)
950	−375.9	−386.1	−635.0 (4.57)
1000	−591.5	−610.8	−958.0 (6.36)

Table 2. Binding energies obtained with $m_{\rm scr} = 1000$ GeV.

$m_{\rm f}$	E_1	E_2	E_3 (δ)
500	−0.230	−0.242	−8.91 (0.23)
550	−0.566	−0.633	−13.8 (0.30)
600	−1.168	−1.595	−21.2 (0.39)
650	−5.757	−6.669	−31.6 (0.48)
700	−13.62	−14.83	−45.5 (0.58)
750	−25.42	−26.92	−63.3 (0.69)
800	−41.44	−43.23	−85.1 (0.80)
850	−61.93	−63.97	−111.0 (0.92)
900	−86.97	−89.27	−141.0 (1.03)
950	−116.7	−119.2	−176.0 (1.15)
1000	−151.0	−153.8	−214.0 (1.26)

Table 3. Binding energies obtained with $m_{\rm scr} = 1200$ GeV.

$m_{\rm f}$	E_1	E_2	E_3 (δ)
500	−0.232	−0.244	−9.13 (0.23)
550	−0.579	−0.653	−14.3 (0.31)
600	−1.272	−1.712	−22.1 (0.40)
650	−6.421	−7.355	−33.4 (0.51)
700	−15.06	−16.34	−48.4 (0.62)
750	−28.05	−29.64	−67.7 (0.73)
800	−45.80	−47.68	−91.5 (0.86)
850	−68.49	−70.65	−120.0 (0.98)
900	−96.38	−98.79	−153.0 (1.11)
950	−129.4	−132.1	−190.0 (1.24)
1000	−167.6	−170.5	−233.0 (1.38)

We also show, in parenthesis, the difference of the two energies given by the Coulomb interaction, that is,

$$\delta = E(q_1 q_2 = -1/9) - E(q_1 q_2 = -4/9). \qquad (21)$$

We turn to the specific content of the tables.

For the sake of clarity and completeness, we give in table 1 the results obtained by solving the semirelativistic wave equation without the relativistic corrections and with no screening in the Yukawa potential (i.e. only with $V_{\rm Y}^{\rm st}(r)$), that is without the two most controversial, or model-dependent, contributions of the present investigation.

The results of tables 2–4 are obtained by solving the complete model, with the relativistic corrections and the screening factor in the potential; the value of the screening mass $m_{\rm scr} = 1000$, 1200 and 1400 GeV for the three tables, respectively.

We can now analyse the physical content of the results.

Our study confirms the main result of ref. [30]: for a fermion pair with mass $m_{\rm f} \geq 500$ GeV, the Higgs interchange interaction is sufficient to form a bound state, as

Table 4. Binding energies obtained with $m_{scr} = 1400$ GeV.

m_f	E_1	E_2	E_3 (δ)
500	−0.233	−0.245	−9.29 (0.24)
550	−0.589	−0.668	−14.7 (0.32)
600	−1.344	−1.794	−22.9 (0.41)
650	−6.935	−7.895	−34.7 (0.52)
700	−16.21	−17.53	−50.7 (0.64)
750	−30.16	−31.80	−71.1 (0.77)
800	−49.30	−51.25	−96.5 (0.90)
850	−73.78	−76.03	−127.0 (1.04)
900	−104.1	−106.6	−162.0 (1.18)
950	−139.8	−142.6	−201.0 (1.33)
1000	−181.1	−184.1	−246.0 (1.48)

shown by the values of E_1 in all the tables. The electromagnetic and strong interactions increase the binding energy of the system, as shown by E_2 and E_3 in all the tables.

A comparison between table 1 and tables 2–4 shows that for $m_f \leq 600$ GeV the binding energies do not depend strongly on the presence of relativistic corrections and screening factor, so that the results can be considered sufficiently reliable, in the sense that they do not depend on the specific form of the relativistic corrections and the screening factor.

But for higher masses (with no screening), due to eq. (1), the interaction grows up and the two fermions, confined in a small spatial region, become very relativistic; in particular, as shown in table 1, the absolute value of the binding energy will be of the same order of the fermion mass. In this situation, a potential model with a fixed number of interacting particles cannot be consistently applied and the introduction of the screening factor is strictly required.

The results of tables 2–4 show that, with the screening factor, the binding energies remain reasonably smaller than the fermion mass. Furthermore, in the calculations, the relativistic corrections are always smaller than 20% of the binding energies. Obviously, the results depend on the screening mass: higher values of m_{scr} give a less screened potential and, as a consequence, a more deeply bound system. Again, this effect is more evident for higher values of fermion mass.

We now analyse in more detail the binding energies E_1, E_2 and E_3 for the three dynamical cases examined. In particular, comparing E_2 and E_1 we note that the effects of the Coulomb interaction are relatively small with respect to the Higgs interchange. The role of the Coulomb interaction is more important at low values of the fermion mass, when the Higgs interaction is weaker.

Finally, for a quark pair, considerably higher values of the binding energies E_3 are found with respect to the lepton pair energies E_1 and E_2. Within the present model, the strong interaction gives a very relevant contribution to the properties of the bound system. On the other hand, the Coulomb quark–antiquark interaction always gives only a small contribution to the binding of the system and, as a consequence, the values of the difference (δ) are always very small.

6. Comments and conclusions

The results shown in the previous section substantially confirm [30] the importance of the Higgs exchange in superheavy bound fermion systems. Furthermore, the Higgs interchange alone is sufficient to bind a neutral fermion pair.

The adopted model, particularly by means of the screening term in the Yukawa Higgs potential, allows to calculate perturbatively, without inconsistencies, all the contributions.

However, specially for high values of the fermion mass, the numerical results show a dependence on the screening mass parameter $m_{\rm scr}$.

The hypothesis of the existence of 4G fermions (with $m_{\rm f} \leq 1$ TeV) must be verified experimentally, in particular at the LHC. Less probably a conclusive answer to this problem will be given in a short time by Dark Matter experiments.

In very simple words, the following three scenarios can be conceived:

- 4G fermions and in particular their bound states are detected. Their properties are reasonably described by the present theoretical investigation that would represent a starting point to develop a more detailed model to study all the interactions of the system also including the effects listed at the end of §3.5. For the $Q\bar{Q}$ case the ground-state splitting and the spectrum of the excited states should be explored.
- 4G fermions and in particular their bound states are discovered but the properties of the bound states are significantly different from those determined in the present theoretical framework. In this case the description of the interaction should be revised considering its relationship with the underlying fundamental theory. Different values for the mass of the exchanged boson could be considered. It should also be verified whether it is possible to apply, in such case, a model consisting of a standard effective potential term and relativistic correction operators.
- 4G fermions are not experimentally found. This work and the other works on the same subject [26–33] can be considered as exercises with no direct application.

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