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On acceleration dependence of Doppler effect in light

SANJAY M WAGH

Central India Research Institute, 34, Farmland, Ramdaspeth, Nagpur 440 010, India
E-mail: waghsm.ngp@gmail.com

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Abstract. Using only the geometric relationships of suitable locations, we analyse Doppler effect in light to show how the acceleration of the source also contributes to the Doppler shift. We further propose that an experiment be performed using cyclotron-type devices to determine the acceleration dependence of the Doppler shift.

Keywords. Doppler effect; acceleration dependence; experiment; cyclotron-type devices.

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1. Introduction

Doppler effect involves [1–3] a shift in the wavelength or the frequency of a wave when its source and the observer are in relative motion.

As is well known [1–3], this effect is analysed, classically, for sound by using the (Newtonian) law of addition of velocities. However, this law of addition of velocities was found inadequate/deficient when applied to phenomena involving light.

Then, appealing to the covariance of Maxwell's equations, Einstein [4] had assumed the constancy of the speed of light (in vacuum) for inertial observers only, and had analysed Doppler effect in light to arrive at its 'transverse' version.

Clearly, the relative motion of the source and the observer can involve their relative acceleration, apart from their relative velocity. If the speed of light is the same for all observers, then we may expect the Doppler shift to naturally depend on the acceleration. But, Doppler effect was incompletely investigated in the past on this account.

As the history shows, the approaches of the past towards understanding Doppler effect can be broadly categorized as:

(1) those [4–7] considering the covariance of Maxwell's equations either in vacuum or in matter to obtain transformations of electric and magnetic fields in relevant frames of reference to obtain the Doppler shift,
(2) those [8,9] considering the Galilean invariance to obtain the Doppler shift,

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(3) those [10,11] extending special relativity, for example, by considering a maximum for the acceleration to obtain the Doppler shift,
(4) those [12–17] postulating ether, modified gravity, conformal covariance of Maxwell's equations, etc. to obtain the Doppler shift.

But, the Lorentz transformation properties of the fields do not show us how to involve the acceleration. Assumptions of extended special relativity need independent justification(s), which are not universally agreeable. Galilean invariance too does not involve acceleration. Also, the concept of ether is inessential [3].

Therefore, none of these approaches can be used in general situations of the source–observer motion and is not a complete explanation of the Doppler shift. Constancy of the speed of light for the involved observers is then not the only assumption of these approaches for deriving the Doppler shift.

With a complete investigation as our aim, we assume only that the speed of light (in vacuum) is the same for all observers. We make no other assumptions, and present the following 'new' analysis that is applicable in all the situations.

2. General case of relative motion with uniform velocity

Let a stationary source S emit a monochromatic wave whose crest meets an observer at point O at time $t = 0$, the initial instant of time (see figure 1a). Then, at the initial instant of time, the subsequent crest is lagging at point M, located at distance cT behind the point O along the line SO, the initial line of sight, with T being the period of the wave and c being the speed of light (in vacuum).

Now, at the aforementioned initial instant of time, let the observer at O move with uniform velocity $\vec{v}$ making an angle θ with the line SO, the initial line-of-sight with the source, as shown in figure 1a. Furthermore, let the wave at M reach the observer at point P after a lapse T' of time.

Actually, the ray from the source S passing through the point Q on the wavefront at M reaches the observer at time T', as is shown in figure 1a. Then, if we denote $SM = SQ = D$, we have $MQ = D\phi$.

But, the angle ϕ made by the rays SM and SQ at the source S is exceedingly small when the source is very distant from the observer. Then, $MP = QP$ to a high degree of accuracy. Thus, we may use MP for QP in further computations.

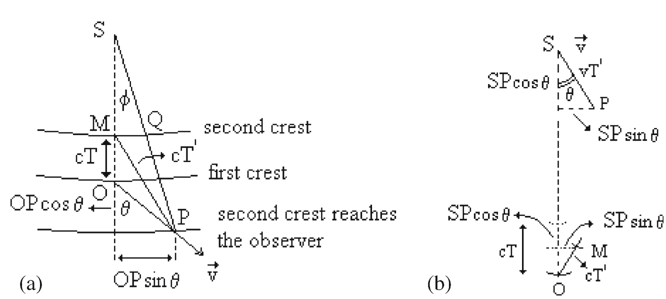

Figure 1. Geometry of the Doppler effect.

Then, from the geometry of figure 1a, we have

$$(cT')^2 = v^2 T'^2 \sin^2\theta + (cT + vT'\cos\theta)^2. \tag{1}$$

By manipulating this equation, we then obtain

$$\nu_o = \nu_e\left[\sqrt{1-\beta^2\sin^2\theta} - \beta\cos\theta\right] \equiv \nu_e\left[\frac{1-\beta^2}{\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}}\right], \tag{2}$$

where $\nu_e = 1/T$ is the frequency of the wave emitted by the source, $\nu_o = 1/T'$ is the frequency of the same wave observed by the observer and $\beta = v/c$.

Figure 1b now shows the observer as stationary and the source as moving (towards the observer) with velocity $\vec{v}$ making an angle θ with the line SO.

From the geometry of figure 1b, eq. (2) is obtained even in this case when we use the same speed c for light here as well. Then, only the relative motion between the source and the observer is significant for the effect.

The following special cases then follow:

- $\theta = 0$. Then, $\nu_o = (1-\beta)\nu_e$. This is the longitudinal Doppler effect with the source and observer moving away from each other.
- $\theta = \pi$. Then, $\nu_o = (1+\beta)\nu_e$. This is the longitudinal Doppler effect with source and observer approaching each other.
- $\theta = \pm\pi/2$. Then, $\nu_o = \nu_e\sqrt{1-\beta^2}$. This is the transverse Doppler effect.

Clearly, the expression obtained here for the transverse Doppler effect is 'identical' with that derived using the special theory of relativity. Evidently, the present derivation then complies with the relevant experimental results [18–22].

We emphasize that we have not used, do not need to use, the Lorentz transformations in this analysis, which uses only the geometric relationships of appropriate locations relevant to Doppler effect. In §3, these same relationships will be seen to permit considerations of relative motion with acceleration.

Let us then note that we can write eq. (1) as

$$(MP)^2 \equiv c^2T'^2 = c^2T^2 + 2cT(OP)\cos\theta + (OP)^2 \tag{3}$$

when the source is at rest as in figure 1a.

When the observer is at rest as in figure 1b, we can write eq. (1) as

$$(MO)^2 \equiv \bar{c}^2\bar{T}'^2 = \bar{c}^2\bar{T}^2 - 2\bar{c}\bar{T}(SP)\cos\theta + (SP)^2, \tag{4}$$

where $\bar{c}$ is the speed of light (in vacuum), $\bar{T}$ is the period of the wave emitted by the source, and $\bar{T}'$ is the measured period, all in a new frame of reference.

Notice that the observer is shown moving away from the source in figure 1a, while the source is shown moving towards the observer in figure 1b; and that accounts for the difference in signs of the $\cos\theta$ terms of eqs (3) and (4).

Furthermore, if eqs (3) and (4) are not identical for $(SP) = (OP)$, the observer will be able to measure self motion using Doppler effect. Then, the (general) principle that no observer measures self-motion using physical effects requires, in the simplest way, that $c = \bar{c}$, $T = \bar{T}$, $T' = \bar{T}'$, which we assume.

Clearly now, we do not have to specify how the observer has travelled distance (OP) in figure 1a or the source has travelled distance (SP) in figure 1b, whether with uniform velocity, constant acceleration or variable acceleration.

We emphasize that only the constancy of the speed of light for all observers is being used in the present analysis, and assumptions like the constancy of the acceleration, as in §3, are incidental in obtaining manageable equations.

3. General case of relative motion with uniform acceleration

It is, now, noteworthy that irrespective of how the distance OP in figure 1a (or SP in figure 1b) is travelled, whether at uniform speed or not, our approach provides for the shift in the frequency.

Then, let the observer move along the line OP in figure 1a (or let the source move along the line SP in figure 1b) with constant acceleration $\vec{a}$ and initial velocity $\vec{u}$. (In general, velocity and acceleration can be differently directed. Also, as we consider only the time interval equal to the period of the wave, constancy of the acceleration should turn out to be a good approximation.) In this case, the geometry now dictates that

$$(cT')^2 = \left(uT' + \frac{1}{2}aT'^2\right)^2 \sin^2\theta + \left[cT + \left(uT' + \frac{1}{2}aT'^2\right)\cos\theta\right]^2. \quad (5)$$

On manipulating this equation, we then obtain the following quartic equation:

$$\alpha^2 T'^4 + 4\alpha\beta T'^3 - 4(1-\beta^2 - \alpha T\cos\theta)T'^2 + (8\beta T\cos\theta)T' + 4T^2 = 0. \quad (6)$$

Then, in terms of the observed frequency ν_o and the emitted frequency ν_e, we have

$$\nu_o^4 + (2\beta\nu_e\cos\theta)\nu_o^3 - [(1-\beta^2)\nu_e^2 - \alpha\nu_e\cos\theta]\nu_o^2 + \alpha\beta\nu_e^2\nu_o + \frac{\alpha^2}{4}\nu_e^2 = 0, \quad (7)$$

where we have set $\alpha = a/c$ and $\beta = u/c$.

We may formally solve eq. (7) for $\nu_o \equiv \nu_o(\nu_e, \alpha, \beta, \theta)$. Note that, for $\alpha = 0$, the solution to this equation needs to reduce to that in eq. (2).

To first order in α and neglecting the term with product $\alpha\beta$, the observed frequency is obtained using eq. (7) as

$$\nu_o = \nu_e\left[\sqrt{1 - \beta^2\sin^2\theta - \left(\frac{\alpha}{\nu_e}\right)\cos\theta} - \beta\cos\theta\right]. \quad (8)$$

Equation (7) is obtained if the source was moving with initial velocity $\vec{u}$ and uniform acceleration $\vec{a}$ (not necessarily in the direction of velocity) along the line SP in figure 1b, provided the speed of light (in vacuum) is the same even in this case.

As is well known, the transverse Doppler shift [4] (for $\theta = \pi/2$) stands verified [18, 19,22] in experiments. So, we propose here the following experiment for verifying the acceleration dependence of the Doppler shift as in eq. (8).

4. Results of experimental significance

If we use an ion of mass m and charge q moving along a circular orbit of radius r in a confining magnetic field B, then we have

$$\beta = \left(\frac{\omega}{c}\right) r, \qquad \alpha = \omega\beta, \qquad \omega = \left(\frac{q}{m}\right) B \equiv 2\pi \nu_c \tag{9}$$

with the velocity being tangentially directed and the acceleration being radially directed towards the centre of the orbit. Here, ν_c is the cyclotron frequency of the ion.

Then, we may rewrite eq. (8) as

$$\theta = \pi, \qquad \nu_o = \nu_e \left(\frac{1+\beta}{2}\right) \left[1 + \sqrt{1 - \frac{4\pi(\nu_c/\nu_e)\beta}{(1+\beta)^2}}\right], \tag{10}$$

where the ratio ν_c/ν_e characterizes an experimental set-up.

When $\nu_c \ll \nu_e$, as is usually the case, we have

$$\theta = \pi, \qquad \nu_o \approx \nu_e \left[1 + \beta - \pi \left(\frac{\nu_c}{\nu_e}\right) \left(\frac{\beta}{1+\beta}\right)\right]. \tag{11}$$

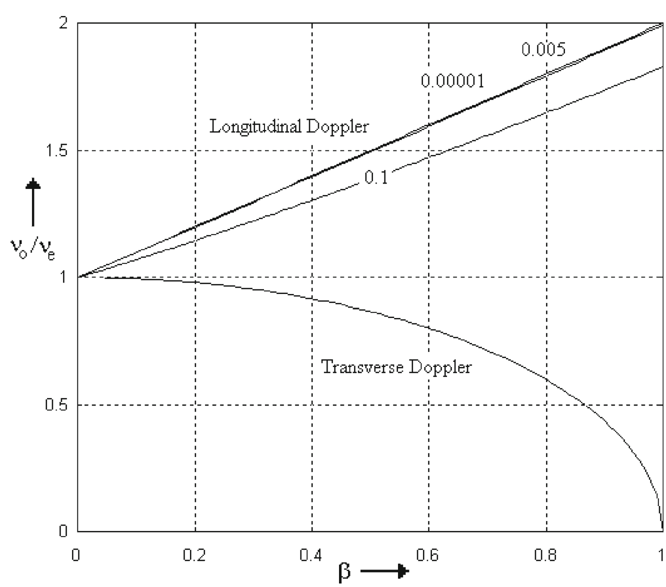

Figure 2. ν_o/ν_e vs. β for different values of ν_c/ν_e.

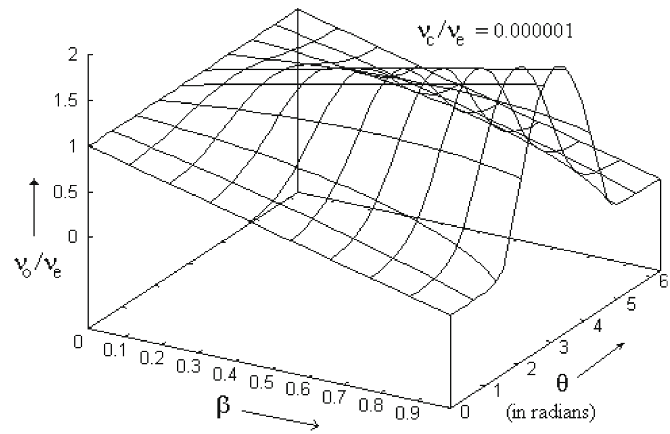

Figure 3. Angular and β dependence of ν_o/ν_e for $\nu_c/\nu_e = 0.000001$.

The dependence of the observed frequency on the cyclotron frequency to its second order is then implied by the presence of acceleration term.

A comparison of the transverse Doppler shift and the longitudinal Doppler shift is then provided in figure 2 for the ready reference.

The angular dependence of ν_o/ν_e obtained, now, from eq. (8) is shown in figure 3 (for a particular value of ν_c/ν_e) as a surface. For particular values of β, the angular dependence of ν_o/ν_e is shown in figure 4.

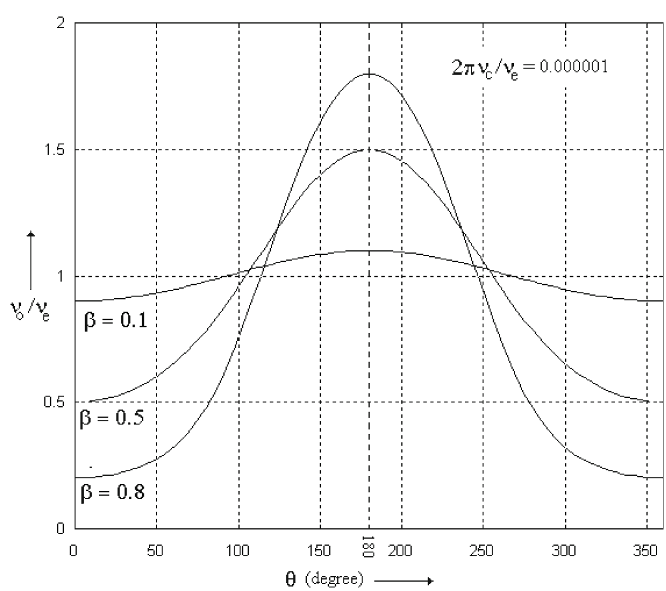

Figure 4. Angular dependence of ν_o/ν_e for $\nu_c/\nu_e = 0.000001$.

5. Possible experimental strategies

Equation (11) implies two experimental strategies, depending on whether ν_c is maintained constant or β is maintained constant within the experiment. In what follows, we consider the details of these two experimental strategies.

5.1 *When ν_c is constant*

For the first experimental strategy, let us consider that the magnetic field is maintained constant within the experimental device like a synchrotron. Then, when ν_c is maintained constant within an experimental set-up, eq. (11) implies that

$$\left.\frac{\Delta\nu}{\nu_e}\right|_{\nu_c} = \left.\frac{\nu_o^{(2)} - \nu_o^{(1)}}{\nu_e}\right|_{\nu_c} \approx (\beta_2 - \beta_1)\left[1 - \frac{\pi(\nu_c/\nu_e)}{(1+\beta_1)(1+\beta_2)}\right]. \tag{12}$$

We may select $\beta_1 \sim 0$, and obtain

$$\left.\frac{\Delta\nu}{\nu_e}\right|_{\nu_c} \approx \beta_2\left[1 - \frac{\pi(\nu_c/\nu_e)}{1+\beta_2}\right] \approx 1 - 1.57 \times \left(\frac{\nu_c}{\nu_e}\right) \quad \text{for} \;\; \beta_2 \sim 1. \tag{13}$$

The resolution of the effect being sought here then depends on the ratio of the cyclotron frequency and the rest frequency of the ionic emission. In synchrotron-type devices, using which $\beta \sim 1$ may be achievable, this strategy may be adoptable.

5.2 *When β is constant*

For the other experimental strategy, let us consider a 'fixed' value of β. For a specific value of β, we then obtain from eq. (11):

$$\Delta\nu|_\beta = \nu_o^{(2)} - \nu_o^{(1)} \approx \pi[\nu_c^{(1)} - \nu_c^{(2)}]\left(\frac{\beta}{1+\beta}\right). \tag{14}$$

For the same value of β, in terms of the confining magnetic field strength $B^{(1)}$ at the radius r_1 of the orbit of the ion and the strength $B^{(2)}$ at the radius r_2, we require

$$\beta = \frac{2\pi}{c}\nu_c^{(1)} r_1 = \frac{2\pi}{c}\nu_c^{(2)} r_2 \qquad \Longrightarrow \qquad B^{(1)} r_1 = B^{(2)} r_2. \tag{15}$$

We therefore obtain

$$\left.\frac{\Delta\nu}{\nu_e}\right|_\beta = 2\pi\left(\frac{\nu_c^{(1)}}{\nu_e}\right)\left(\frac{r_2 - r_1}{2r_2}\right)\left(\frac{\beta}{1+\beta}\right). \tag{16}$$

Then, on comparing the observed frequencies at two suitable radii, r_1 and r_2, and for differing strengths B_1 and B_2 of the magnetic field, we may establish the acceleration-type cyclotron frequency dependence of the frequency of the ionic emission.

For this experiment, the ratio $\nu_c^{(1)}/\nu_e$ is also important. It needs to be suitably large for establishing the effect under considerations. For largeness of the β-term in eq. (16), we may arrange to inject ions, possessing suitably high energy, within two different strengths of the confining magnetic field.

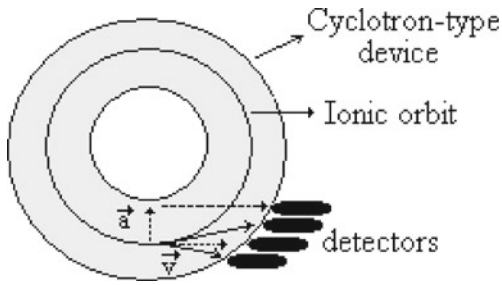

Figure 5. Detectors in place in a cyclotron-type device.

6. Discussion

Thus, within cyclotron-type devices, we may place radiation detectors on positions that are tangential to the ionic orbits (see figure 5).

On comparing radiation emitted at suitable radii of ionic orbits, it should be possible to verify the cyclotron frequency dependence of the frequency of the ionic emission of the type implied by the acceleration-dependent term in eq. (11).

The effect to be expected is a shift of the $1 + \beta$ line below it by the order of ν_c/ν_e in the ν_o/ν_e vs. β (or, equivalently, r) plot.

For $\nu_c \sim 10^8$ Hz and observations at optical frequencies, $\nu_e \sim 10^{14}$ Hz, we expect the shift to be $\sim 10^{-6}$. This is illustrated in figure 6 for the moderate range of the parameter

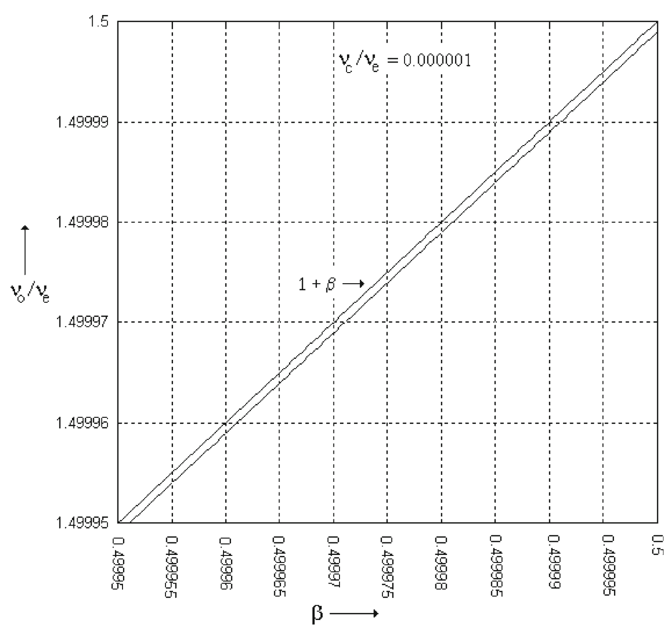

Figure 6. Shift of ν_o/ν_e below the line $1 + \beta$.

β, which is linearly related to the radius r of the orbit via the cyclotron frequency of the ion as in eq. (9).

We leave out of this discussion issues of data contamination, for these are also related to specifics of the experimental set-up. Nevertheless, the primary source of data contamination will be the nearby orbits sending the same frequency to detectors due to the angular dependence of ν_o/ν_e as shown in figure 3 or figure 4. The rate of increase of the radius of orbit of ions (spiralling out of the centre) per cycle of revolution under the applied electric field determines the data contamination of this kind. Shielding detector(s) from certain angles may then be an issue for the experimental set-up. This can be achieved using values of θ implied by eq. (8) or its approximate form given by

$$\nu_o \approx \nu_e \left[1 - \frac{\beta^2}{2} \sin^2\theta - \beta\cos\theta\right] - \pi\nu_c\beta\cos\theta. \quad (17)$$

As these and other issues are related to specifics of the experimental set-up, we do not discuss them any further.

Now, the proposed explanation of Doppler effect has the following features:

(1) It crucially assumes only the constancy of the speed of light for all observers; whether moving with uniform velocity or with acceleration. It implies then the acceleration dependence of the Doppler shift.
(2) The approximation that the distance between the source and the observer is much larger than the wavelength of light is incidental for it. Without this kind of approximation, the present method leads to an equation involving $D\phi$.
(3) It does not involve the transformation of the time coordinate between the frames of reference: T' or ν_o is, and so is T or ν_e, identical in them. This is required to ensure that the observer does not detect self-motion.
(4) Assumed constancy of the acceleration is also incidental, and it should be a good approximation in most situations as it is over the period of the light wave. Without this assumption, the present method can, in principle, provide for the Doppler frequency shift, also.

In summary, any experiment of the proposed type will then establish the reality or otherwise of the acceleration dependence of the Doppler effect, which has hitherto been neglected. Any such experiment will also be an experimental test of the hypothesis that the speed of light (in vacuum) is independent of the state of motion of the observer. This is because our analysis of the Doppler shift is explicitly based only on it.

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References

[1] E T Whittaker, *A history of the theories of aether and electricity – The classical theories* (Thomas Nelson and Sons, London, 1958)
[2] R P Feynman, R B Leighton and M Sands, *The Feynman lectures on physics* (Addison-Wesley, Reading, Massachusetts, 1977) Vol. 1
[3] S M Wagh and D A Deshpande, *Essentials of physics* (Prentice-Hall India, New Delhi, 2013) Vol. 2
[4] A Einstein, *Ann. d. Phys.* 17 (1905)
[5] B Rothenstein and A De Sabata, *Eur. J. Phys.* **19**, 589 (1998)
[6] B Rothenstein and A De Sabata, *Electrodynamics in material media* (2005), available as: http://arxiv.org/physics/0501159
[7] H Kim, *J. Mod. Opt.* **59**, 1330 (2012)
[8] H Poincare, *R. Gen. des Sciences Pures et Appl.* **19**, 386 (1908)
[9] Y Pierseaux, *Annales de la Fondation de Broglie*, **30**, 3–4 (2005)
[10] Y Friedman, *Ann. d. Phys.* **523**, 408 (2011)
[11] Y Friedman and I Nowik, *Phys. Scr.* **85(6)**, 065702 (2012)
[12] W Cochran, *Am. J. Phys.* **57**, 1039 (1989)
[13] C Neipp *et al*, *Eur. J. Phys.* **24**, 497 (2003)
[14] R Wayne, *African Phys. Rev.* **4**, 43 (2010)
[15] R T Cahill, *Infinite Energy* **10**, 28 (2005)
[16] R T Cahill, *Einstein and Poincare: The physical vacuum* edited by V Dvoeglazov (Apeiron, Montreal, 2006) pp. 129–141
[17] W Moreau, *Am. J. Phys.* **62**, 5 (1994)
[18] H E Ives and G R Stilwell, *J. Opt. Soc. Am.* **28(7)**, 215 (1938)
[19] H E Ives and G R Stilwell, *J. Opt. Soc. Am.* **31**, 369 (1940)
[20] H I Mandelberg and L Witten, *J. Opt. Soc. Am.* **52(5)**, 529 (1962)
[21] W Kündig, *Phys. Rev.* **126(6)**, 2371 (1963)
[22] D Hasselkamp, E Mondry and A Scharmann, *Z. Phys. A* **289(2)**, 151 (1979)