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Localized structures for (2+1)-dimensional Boiti–Leon–Pempinelli equation

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Abstract. It is shown that Painlevé integrability of (2+1)-dimensional Boiti–Leon–Pempinelli equation is easy to be verified using the standard Weiss–Tabor–Carnevale (WTC) approach after introducing the Kruskal's simplification. Furthermore, by employing a singular manifold method based on Painlevé truncation, variable separation solutions are obtained explicitly in terms of two arbitrary functions. The two arbitrary functions provide us a way to study some interesting localized structures. The choice of rational functions leads to the rogue wave structure of Boiti–Leon–Pempinelli equation. In addition, for the other choices, it is observed that two solitons may evolve into breather after interaction. Also, the interaction between two kink compactons is investigated.

Keywords. Rogue waves; Painlevé test; a singular manifold method; soliton.

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1. Introduction

Since the soliton concept was introduced by Zabusky and Kruskal in 1965 [1], many integrable systems have been discovered in the natural and applied sciences [2–19]. Integrable systems exhibit richness and variety of exact solutions such as soliton solutions, periodic solutions, rational solutions, and complexiton solutions (see [20,21]).

In recent years, many localized structures, like dromions, lumps, ring soliton and oscillated dromion, breather solution, fractal dromion, and fractal lump soliton structures [22], were discovered. Besides the usual localized structures, some new localized excitations like peakons, compactons, folded solitary waves, and foldon structures were found by choosing some types of lower-dimensional appropriate functions [22–30]. The interaction

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properties of peakon–peakon, dromion–dromion, and foldon–foldon interactions have also been investigated [23,24]. Recently, some novel localized structures referred to as rogue waves solutions [25–27], have also been studied. In this work, we study rogue waves and novel interactions among multiple kink compactons and solitons in the framework of (2+1)-dimensional Boiti–Leon–Pempinelli (BLP) equation. The BLP equation is given by [31]

$$v_t - v_{xx} - 2uv_x = 0, \tag{1}$$

$$u_{yt} - (u^2 - u_x)_{xy} - 2v_{xxx} = 0, \tag{2}$$

which describes the evolution of the horizontal velocity component of the water waves propagating in the x–y plane in an infinite narrow channel of constant depth. It is also thought to be a (2+1)-dimensional generalization of the sinh-Gordon equation. By suitable transformation and reduction [31,32], BLP equation can be reduced to one-dimensional dispersive long-wave equation, the Burgers or anti-Burgers equation. Additionally, various interesting properties of (2+1)-dimensional BLP equation have been studied by many authors [31–34]. For example, the Hamiltonian structure, Lax pair, and Bäcklund transformation have been discussed in [31]. Besides, the bilinear form of the BLPE is obtained by virtue of the binary Bell polynomials in [33]. With the help of extended tanh-function method, different kinds of localized coherent structures have been obtained in [34].

The primary purpose of the work is to explore novel localized structures of BLP equation by using a singular manifold method based on Painlevé truncation. In §2, we carry out the Painlevé analysis to verify the integrability using WTC–Kruskal approach [4–7]. In §3, we search for variable separation solutions. In §4, we study three types of novel localized structures with the help of arbitrary functions. The conclusions are given in §5.

2. Painlevé test

In this section, we give out the standard test with the Kruskal's simplification, which is constituted by three steps, the leading order analysis, the resonance determination, and the resonance conditions' verification. According to the standard WTC approach [4–7], we investigate the general solution of (1)–(2) of the following form:

$$u = \sum_{j=0}^{\infty} u_j \phi^{(j+\alpha)}, \qquad v = \sum_{j=0}^{\infty} v_j \phi^{(j+\beta)}, \tag{3}$$

substituting $u = u_0\phi^{\alpha}$, $v = v_0\phi^{\beta}$ into eqs (1) and (2), and balancing the leading term of ϕ, leads to

$$\alpha = \beta = -1, \tag{4}$$

with the condition

$$u_0 = \phi_x, \qquad v_0 = \phi_y. \tag{5}$$

Inserting (3) into (1) and (2) and vanishing the coefficients of (ϕ^{j-3}, ϕ^{j-4}) produce resonances

$$j = -1, 1, 2, 3, 4. \tag{6}$$

It is noted that the resonance at $j = -1$ corresponds to the arbitrary singularity manifold ϕ. To verify the resonance conditions, we introduce the Kruskal's simplification

$$\phi(x, y, t) = x + f(y, t), \qquad u_j = u_j(y, t), \qquad v_j = v_j(y, t), \tag{7}$$

where $f(y, t)$ is assumed to be an arbitrary function of y, t. Using this simplification, eq. (5) is reduced to

$$u_0 = 1, \qquad v_0 = f_y. \tag{8}$$

For $j = 1$, vanishing the coefficients of (ϕ^{-2}, ϕ^{-3}) leads to

$$u_1 = \frac{1}{2} f_t, \tag{9}$$

v_1 is an arbitrary function. Obviously, the resonance condition at $j = 1$ is satisfied.

For $j = 2$, vanishing the coefficients of (ϕ^{-1}, ϕ^{-2}), with the help of conclusion (9), we have

$$u_2 = \frac{2v_2 - f_{yt}}{2 f_y}, \tag{10}$$

where v_2 is an arbitrary function. Thus, the resonance condition at $j = 2$ holds.

For $j = 3$, vanishing the coefficients of (ϕ^0, ϕ^{-1}), with the help of the conclusions (9) and (10), we find that

$$u_3 = \frac{6v_3 - v_{1t}}{2 f_y}, \tag{11}$$

where v_3 is an arbitrary function. Similarly, the resonance condition at $j = 3$ is identically satisfied.

For $j = 4$, vanishing the coefficients of (ϕ^1, ϕ^0), with the help of the previous condition, it reads that

$$u_4 = -\frac{1}{2} \frac{v_{2t} f_y - 12 v_4 f_y + v_2 f_{yt} - 2v_2^2}{f_y^2}, \tag{12}$$

where v_4 is an arbitrary function. As a result, the resonance conditions at $j = 4$ is also satisfied. This shows that (2+1)-dimensional BLP equation can pass Painlevé test. This is in agreement with the modification of the Painlevé test of Garagash [39].

3. Variable separation solutions

In this section, we concentrate on seeking for variable separation solutions of (2+1)-dimensional BLP equation. To this end, we truncate the Laurent series of (2+1)-dimensional BLP equation at the constant level term, and we obtain the following Bäcklund transformation:

$$u = \frac{u_0}{\phi} + u_1, \qquad v = \frac{v_0}{\phi} + v_1. \tag{13}$$

Following general procedure, we consider a seed solution

$$u_1 = u_1(x, t), \qquad v_1 = 0. \tag{14}$$

Now, inserting eq. (13) with the seed solution (14) into (1) and (2), then vanishing the coefficients of (ϕ^{-3}, ϕ^{-4}), we have

$$v_0\phi_x^2 - u_0 v_0 \phi_x = 0, \tag{15}$$

$$-6u_0\phi_x^2\phi_y + 12v_0\phi_x^3 - 6u_0^2\phi_y\phi_x = 0. \tag{16}$$

Solving the above system, we obtain the leading-order coefficients u_0, v_0 in agreement with (5). Subsequently, equating the coefficients of (ϕ^{-2}, ϕ^{-3}) to zero we get

$$-v_0\phi_t - 2u_0 v_{0x} + v_0\phi_{xx} + 2v_{0x}\phi_x + 2u_1 v_0\phi_x = 0, \tag{17}$$

$$\begin{aligned} &2u_0\phi_{xx}\phi_y - 12v_0\phi_x\phi_{xx} + 2u_0\phi_y\phi_t + 2u_{0y}\phi_x^2 - 12v_{0x}\phi_x^2 + 4u_{0x}\phi_x\phi_y \\ &\quad - 4u_1u_0\phi_x\phi_y + 4u_0\phi_x\phi_{yx} + 4u_0\phi_y u_{0x} + 2u_0^2\phi_{yx} + 4u_{0y}u_0\phi_x = 0. \end{aligned} \tag{18}$$

With eq. (5), the above set of overdetermined system of equations can be consistently solved as follows:

$$u_1(x, t) = \frac{\phi_{1t} + \phi_{2t} - \phi_{1xx}}{2\phi_{1x}}, \qquad \phi = \phi_1(x, t) + \phi_2(y, t). \tag{19}$$

Next, from the coefficients of (ϕ^{-1}, ϕ^{-2}), we have

$$v_{0t} - v_{0xx} - 2u_1 v_{0x} = 0, \tag{20}$$

$$\begin{aligned} &2v_0\phi_{xxx} - u_{0y}\phi_t - u_{0t}\phi_y - 2u_{0y}u_{0x} + 6v_{0x}\phi_{0xx} - u_{0y}\phi_{xx} - u_0\phi_{yt} \\ &\quad + 6v_{0xx}\phi_x + 2u_1u_{0x}\phi_y + 2u_1u_{0y}\phi_x - u_0\phi_{yxx} + 2u_0\phi_y u_{1x} - 2u_{0yx}\phi_x \\ &\quad - 2u_{0x}\phi_{yx} + 2u_1u_0\phi_{yx} - u_{0xx}\phi_y - 2u_0u_{0yx} = 0. \end{aligned} \tag{21}$$

The compatibility condition of the above equations is that ϕ_2 should only be a function of the variable y

$$\phi_2 = \phi_2(y). \tag{22}$$

On the above conditions, it is easily found that the coefficients (ϕ^0, ϕ^{-1})

$$-2v_{0xxx} + u_{0yt} + u_{0yxx} - 2u_1u_{0yx} - 2u_{0y}u_{1x} = 0 \tag{23}$$

hold naturally. Thus, we obtain variable separation solution of (2+1)-dimensional BLPE as

$$u = \frac{\phi_{1x}}{\phi_1 + \phi_2} + \frac{\phi_{1t} - \phi_{1xx}}{2\phi_{1x}}, \quad v = \frac{\phi_{2y}}{\phi_1 + \phi_2}, \tag{24}$$

where $\phi_1 = \phi_1(x, t)$ and $\phi_2 = \phi_2(y)$ are arbitrary functions. The corresponding potential field reads as

$$w = -u_y = -v_x = \frac{\phi_{2y}\phi_{1x}}{(\phi_1 + \phi_2)^2}. \tag{25}$$

4. Novel localized excitations

So far, localized coherent structures such as the dromions, solitoff solutions, and lump solutions for the higher-dimensional NLEEs have been proposed [36]. In addition, it is possible to obtain more and more interesting localized structures with the help of arbitrary functions in (25). In this paper, after many attempts, we pick out three choices of arbitrary functions $\phi_1(x, t)$ and $\phi_2(y)$ to construct novel localized structures as follows.

(I) *Rogue wave*

If the arbitrary functions $\phi_1(x, t)$ and $\phi_2(y)$ are chosen as

$$\phi_1(x, t) = \frac{x^2}{2} + t^2, \tag{26}$$

$$\phi_2(y) = a^2 + \frac{y^2}{2}, \tag{27}$$

this case yields

$$w(x, y, t) = \frac{xy}{(a^2 + (x^2/2) + t^2 + (y^2/2))^2}. \tag{28}$$

Here, a is an arbitrary nonzero real parameter. At $t = 0$, the function $w(x, y, 0)$ becomes

$$w(x, y, 0) = \frac{xy}{(a^2 + (x^2/2) + (y^2/2))^2}.$$

In order to obtain the maximum values of the function $w(x, y, 0)$, it is needed to calculate the necessary condition

$$\frac{\partial w(x, y, 0)}{\partial x} = \frac{-4y\left(2a^2 - 3x^2 + y^2\right)}{\left(2a^2 + x^2 + y^2\right)^3} = 0,$$

$$\frac{\partial w(x, y, 0)}{\partial y} = \frac{-4x\left(2a^2 - 3y^2 + x^2\right)}{\left(2a^2 + x^2 + y^2\right)^3} = 0.$$

Thus, solving these two conditions leads to five points

$$(a, a), \quad (a, -a), \quad (-a, a), \quad (-a, -a), \quad (0, 0).$$

After calculations, it is easily verified that maximum values of the function $w(x, y, 0)$ occur at the points $(-a, a)$ and $(a, -a)$. Furthermore, one can get the maximum value as

$$w_{\max} = \frac{1}{4a^2}.$$

From figure 1, it is observed that the three-dimensional spatial structure of the function $w(x, y, 0)$ has similar structure of the rogue waves which is a point of hot discussion in recent years [26]. The maximum amplitude of the rogue wave solution (28) increases inversely with the values of the parameter a. In figure 2, when $a = 1/4$, the maximum amplitude of this rogue wave solution is equal to 4. Meanwhile, we note that this rogue wave has two peaks whose amplitudes are equal to the amplitudes of the one-dimensional rogue waves observed in [25]. Indeed, it is interesting that 2D rogue waves have more diversity than 1D rogue waves as in an integrable 2D Schödinger equation proposed by Kundu's group [27]. Thus, it indicates that 2D rogue wave which is localized in both x and y directions may also exist in the framework of BLP equation.

(II) *Two solitons evolve into breather after collision*

If we choose the arbitrary functions $\phi_1(x, t)$ and $\phi_2(y)$ as

$$\phi_1(x, t) = a_0 + a_1 \cosh(k_1 x + c_1 t), \tag{29}$$

$$\phi_2(y) = b_0 + b_1 \tanh(k_2 y) + b_2 \exp(k_3 y) + b_3 \cos(k_4 y), \tag{30}$$

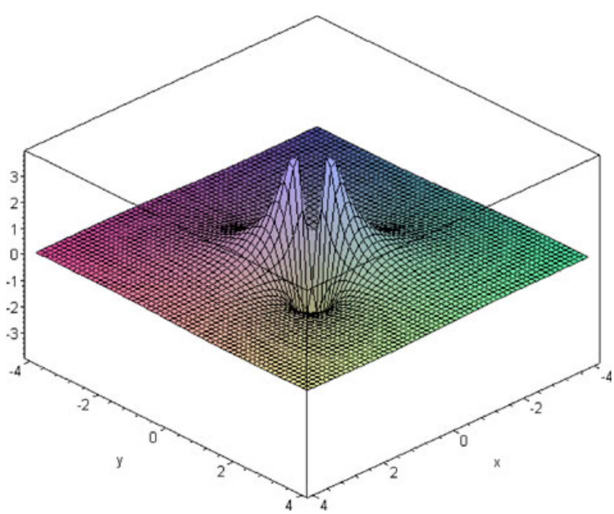

Figure 1. The spatial structure of 2D rogue wave for eq. (28) at $t = 0$ for $a = 1/4$.

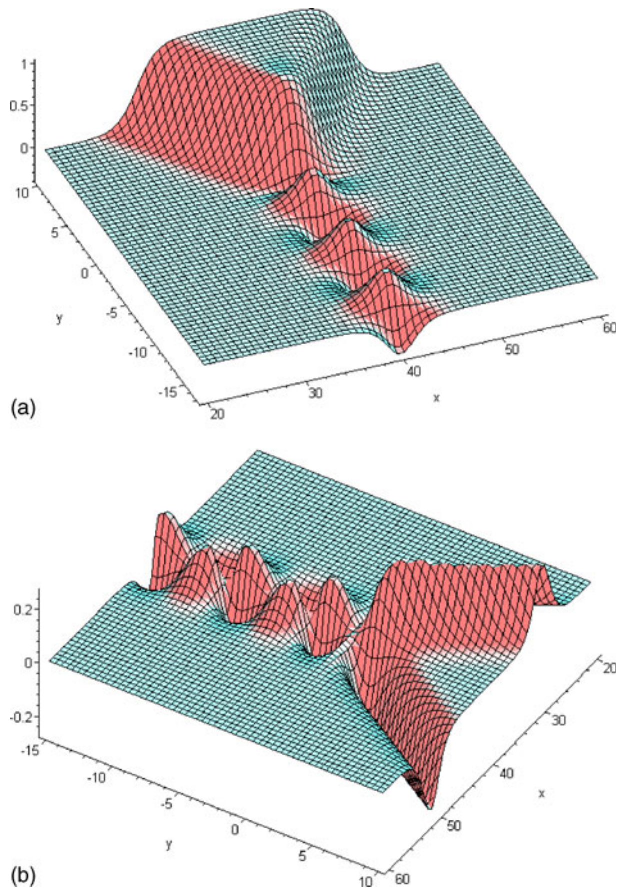

Figure 2. A spatial structure in x–y plane of (**a**) v in (24) and (**b**) w in (25) under conditions (29) and (30).

figure 2 exhibits breather generation after two soliton collision for the parameters $a_0 = 1, a_1 = 10, k_2 = k_3 = k_4 = k_1 = c_1 = b_0 = 1, b_1 = -10, b_2 = 10, b_3 = 8, t = -40$.

(III) *Multiple kink–compacton interactions*

Rosenau and Hymann have found compactons which vanish identically outside a finite region and retain their identity after multiple collisions [37,38]. Then, we choose the function $\phi_1(x,t)$ as compacton

$$\phi_1(x,t) = b_0 + \sum_{i=1}^{N} \begin{cases} 0, & x + v_i t \le x_{0i} - \dfrac{\pi}{2k_i} \\ b_i \sin(k_i(x + v_i t - x_{0i})) + b_i, & x_{0i} - \dfrac{\pi}{2k_i} < x + v_i t \le x_{0i} + \dfrac{\pi}{2k_i} \\ 2b_i, & x + v_i t > x_{0i} + \dfrac{\pi}{2k_i} \end{cases} \tag{31}$$

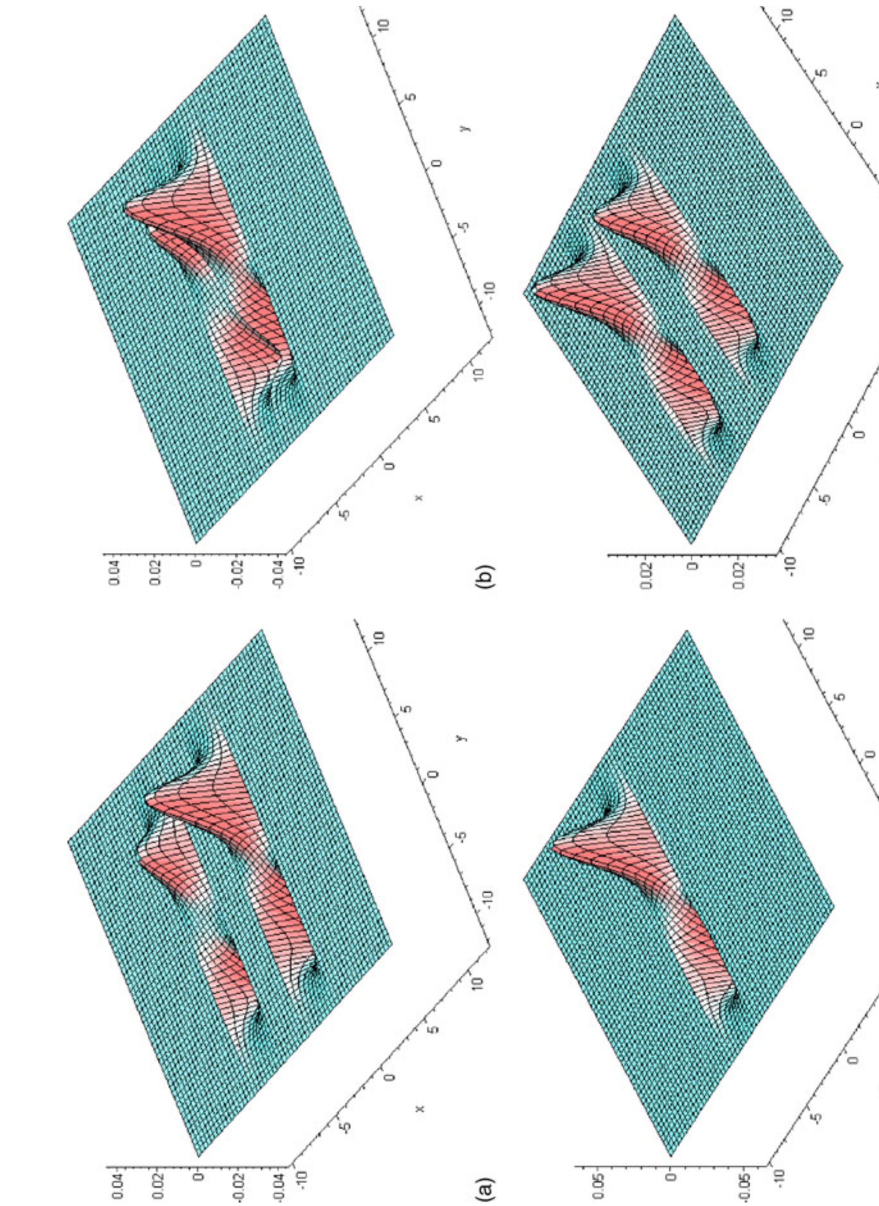

Figure 3. The evolution plots of interactions between kink and compactons corresponding to (25) with (31) and (32), **(a)** $t = -2$; **(b)** $t = -0.8$; **(c)** $t = 0$; **(d)** $t = 2$.

and $\phi_2(y)$ as a combination of hyperbolic cosh function in the following form:

$$\phi_2(y) = c_0 + \sum_{i=1}^{M} c_i \cosh(l_i(y + y_{0i})), \tag{32}$$

where $b_i, x_{0i}, k_i, c_i, y_{0i}$, and v_i are arbitrary constants. For $M = 1, N = 2, b_0 = 80, b_1 = -2, b_2 = -10, k_1 = k_2 = 1, v_1 = -1, v_2 = 2, c_0 = 0, c_1 = 1, l_1 = 1, x_{01} = x_{02} = y_{01} = 0$, figure 3 shows that interaction between two kink–compactons is inelastic.

5. Conclusions

In this work, Painlevé integrability of (2+1)-dimensional BLP equation has been effectively demonstrated using the standard WTC–Kruskal approach. This outcome has been compared with the modification of the Painlevé test of Garagash [39], and then by using a singular manifold method based on Painlevé truncation, variable separation solutions with two arbitrary functions are obtained. Based on (24), new types of interactions among solitons as well as kink–compactons of BLPE are investigated both analytically and graphically. Meanwhile, the arbitrary function allows us to construct rogue wave solution of BLP equation. It is expected that the interaction among rogue wave also can be studied by the proper choice of the arbitrary functions in the future. To the best of our knowledge, these novel properties and interesting localized structures are investigated for the fist time in BLP equation.

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