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A class of exact strange quark star model

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Abstract. Static spherically symmetric space-time is studied to describe dense compact star with quark matter within the framework of MIT Bag Model. The system of Einstein's field equations for anisotropic matter is expressed as a new system of differential equations using transformations and it is solved for a particular general form of gravitational potential with parameters. For a particular parameter, as an example, it is shown that the model satisfies all major physical features expected in a realistic star. The generated model also smoothly matches with the Schwarzschild exterior metric at the boundary of the star. It is shown that the generated solutions are useful to model strange quark stars.

Keywords. Einstein's field equations; anisotropic matter; equation of state; quark star.

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1. Introduction

The modelling and description of compact astrophysical objects have been a critical issue in relativistic astrophysics, and strange quark stars (SQS) have long been proposed as an alternative to neutron star models. Recent experimental observations of radio timing measurements such as a strong Shapiro delay signature in the binary millisecond pulsar PSR J1614-2230 [1] and of the three post-Keplerian effects in the binary pulsar PSR J1903+0327 [2] suggest that the densities of these compact objects exceed by far the ground state density of atomic nuclei, $\rho_0 \sim 0.16$ nucleons/fm^3 ($\sim 2.7 \times 10^{14}$ g cm^{-3}) or the so-called neutron stars. These experimental results, together with many others [3–6], suggest that these compact stars might contain a deconfined and chirally restored quark phase. The so-called hybrid stars contain quarks only in their interior either in the form of a pure quark matter core or as a quark–hadron mixed phase. The SQS [7] is realized for the special scenario of absolutely stable strange quark matter (SQM) that forms the true ground state of nuclear matter [8] and occupies the entire compact star. These highly dense compact objects are not compatible with the standard neutron star models. A SQS would be the bulk SQM phase consisting of almost equal number of up, down and strange quarks, plus a small number of electrons to ensure charge neutrality. SQM would have a lower charge-to-baryon ratio compared to the nuclear matter and can show itself in the form of an SQS [8]. As SQM is more stable and bound at finite density, one expects configurations of quark stars with macroscopic properties quite different from neutron stars [9–11]. Strange stars are expected to form during the collapse of the core of a massive star after the supernova explosion. Neutron stars with 1.5–1.8$M_{\odot}$ masses having rapid spins are theoretically the best candidates for conversion to an SQS.

The SQS, founded from quark matter theory consists of too many unsolved puzzles which are usually involved in the physics of these high-density relativistic objects and hence prohibits considering all physical and astrophysical properties simultaneously. Therefore, most of the SQS studies have been performed within the framework of quark matter MIT Bag Models [9,11–16]. In the quark bag model [17], it is assumed that breaking of physical vacuum takes place inside hadrons. As a result, vacuum energy densities inside and outside a hardron becomes essentially different and the vacuum pressure on the bag wall equilibrates the quark pressure thus stabilizing the system. In the simplified version of the bag model, assuming quarks are massless and non-interacting, we have quark pressure $p_{\mathrm{q}} = \frac{1}{3}\rho_{\mathrm{q}}$, the total energy density $\rho = \rho_{\mathrm{q}} + B$ and the total pressure $p = p_{\mathrm{q}} - B$, where ρ_{q} is the quark energy density. Thus, one gets the equation of state (EOS) for SQM as $p = \frac{1}{3}(\rho - 4B)$ [18], where B is known as the bag constant, which is the universal pressure on the surface of any region containing quarks. The stability of the SQS is due to the long-range effects of confinement of quarks, represented by the bag constant B [9,17]. The EOS formulated by Dey *et al* [6] can also be approximated to a linear form [19] that describes the quark interaction in a SQS by an interquark vector potential originating from gluon exchange and a density-dependent scalar potential which restores chiral symmetry at high density. Therefore, linearity in EOS seems to be a feature in the composition of compact objects such as SQS. Nevertheless, there have been a few works reported with non-linear EOS like by Varela *et al* [20], Feroze and Siddiqui [21] and Maharaj and Mafa Takisa [22] on anisotropic matter in the presence of electromagnetic field. Recent work of Maharaj and Takisa provides a new class of physically reasonable exact solutions to the Einstein–Maxwell system of equations in the static spherical symmetry, utilizing a quadratic equation of state relating the radial pressure to the energy density.

However, as densities within SQS are normally beyond nuclear matter density, one expects anisotropy to play a crucial role in the modelling of ultracompact stars such as SQS. Early work of Ruderman showed that nuclear matter may become anisotropic in the high-density region of order 10^{15}g cm^{-3}, where nuclear interactions need to be treated relativistically [23]. Since the pioneering work of Bowers and Liang [24], there has been extensive literature devoted to the study of anisotropic, spherically symmetric, static relativistic matter distributions in which radial pressure (p_{r}) is not equal to tangential pressure (p_{t}). The difference $\Delta = p_{\mathrm{t}} - p_{\mathrm{r}}$ is crucial for calculating surface tension of a compact star. The anisotropy will be directed outward (repulsive) when $p_{\mathrm{t}} > p_{\mathrm{r}}$ (i.e., $\Delta > 0$) and inwards when $p_{\mathrm{t}} < p_{\mathrm{r}}$ (i.e., $\Delta < 0$). Chan *et al* [25] studied in detail the role played by the local pressure anisotropy in the onset of instabilities and showed that small anisotropies might in principle drastically change the stability of the system. Various factors may contribute to pressure anisotropy [26] such as exotic phase transitions during gravitational collapse [27,28], the existence of a solid core or the presence of a type-3A superfluid [29], strong electromagnetic fields [30–32], viscosity [33] as well as the slow rotation of a fluid [34]. Though we lack a complete understanding of the microscopic origin of the pressure anisotropy, the role of pressure anisotropy in modelling compact stars is a field of active research. There have been different solutions of Einstein's field equations for anisotropic fluid distribution with spherical geometry [35–43], in particular, results on modelling for anisotropic SQM [37–39,42,43] with a barotropic EOS. However, relatively few interior solutions have been reported in the literature that satisfy all the physical requirements of a realistic star.

We have however done a study on an anisotropic compact static sphere with polytropic EOS [42] for a particular choice of gravitational potential. In this work we have treated the anisotropic compact spherical matter with a linear EOS by considering a generalized form of the gravitational potential chosen in [42]. In order to compare with the results in [42], a particular example with the same potential has been chosen in §4. A particular distinction in the linear EOS of SQM is the non-zero surface density where the binding effect is represented by the bag constant, whereas with polytropic EOS in the model studies [42] the surface becomes indistinguishable with vanishingly small density. In fact, a quark star can be bound not only by gravity but also by additional strong interaction due to strong confinement between quarks. The quark phase inside the core is best described by linear bag model whereas a polytropic EOS may represent a hybrid star which have cores composed of unconfined quarks [44,45] that describe normal phase (composed of normal baryons) and mixed phase (composed of denser baryon matter).

Our objective in this paper is to generate a new class of models for anisotropic SQM with linear EOS using a systematic way that satisfies the physical criteria [46]: regularity of the gravitational potentials at the origin, positive definiteness of the energy density and the radial pressure at the origin, vanishing of the radial pressure at some finite radius, monotonic decrease of the energy density and the radial pressure with increasing radius and the speed of the sound is less than the speed of the light. In addition, the generated interior metric matches smoothly with the Schwarzschild exterior metric at the boundary of stellar object. Therefore, the generated models are expected to give a satisfactory description of realistic astrophysical compact objects like SQS.

2. The field equations

The gravitational field should be static and spherically symmetric for describing the interior of a relativistic dense star. Consequently, we can find coordinates $(x^a) = (t, r, \theta, \phi)$ such that the line element is of the form

$$\mathrm{d}s^2 = -\mathrm{e}^{2\nu(r)}\mathrm{d}t^2 + \mathrm{e}^{2\lambda(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{1}$$

We assume the energy–momentum tensor for an anisotropic imperfect fluid sphere to be of the form

$$T_{ij} = \mathrm{diag}(-\rho, p_\mathrm{r}, p_\mathrm{t}, p_\mathrm{t}). \tag{2}$$

The energy density ρ, the radial pressure p_{r} and the tangential pressure p_{t} are measured relative to the comoving fluid velocity $u^i = \mathrm{e}^{-\nu}\delta^i_0$. For the line element (1) and matter distribution (2) the Einstein's field equations can be expressed as

$$\rho = \frac{1}{r^2}[r(1-\mathrm{e}^{-2\lambda})]', \tag{3}$$

$$p_{\mathrm{r}} = -\frac{1}{r^2}(1-\mathrm{e}^{-2\lambda}) + \frac{2\nu'}{r}\mathrm{e}^{-2\lambda}, \tag{4}$$

$$p_{\mathrm{t}} = \mathrm{e}^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right), \tag{5}$$

where primes denote differentiation with respect to r. In the field equations (3)–(5), we are using units where the coupling constant $(8\pi G/c^4) = 1$ and the speed of light $c = 1$. This system of equations determines the behaviour of the gravitational field for an anisotropic imperfect fluid sphere.

The mass contained within a radius r of the sphere is defined as

$$m(r) = \frac{1}{2}\int_0^r \omega^2 \rho(\omega)\mathrm{d}\omega. \tag{6}$$

With the use of the transformation

$$x = Cr^2, \quad Z(x) = \mathrm{e}^{-2\lambda(r)} \quad \text{and} \quad A^2 y^2(x) = \mathrm{e}^{2\nu(r)}, \tag{7}$$

where A and C are arbitrary constants and we introduce the linear EOS for SQM as motivated in the introduction:

$$p_{\mathrm{r}} = \alpha\rho - \beta, \tag{8}$$

where α and β are constants; the systems (3)–(5) have the simple form

$$\frac{\rho}{C} = \frac{1-Z}{x} - 2\dot{Z}, \tag{9}$$

$$p_{\mathrm{r}} = \alpha\rho - \beta, \tag{10}$$

$$p_{\mathrm{t}} = p_{\mathrm{r}} + \Delta, \tag{11}$$

$$\frac{\Delta}{C} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x}, \tag{12}$$

$$\frac{\dot{y}}{y} = (1+\alpha)\frac{(1-Z)}{4xZ} - \frac{\alpha}{2}\frac{\dot{Z}}{Z} - \frac{\beta}{4CZ}, \tag{13}$$

where the quantity $\Delta = p_{\mathrm{t}} - p_{\mathrm{r}}$ is the measure of anisotropy in this model and dots denote the differentiation with respect to the variable x. The mass function (6) becomes

$$m(x) = \frac{1}{4C^{3/2}}\int_0^x \sqrt{w}\rho(w)\mathrm{d}w, \tag{14}$$

in terms of the new variables in (7).

3. Generating exact solutions

To solve the systems (9)–(13), we use a similar approach that we used in [42]. However, in this treatment we have chosen a form for the gravitational potential Z as

$$Z = (1 - ax)^n, \tag{15}$$

where a is a real constant and n is any natural number, which generalizes our previous choice in [42]. It is noted that the case $n = 2$ has been studied previously [42,47], and shown to be physically reasonable with polytropic EOS [42]. On substituting (15) in (13) we obtain

$$\frac{\dot{y}}{y} = \frac{(1+\alpha)}{4x(1-ax)^n} - \frac{(1+\alpha)}{4x} + \frac{\alpha}{2}\frac{an}{(1-ax)} - \frac{\beta}{4C(1-ax)^n} \tag{16}$$

which is a linear equation in the gravitational potential y. Note that the integration of (16) depends on the expansion of the first term of the right-hand side. Using the knowledge of partial fraction we can establish a general expansion for this term as

$$\frac{1}{x(1-ax)^n} = \frac{1}{x} + \sum_{i=1}^{n} \frac{a}{(1-ax)^i}. \tag{17}$$

It is possible to establish that the result (17) holds for all positive integers using the principle of mathematical induction. With the help of (17), eq. (16) can be written as

$$\frac{\dot{y}}{y} = \frac{(1+\alpha)}{4}\frac{a}{(1-ax)} + \frac{(1+\alpha)}{4}\sum_{i=2}^{n}\frac{a}{(1-ax)^i} + \frac{\alpha}{2}\frac{an}{(1-ax)} - \frac{\beta}{4C(1-ax)^n}. \tag{18}$$

For the integration of (18) we consider the following two cases.

3.1 *The case* $n = 1$

In this case (18) becomes

$$\frac{\dot{y}}{y} = \frac{[(1+3\alpha)aC - \beta]}{4aC}\frac{a}{(1-ax)}. \tag{19}$$

On integrating (19) we get

$$y = D(1-ax)^{-[(1+3\alpha)aC-\beta]/4aC}, \tag{20}$$

where D is the constant of integration. In this case the energy density becomes constant and is useful to study a compact anisotropic sphere with negligible energy density variation along the radius r. It can be shown [39] that for suitable values of the parameters α and β, the measure of anisotropy Δ vanishes and hence the corresponding line element reduces to the familiar isotropic de Sitter metric and Einstein metric.

3.2 *The case* $n > 1$

In this case, integrating (18) we get

$$y = d(1-ax)^{-[1+(2n+1)\alpha]/4} \times \exp\left[\frac{1+\alpha}{4}\sum_{i=2}^{n}\frac{1}{(i-1)(1-ax)^{i-1}} - \frac{\beta}{4aC(n-1)(1-ax)^{n-1}}\right], \tag{21}$$

where d is the constant of integration.

4. Example

In this section we discuss a particular choice $n = 2$ in detail and show that it is physically viable to model an anisotropic dense star in the context of general relativity theory. This choice enables us to compare the model with polytropic EOS [42]. For $n = 2$, eq. (21) becomes

$$y = d(1-ax)^{-(1+5\alpha)/4}\exp\left[\frac{aC(1+\alpha)-\beta}{4aC(1-ax)}\right]. \tag{22}$$

4.1 *Physical quantities*

For $n = 2$ we can generate the physical quantities from systems (9)–(13) as follows:

$$\mathrm{e}^{2\lambda} = \frac{1}{(1-ax)^2}, \tag{23}$$

$$\mathrm{e}^{2\nu} = A^2 d^2 (1-ax)^{-(1+5\alpha)/2}\exp\left[\frac{aC(1+\alpha)-\beta}{2aC(1-ax)}\right], \tag{24}$$

$$\rho = aC(6-5ax), \tag{25}$$

$$p_{\mathrm{r}} = \alpha\rho - \beta, \tag{26}$$

$$p_{\mathrm{t}} = p_{\mathrm{r}} + \Delta, \tag{27}$$

$$\Delta = \frac{x}{4C(1-ax)^2}\left\{a^2C^2\left[5a^2x^2(1+\alpha(2+5\alpha)) - 4ax(4+3\alpha(3+5\alpha)) + 4(3+\alpha(7+9\alpha))\right] - 2aC\beta(4+6\alpha-ax(3+5\alpha)) + \beta^2\right\}. \tag{28}$$

The solutions (23)–(28) is given in simple elementary function so that it is more convenient to study the physical behaviour of SQS.

The mass function (14) takes the form

$$m(x) = \frac{ax^{3/2}(2 - ax)}{2\sqrt{C}}. \tag{29}$$

4.2 *Physical analysis*

In this section we briefly discuss the physical properties that have to be satisfied by the realistic star [46] for the model generated in §4.1. The gravitational potentials are regular at the origin since $\mathrm{e}^{2\nu(0)} = A^2 d^2 \exp[(aC(1+\alpha) - \beta)/2aC]$ and $\mathrm{e}^{2\lambda(0)} = 1$ are constants and also $(\mathrm{e}^{2\nu(r)})' = (\mathrm{e}^{2\lambda(r)})' = 0$ at $r = 0$. Moreover, the energy density and radial pressure at the origin are positive for $a > 0$ and $\beta < 6\alpha aC$. It is noted that the radial pressure vanishes at the finite boundary

$$r = R = \frac{1}{aC}\sqrt{\frac{6\alpha aC - \beta}{5\alpha}}$$

for the above parameter restrictions. Since $\mathrm{d}\rho/\mathrm{d}r = -10a^2C^2r < 0$ and $\mathrm{d}p_\mathrm{r}/\mathrm{d}r = -10\alpha a^2C^2r < 0$ for all $0 < r < R$, the energy density ρ and the radial pressure p_r decrease monotonically from the centre to the boundary of the star $r = R$. To maintain the casuality, the speed of sound is less than the speed of light throughout the interior of star, and we must choose α such that $0 < \alpha < 1$.

We shall now demonstrate graphically that the matter variables are well behaved throughout the interior of the star. Figures 1–4 represent the energy density, the radial pressure, the tangential pressure and the measure of anisotropy, respectively. To plot the graphs we choose parameters $a = 0.152, \alpha = \frac{1}{3}$ and $\beta = 0.149$ with stellar boundary set at $r = 2$. From figures 1 and 2 we see that both the energy density ρ and the radial pressure p_r are continuous throughout the interior and decreasing monotonically from the centre to the boundary, a feature characteristic to linear equation of state. The degree of anisotropy of the model is demonstrated in figures 2–4. Figure 3 illustrates that the tangential pressure p_t is continuous and well behaved in the interior region and show a complete contrast over its behaviour to that of the radial pressure. Although the radial and tangential pressures are equal at the centre ($p_\mathrm{r}(0) = p_\mathrm{t}(0)$), but in contrast to the monotonic decrease of p_r, p_t increases up to a maximum of about $1.7p_\mathrm{t}(0)$ at around $r = 1.65$ and then shows a rapid decrease and vanishes at the stellar boundary of the star for our choice of parameter values. The measure of anisotropy $\Delta = (p_\mathrm{t} - p_\mathrm{r})$ for the model is illustrated in figure 4 and is maximum at about $r = 1.7$ of the radial distance from the stellar boundary and vanishes at the centre and boundary. Although p_r and p_t vanish at the surface of the star in this model, a non-vanishing surface tangential pressure is also acceptable from a physical point of view [48]. As seen in figure 4, the measure of anisotropy remains finite and continuous in the interior and shows a similar profile to the relativistic dense stars modelled for quark matter with linear equation of state by Sharma and Maharaj [38] and Mak and Harko [43]. In fact, Mak and Harko [43] show that the anisotropic pressure distribution leads to an increase in the maximum radius and mass of the quark star, which in the particular model is shown to be around three solar masses.

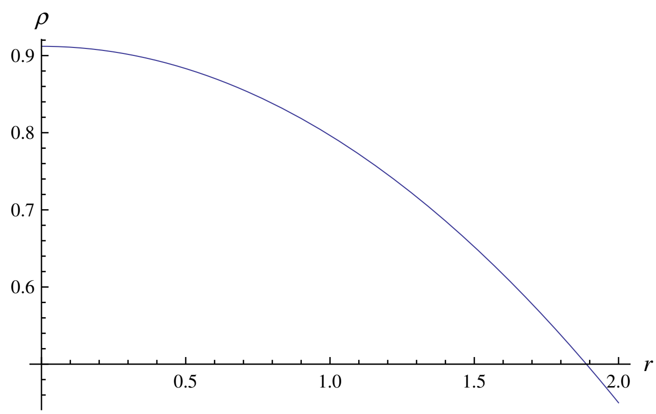

Figure 1. Energy density.

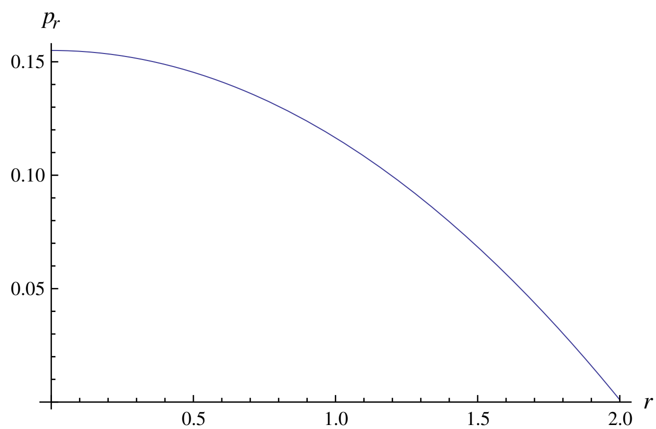

Figure 2. Radial pressure.

4.3 *Junction condition*

Interior metric matches smoothly with the Schwarzschild exterior metric:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

across the boundary $r = R$, where M is the total mass of the sphere.

From eq. (29), the total mass of the star (M) becomes

$$M = m(R) = \frac{(4\alpha aC + \beta)(6\alpha aC - \beta)^{3/2}}{50\sqrt{5}\alpha^{5/2}a^3C^3}.$$

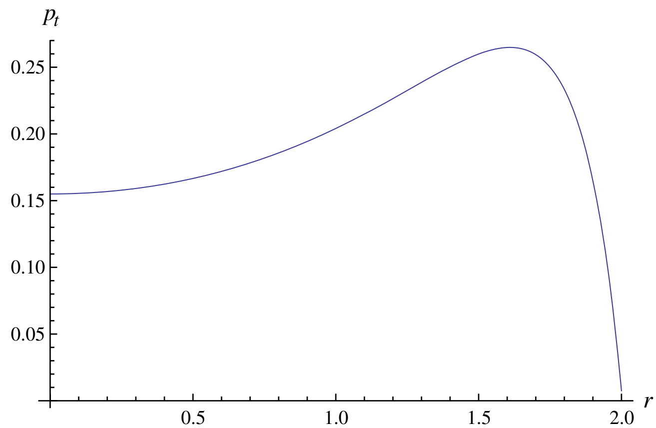

Figure 3. Tangential pressure.

The junction (matching) condition for the model generated in §4.1 becomes

$$\left(1-\frac{2M}{R}\right) = (1-aCR^2)^2, \tag{30}$$

$$\left(1-\frac{2M}{R}\right) = A^2y^2(CR^2). \tag{31}$$

The condition (30) does not impose any restrictions on the parameters. However, the condition (31) imposes the restriction on the parameters β:

$$\beta = \alpha a C.$$

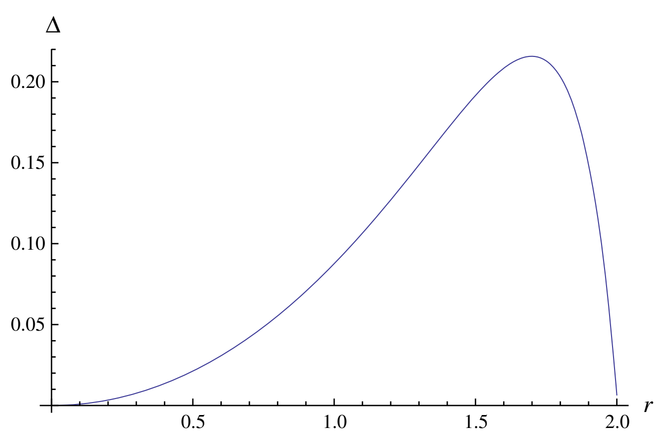

Figure 4. Measure of anisotropy.

4.4 *Stellar structure*

Now we shall show that the solution obtained in §4.1 can be used to describe realistic compact objects. In this model, the parameter a has the dimension of length^{-2}. For simplicity, we introduce the transformation

$$\tilde{a} = aS^2,$$

where S is a parameter which has the dimension of length. Under this transformation the energy density becomes

$$\rho = \frac{\tilde{a}}{S^2}(6 - 5\tilde{a}\tilde{x}), \tag{32}$$

where we have set $C = 1$ and $\tilde{x} = r^2/S^2$. When $\beta = \alpha a C$ the mass contained within a radius R has the form

$$M = \frac{1}{2}\frac{S}{\sqrt{\tilde{a}}}. \tag{33}$$

It is now possible to calculate the central density and mass for particular parameter values from (32) and (33). For example, if we set $\tilde{a} = 0.02$, $S = 1$ km, for $R = 7.07$ km, we obtain the central density $\rho_0 = 6.43497 \times 10^{15}$ g cm^{-3} and the mass $M = 2.39245 M_\odot$. If we set $\tilde{a} = 0.00592$, $S = 1$ km, as another example, we obtain the central density $\rho_0 = 1.903 \times 10^{15}$ g cm^{-3} and the mass $M = 4.3991 M_\odot$ for $R = 13$ km.

5. Discussion

We have generated solutions for anisotropic SQM with the linear EOS in spherically symmetric static space-time. We have expressed the system of Einstein's field equations as a new system of differential equations using transformations and solved the system for gravitational potential of the form $Z = (1 - ax)^n$. The solution is possible by expanding the general term involved in the integration of eq. (16) that can be proven by mathematical induction. As an example, for a particular parameter $n = 2$, we show that the generated anisotropic model satisfies all the physical features of a realistic star: regularity at the origin of gravitational potential, positive definiteness of energy density and the radial pressure at the origin, vanishing of radial pressure at some finite radius and monotonic decrease of the energy density and the radial pressure with increasing radius. Considering a linear equation of state for radial pressure inside the star, the tangential pressure of the model for particular parameter values is shown to increase up to a maximum and then decrease rapidly and vanish at the surface, thus illustrating a significant pressure anisotropy that is quite similar to some reported pressure anisotropy of dense quark star models with linear equation of state [38,43]. We have also shown that the models match smoothly with the Schwarzschild exterior line element at the boundary which restricts the bag constant in the model.

The values for mass and central densities for $n = 2$ were generated in previous section for compact spheres of 7.07 km and 13.0 km radii in order to compare with some experimental observations and theoretical studies. The estimated mass-to-radius ratio for both of these radii are greater than 0.3 suggesting that the model is best to describe a SQS [49].

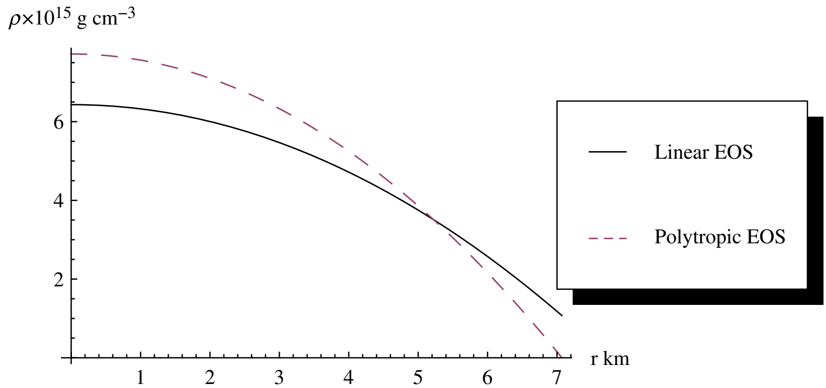

Figure 5. Comparison of energy density.

A comparative study of the mass–radius relationship of the compact star SAX J1808.4-3658 with theoretical EOS models of both neutron and strange stars gives mass-to-radius ratio 0.3 and radii 7.07 and 6.53 km respectively [4]. Our model with $n = 2$ for 7.07 km radius gives a mass about 1.7 times that of SAX J1808.4-3658 and an average density $12\rho_{\rm nu}$ which monotonically decreases from the highest value of $24\rho_{\rm nu}$ at the centre to $4\rho_{\rm nu}$ at the surface, where $\rho_{\rm nu} = 2.7 \times 10^{14}$ g cm^{-3} is the nuclear saturation density.

Now we shall compare the density profile of this model with that of the reported polytropic model [42] for the gravitational potential $Z = (1 - ax)^2$. At the centre in both models, the density $\rho_0 = 6(\tilde{a}/S^2)$. However, the numerical values for the two models vary because the parameter $\tilde{a}$ would be different when we impose the junction condition and the condition for vanishing radial pressure at the boundary of the star. In polytropic case the surface density vanishes but non-zero ($\rho_{\rm s} = \beta/\alpha$) with linear EOS. To illustrate this feature geometrically, we plot the behaviour of density profiles in figure 5 for 7.07 km radius.

Therefore, solutions generated in this linear model satisfy all major properties of a realistic star and gives mass and densities that are physically reasonable compared to experimental observations and be useful to model realistic SQS that are about an order of magnitude denser than nuclear matter. Moreover, the model generated in this paper with linear EOS that describes a quark phase gives a good comparison with the model having the same gravitational potential but with polytropic EOS that might represent a hybrid star which have cores composed of unconfined quarks [44,45].

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