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New perspective in the use of soft rotor formula for K = 2 γ-band

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Abstract. The use of soft rotor formula (SRF) for the level energies of $K = 2$ γ-band for the shape transitional even Z even N nuclei in the medium mass region is illustrated. With proper treatment, we obtained positive values of the moment of inertia and softness parameter, as opposed to negative values reported in literature. The moments of inertia of the γ-band are almost equal to the ground state band values. The systematic dependence of the softness parameter on energy ratio $R_{4/2}$ is studied. The effect of the odd–even spin staggering on these parameters is studied in detail. In deformed nuclei, the same parameters for odd and even spin members yield fair energy values.

Keywords. Nuclear structure; γ-band; even Z even N nuclei; soft rotor formula; softness parameter; moment of inertia.

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1. Introduction

In shape transitional nuclei of the medium mass region, the level energies in the ground state collective band of even Z even N nuclei, with spin $I = 0, 2, 4, 6, \ldots$ deviate from the ideal rotor formula [1]. In the last few decades, several empirical expressions were used to empirically fit the level energies with varying degrees of successes in different regions of the nuclide chart. For example, the Bohr–Mottelson expansion in powers of $X = I(I + 1)$ including two or more terms is adequate for good rotors. The extended form of the two-term Ejiri formula [2]

$$E(I) = aI + bI(I + 1) + cI^2(I + 1) \tag{1}$$

was used to study medium mass nuclei in ref. [3]. The *ab* formula [4]

$$E(I) = a\left[(1 + bI(I + 1))^{1/2} - 1\right] \tag{2}$$

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was also used for transitional nuclei. Recently, the soft rotor formula (SRF) [5]

$$E(I) = \frac{\hbar^2 I(I+1)}{2\theta_0(1+\sigma I)} \tag{3}$$

based on the concept of increase of moment of inertia (MoI) θ_I with increasing spin I, due to centrifugal stretching and coriolis antipairing effects, was found equally successful [5,6]. For high spins, the variable moment of inertia (VMI) model expression [7]

$$E(I) = \frac{\hbar^2 I(I+1)}{2\theta_I} + \frac{C(\theta_I - \theta_0)^2}{2} \tag{4}$$

is often used, where θ_I and θ_0 are the moment of inertia at spin I and ground state ($I = 0$) and C is the stiffness parameter. A combination of eqs (3) and (4), called the VMINS3 model, provided a simpler procedure [8] to solve the VMI equation for the ground state band. It was found useful for ground band of all nuclei [8], especially if the softness parameter σ is small (<1).

However, studies are very limited in the empirical evaluation of the level energies of $K = 2$ γ-band of the shape transitional nuclei. In these bands, one often notices the odd–even spin staggering [9,10]. The band mixing interaction pushes the even spin members in γ-band relative to the odd spin members, due to the interaction with even spin members of the ground band [11]. Hence, the even spin members have to be treated separately, or an additional term, to take this effect into account, is employed [1,12].

As in most cases only the level energies of up to $I = 8$ or 10 are available and the energy of the band head has to be accounted for, a two-parameter formula is better suited for the study of γ-band. The SRF eq. (3) involves the softness parameter σ of the nuclear core and the moment of inertia θ_0, which are useful attributes of a collective band.

The SRF formula was used to evaluate the MoI and the softness parameter of the γ-band in ref. [13]. However, both positive and negative values of σ and θ_0 were obtained in the isotopes of Ru, Pd, Xe, Ba, Gd and Dy. It is difficult to justify the negative values of the MoI. Also, large values of the softness parameter ($|\sigma| \gg 1$) were obtained. The softness parameter in SRF is only a perturbation correction for the spin dependence of MoI [5]. So $\sigma < 1$ is expected and should be positive.

In the present work, we resolve the anomaly of negative θ_0 and the negative softness parameter σ. We show that with proper application of SRF expression (eq. (3)), one can obtain reasonable positive values of σ and θ_0. Also, SRF MoI for γ-band in our treatment is close to the approximate value of MoI given by $3/[E(3_\gamma) - E(2_\gamma)]$ and ground band MoI $3/E(2_g)$. Our procedure would be useful for all excited bands. In §2, we describe the details of our method and give the results for a large number of deformed and shape transitional nuclei. In §3, we state the conclusion and provide a discussion.

2. Method and results

2.1 *Method*

For an excited rotational band, eq. (3) for the level energies is modified as

$$E(I) = EK + \frac{[I(I+1)]}{2\theta_0(1+\sigma I)}. \tag{5}$$

To eliminate the constant term *EK*, one needs to employ the subtraction method for determining the two parameters σ and θ_0. There is also the problem of odd–even spin staggering. So initially, we used only even spins to determine σ and θ_0 parameters.

$$E_4 - E_2 = \left(\frac{1}{\theta_0}\right)\left[\frac{10}{(1+4\sigma)} - \frac{3}{(1+2\sigma)}\right] \tag{6a}$$

$$E_6 - E_2 = \left(\frac{1}{\theta_0}\right)\left[\frac{21}{(1+6\sigma)} - \frac{3}{(1+2\sigma)}\right]. \tag{6b}$$

By dividing eq. (6b) by eq. (6a), θ_0 is eliminated. The resulting expression yields a quadratic equation in σ. Then the positive root σ is used to determine θ_0 from eq. (6a). Using these parameters, one can calculate the level energies in eq. (6) and similar expressions for higher spin. To minimize the effect of odd–even spin staggering on the calculation, to start with, we applied our method to the well-deformed nuclei, such as ^{166,168}Er, ^{156}Gd and ^{152}Sm, which have little or no staggering. This enabled us to use the same θ_0 and σ for all even and odd spin I.

2.2 *Results*

The calculated level energies are compared with experimental values [14] (upper row) in table 1. For ^{166,168}Er, ^{156}Dy ($N = 90$), ^{154,156}Gd ($N = 92$, 90) and ^{152}Sm ($N = 90$), little or no effect of odd–even spin staggering is seen. All energy values are reproduced equally well. The deviations for odd and even spins lie within the same range. In our method, the energy of $I = 2, 4, 6$ are reproduced exactly as indicated by open spaces in the middle and third rows. The calculated energies for $I = 8, 10$ and $I = 3, 5, 7$ are generally close to experiment (few % deviation), (except for $N = 88$ isotones), indicating the validity of our assumptions. As an alternative, we determined σ and θ_0 from $I = 3, 5, 7$ and then evaluated all level energies as above. Similar or better agreement with experiment is obtained.

2.3 *Comparison with ground band and rotor model values*

In a first-order estimate, the rotor model value of MoI for any K-band serves as a reference value. For ground band, it is given by $\theta = 3/E(2_g)$ and for γ-band by $\theta = 3/[E(3_\gamma) - E(2_\gamma)]$. In a global study of the character of γ-bands [12], it was shown that generally the MoI of the γ-band is almost equal to the ground band MoI of the same nucleus. However, it would be useful to take into account the shape transition of the core and to see the spin dependence of MoI in the γ-band explicitly.

In table 2, besides θ and σ for the γ-band (columns 4, 5) from SRF, we also list the MoI of ground band $\theta_{\text{SRF-g}}$ (column 6) using eq. (3). Based on ideal axial symmetric rotor, MoI of the ground band $\theta = 3/E(2_g)$ and the γ-band $\theta = 3/[E(3_\gamma) - E(2_\gamma)]$ are also listed (columns 7 and 8) for comparison. The other relevant parameters $R_{4/2}(= E_4/E_2)$ and $R_\gamma = E(2_\gamma)/E(2_g)$, determining the character of the nucleus are also listed in table 2.

For $N > 90$ nuclei, the SRF values of θ_0 for the γ-band (column 4) lie within 3–10% of the ground band MoI $\theta_{\text{SRF-g}}$. At $N = 90$, in Dy, Gd and Sm, θ_0^{SRF} for the γ-band differ

Table 1. Level energies (keV) in $K^\pi = 2^+\gamma$-band. Upper row denotes experimental values [14]. Middle row is for calculated values in keV from SRF. Lower row is the difference between experimental and SRF values. The blank spaces for $I = 2, 4, 6$ correspond to input values reproduced exactly.

Z N	$E2_2$	$E3_1$	$E4_2$	$E5_1$	$E6_2$	$E7_1$	$E8_2$	$E9_1$	$E10_2$
68 100	821	896	995	1118	1264	1433	1625		2070
SRF		896		1118		1434	1626	1841	2079
Diff.		0		0		+1	+1		+9
68 98	786	859	956	1075	1216	1376	1556	1751	1964
SRF		859		1075		1378	1560	1762	1983
Diff.		0		0		+2	+4	+11	+19
66 90	891	1022	1169	1335	1525	1729	1959	2192	2448
SRF		1017		1339		1724	1934	2153	2380
Diff.		−5		+4		−5	−25	−39	−68
64 92	1154	1248	1355	1506	1644	1850	2011	2250	2442
SRF		1243		1490		1816	2004	2208	2425
Diff.		−5		−16		−34	−7	−42	−17
64 90	996	1128	1264	1433	1607				
SRF		1118		1428					
Diff.		−10		−5					
62 90	1086	1234	1372	1560	1728	1946	2139		
SRF		1218		1543		1924	2129		
Diff.		−16		−17		−22	−10		
66 88	1027	1334	1442	1740	1885	2183			
SRF		1229		1662					
Diff.		−105		−78					
64 88	1109	1434	1550	1862	1998				
SRF		1328		1774					
Diff.		−106		−88					

from $\theta_{\text{SRF-g}}$ (by larger amount), which itself differ from $\theta_{\text{gsb}}^{\text{RM}}$. The $N = 90$ isotones are examples of $X(5)$ symmetry [15] and lie at the border of spherical to deformed region. For $N = 88$ isotones ^{154}Dy, ^{152}Gd and for lighter nuclei of Ba, Xe, Pd and Ru, the MoI are small. In a plot of MoI of γ-band vs. ground band (figure 1), the SRF data (from table 2) lie on or near the diagonal. The data points near the middle correspond to $N = 90$ nuclei. The datum of ^{152}Sm has the maximum deviation. In figure 2, we exhibit the MoI of γ-band (column 8 of table 2) against the ground band MoI $3/E(2_g)$. Again the data points lie on or near the diagonal. The approximate equality of the MoI of $K = 2$ γ-band and the ground band is exhibited in both figures. This confirms the validity of our method.

In figure 3, we have illustrated the correlation of the softness parameter σ for the $K = 2$ γ-band (column 5, table 2) vs. σ for ground bands of the nuclei studied here. In spite of some scatter, there is a proportionality relation between the y and x values. The maximum deviation from the diagonal curve is for $N = 88$ in ^{152}Gd.

Table 2. The softness parameter σ (even), MoI $\theta_{\gamma\text{-band}}$ and $\theta_{\text{SRF-g}}$ (MeV^{-1}) are from SRF. The rotor model MoI $\theta_{\text{gsb}} = 3/E(2_g)$ for ground band and $3/[E(3_\gamma) - E(2_\gamma)]$ for γ-band are listed for comparison. Rigid rotor energy ratio $R_\gamma = E(2_\gamma)/E(2_g)$ and staggering index $S(4)$ are also listed.

Z	N	$R_{4/2}$	R_γ	$\theta_{\gamma\text{-band}}$	σ (even) $\times$ 10	$\theta_{\text{SRF-g}}$	θ_{gsb}	$3/[E(3_\gamma) - E(2_\gamma)]$	$S(4)$
68	100	3.309	10.29	39.4	0.046	37.3	37.6	38.1	0.30
68	98	3.289	9.75	39.1	0.106	36.8	37.2	40.8	0.30
66	90	2.933	6.46	17.3	0.928	18.8	21.8	22.8	0.11
64	92	3.239	12.97	29.6	0.368	32.7	33.7	32.0	0.15
[13]				−4.4	−1.09				
64	90	3.015	8.10	17.1	1.05	21.8	24.4	22.8	0.05
62	90	3.009	8.917	14.8	1.27	22.0	24.6	20.3	−0.08
66	88	2.234	3.09	5.2	4.10	4.5	9.0	9.76	−0.60
64	88	2.194	3.22	3.0	7.40	4.2	8.7	9.23	−0.61
44	58	2.329	2.32	4.3	2.54	3.6	6.3	7.2	−0.30
44	60	2.482	2.495	5.0	2.41	5.5	8.4	8.5	−0.25
44	62	2.647	2.933	6.6	2.01	8.0	11.1	10.0	−0.31
44	64	2.746	2.922	8.3	1.50	10.0	12.4	11.2	−0.24
44	66	2.757	2.546	9.5	1.10	10.0	12.5	12.1	−0.09
44	68	2.727	2.213	10.2	1.00	10.0	12.7	13.4	+0.04
46	64	2.465	2.178	5.1	2.49	5.2	8.0	7.5	−0.57
46	66	2.536	2.111	2.4	6.50	5.9	8.6	8.3	−0.27
46	68	2.564	2.088	3.3	4.29	6.3	9.0	13.8	−0.03
46	70	2.580	2.167	5.0	2.30	6.2	8.8	9.1	−0.06
54	64	2.404	2.75	4.6	3.57	5.5	8.9	5.8	−1.08
54	66	2.472	2.717	5.2	2.87	6.1	9.3	7.6	−0.83
54	68	2.502	2.545	6.1	2.01	6.0	9.1	8.1	−0.55
54	70	2.483	2.39	6.2	1.74	5.6	8.5	7.5	−0.60
54	72	2.422	2.264	6.0	1.74	4.8	7.7	6.9	−0.69
54	74	2.332	2.19	3.4	4.03	3.9	6.8	6.3	−0.62
56	68	2.834	3.80	7.9	1.85	10.8	13.1	10.3	−0.55
56	70	2.778	3.410	7.0	2.19	9.4	11.7	8.2	−0.99
56	72	2.688	3.114	6.8	2.08	8.0	10.6	6.3	−1.38
56	74	2.524	2.540	4.4	3.25	5.7	8.4	6.62	−0.94

2.4 *Odd–even staggering index*

A commonly used index of odd–even spin staggering $S(4)$ is defined [9] as

$$S(4) = \frac{\left[E(4_\gamma) + E(2_\gamma) - 2E(3_\gamma)\right]}{E(2_g)}. \tag{7}$$

It reflects the displacement of even spin members of γ-band relative to odd spin members. The staggering index $S(4)$ is 0.33 for axially symmetric rotor and 1.67 for rigid triaxial rotor. For γ-soft rotor or $O(6)$, $S(4) = -2.0$ and for spherical vibrator it is $= -1.0$. The

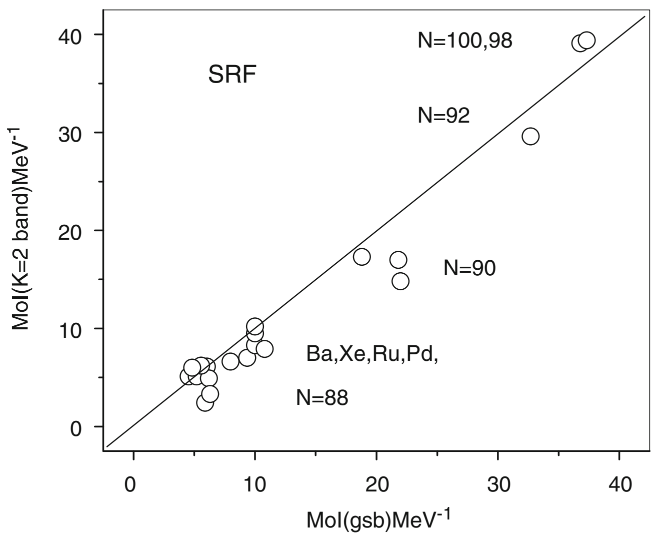

Figure 1. Moment of inertia of $K = 2$ γ-band vs. ground band using eq. (6) for the nuclei listed in table 2.

values of $S(4)$ for the nuclei studied here are listed in table 2. For ^{166,168}Er and ^{156}Gd, $S(4)$ are in the vicinity of axially symmetric deformed rotor value of 0.33. For $N = 90$ isotones ^{156}Dy, ^{154}Gd and ^{152}Sm, $S(4)$ is close to zero. Only at $N = 88$, $S(4)$ is negative and slightly away from the spherical vibrator value of -1.0. Ignoring the sign of $S(4)$, $|S(4)|$ is small for $N = 90$ or $N > 90$ Er, Dy, Gd and Sm nuclei. So one may use the same

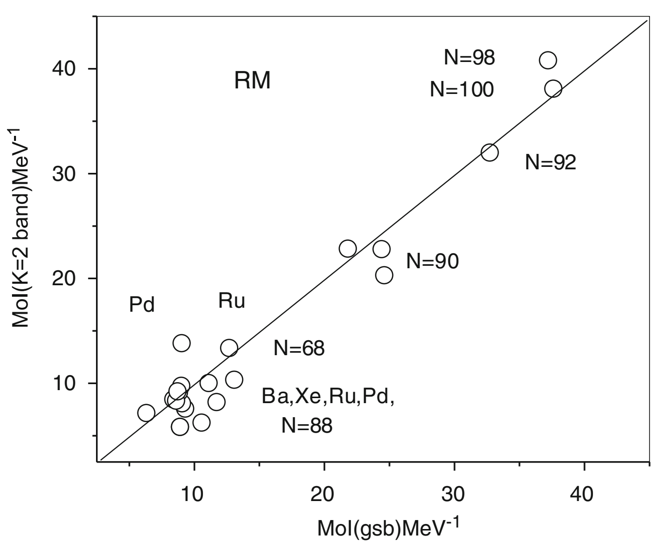

Figure 2. The plot of MoI of $K = 2$ γ-band vs. the MoI of ground band of nuclei listed in table 2 in rotor model.

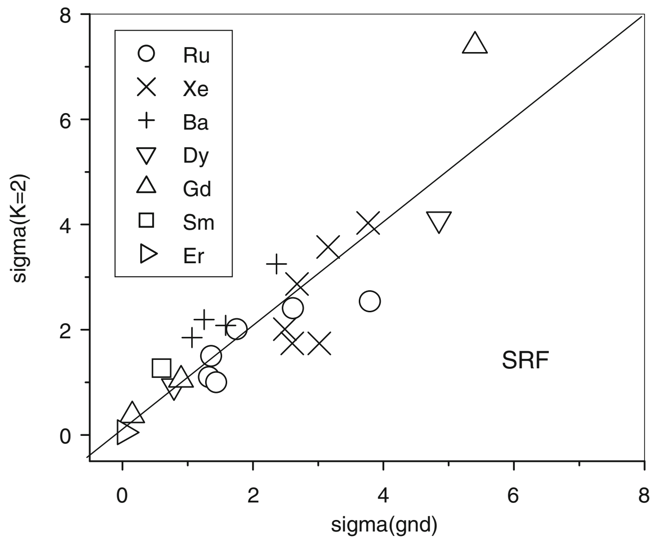

Figure 3. The plot of softness parameter σ ($K = 2$ band) (table 2) vs. σ (ground band). The values have been multiplied by a factor of 10.

SRF θ_0 and σ for both odd and even spin levels. At $N = 88$, $|S(4)| = 0.6$ is relatively larger and greater deviation of SRF value of level energies for odd spin from experiment is expected, as observed for odd spin energies of Dy and Gd with ~100 keV deviation.

2.5 *Light* ($N < 82$) *nuclei with large* γ *and* $S(4)$

When $|S(4)|$ is large, the calculation of the energy of odd, even spin levels has to be done separately. This is a well recognized situation [12,13,16]. It arises on account of the large triaxiality effect (static or dynamic) on even spin members of the γ-band, which pushes these levels closer to the respective odd spin $(I + 1)$ levels. Toki and Faessler [16] studied this aspect of $K = 2$ bands in rigid triaxial rotor model and calculated the level energy, quadrupole moment (Q) and transition rates supporting the internal consistency of this separation. They used extended VMI for the energies in γ-band.

In the national nuclear data centre (nndc) tables [14] also, in such cases, the odd and even spin members are shown in separate bands. In fact, in some cases the higher odd spin level may even go below the lower spin (even spin) level. Obviously, in such cases, the two bands have to be treated as separate bands for determining energies.

We have extended the calculation to $N < 82$ nuclei of Xe, Ba, Ru and Pd ($\gamma > 15°$) isotopes. For Ru and Pd isotopes, $|S(4)| < 0.5$ (table 2). The same parameters for odd–even spin yield fairly good values for Ru and Pd (table 3). The alternative fit to odd spin $I = 3, 5, 7$ also yields a fair fit to both odd and even spins (see third row in each case).

In Xe and Ba isotopes, $|S(4)| > 0.5$ (table 2). Here, initially, we discuss the results of SRF for even spin. With $E(I)$ for $I = 2, 4, 6$ as input, at $I = 8$ in Ba and Xe, there is deviation of up to 3%, where a separate fit is required (table 4). Casten and von Brentano [17] identified the large odd–even staggering in Xe and Ba isotopes and suggested the use of cubic interaction in the treatment of interacting boson model-1 [18].

Table 3. SRF energies (keV) in Ru and Pd. The upper row is for a fit to $I = 2, 4, 6$ and the lower row is for a fit to $I = 3, 5, 7$. The blank spaces signify input energies of $I = 2, 4, 6$ reproduced exactly, or of $I = 3, 5, 7$.

Z N	$E2_2$	$E3_1$	$E4_2$	$E5_1$	$E6_2$	$E7_1$	$E8_2$	$E9_1$	$E10_2$
44 60	893	1242	1503	1872	2197	2624	2848		
Even		1183		1843		2561	2933		
Odd	893		1538		2236		3032		
44 62	792	1092	1307	1641	1908	2284			
Even		1035		1600		2227	2555		
Odd	792		1351		1953		2629		
44 64	708	975	1183	1496	1762	2133	2420	2844	3150
Even		928		1463		2076	2402	2738	3082
Odd	708		1218		1802		2483	2850	
44 66	613	860	1084	1375	1684	2021	2397	2777	
Even		829		1372		2017	2367		
Odd	613		1099		1684		2382	2763	
44 68	524	748	981	1236	1570	1841	2263	2534	
Even		732		1262		1900	2248		
Odd	524		975		1526		2178	2533	
Pd									
46 62	931	1335	1624	2084	2259	2919	2954		
Odd	931		1695		2493		3356		
46 64	814	1212	1398	1759	1987		2651		
Even		1105		1692		2282	2577		
46 66	737	1097	1363		2004				
Even		1047		1683		2327	2650		
46 68	695	1012	1320	1631	1984	2290	2655	2906	3338
Even		1000		1649		2322	2664	3007	3352
Odd	695		1314		1957		2628	2970	
46 70	738	1066	1373	1719	2101	2492	2840	(3255)	
Even		1039		1729		2485	2877		
Odd	738		1374		2093		2910	3344	

2.6 *Problem of negative σ and θ*

To resolve the anomaly of negative softness parameter σ and θ obtained in the previous study [13], we have repeated the calculation using the method of [13]. Here, the *EK* term in eq. (5) was ignored and the value of σ was obtained after the cancellation of θ by dividing energy $E(4)$ by $E(2)$ and equating it with $(10/3)[(1+2\sigma)/(1+4\sigma)]$. This led to the negative θ and negative softness parameter σ; as in [13]. In ^{104}Ru, σ is positive (=27) and in ^{106}Ru, it is negative ($= -25$). Same change of sign is obtained for the MoI ($= +0.06$ and -0.08 MeV^{-1} respectively). For ^{108}Ru, σ increases to +88 yielding the MoI $= +0.025$ MeV^{-1}. Such large values of softness parameter σ (and small MoI) with wide variations do not correspond to any physical reality of a collective nuclear structure. In fact, σ should be positive and less than one.

Table 4. The SRF values of level energies (keV) (middle row) for light ($N < 82$) nuclei, are compared with experiment (upper row). The differences from experiment are listed in third row. The blank spaces for $I = 2, 4, 6$ correspond to input values reproduced exactly.

Z N	$E2_2$	$E3_1$	$E4_2$	$E5_1$	$E6_2$	$E7_1$	$E8_2$	$E9_1$	$E10_2$
56 68	873	1162	1325	1672	1858	2285	2479	2975	3177
SRF		1085		1584		2143	2437	2738	3044
Diff.		−77		−88		−142	−42	−237	−133
56 70	874	1236	1346	1808	1890	2485	2530	3243[a]	3262
SRF		1097		1612		2178	2473	2773	3078
Diff.		−139		−196		−307	−57	–	−184
56 72	885	1324	1372	1931	1939	2631	2600	3387	3346
SRF		1115		1649		2240	2548	2863	3183
Diff.		−209		−282		−391	−52	−524	−163
56 74	908	1361	1478	2013	2101		2800		3603
SRF		1183		1785		2423	2750	3080	3412
Diff.		−178		−228			−50		−191
54 64	928	1366	1441	1922	1997	2560	2625	3240	3255
SRF		1177		1716		2283	2572	2863	3157
Diff.		−189		−206		−277	−53	−377	−98
54 66	876	1272	1401	1817	1986	2461	2654	3174	3326
SRF		1128		1689		2289	2598	2911	3226
Diff.		−144		−128		−172	−56	−263	−100
54 68	843	1214	1403	1775	2057	2459	2795	3216	3609
SRF		1107		1721		2404	2761	3126	3497
Diff.		−107		−54		−55	−34	−90	−112
54 70	847	1248	1438	1837	2144	2575	2912	3344	3670
SRF		1123		1781		2523	2911	3315	3725
Diff.		−125		−56		−52	+1	−29	+55
54 72	880	1318	1488	1903	2214	2661	3062		
SRF		1165		1841		2604	3006		
Diff.		−153		−62		−57	−56		
54 74	970	1430	1604	1997	2281	2731			
SRF		1278		1939		2627	2977		
Diff.		−152		−58		−104			

[a]In ^{126}Ba, a 3243 keV level is shown as part of the $K = 2$ band, without a spin assignment.

Further arithmetic analysis shows that if one writes $X = [(1 + 2\sigma)/(1 + 4\sigma)] = [E(4_\gamma)/E(2_\gamma)]/(10/3)$ and solves for $2\sigma = (1 - X)/(2X - 1)$, then $X > 1/2$ yields positive value of σ and $X < 1/2$ yields negative value of σ. The numerator is always positive ($X < 1$), but the denominator is positive only for $X > 1/2$.

Thus, $X > 0.500$ yields positive σ (and MoI). If $X < 0.500$, it yields negative σ and negative MoI. Note that $X = 0.50453$, 0.4948 and 0.50141 in ^{104,106,108}Ru respectively. The $(2X - 1)$ factor in the denominator easily explains the large variation in the values cited above for Ru isotopes. (For $X = 1/2$, the denominator $(2X - 1)$ reduces to zero

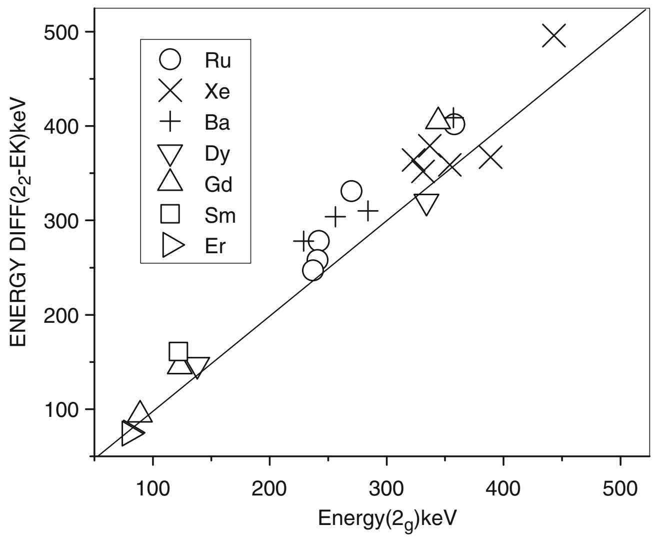

Figure 4. The plot of energy difference $[E(2_2) - EK]$ vs. $E(2_g)$.

and yields very large σ). Further, with fixed $E(4_\gamma)$ and $E(2_\gamma)$ values one should get back these values. The slightly different values of $I = 2, 4$ energies in γ-band, obtained in [13], indicate some fitting effort in [13].

Also, they have treated the even spin and odd spin separately [13]. Separate calculation is required only if there is finite odd–even staggering. For ^{156}Gd, their values of θ and σ are listed in table 2, which have no correspondence with our values for γ-band or with rotor values for ground band or γ-band. The SRF eq. (3) is valid for ground band. For a non-zero band head, one has to work with energy differences.

2.7 *Constant energy EK*

It may be interesting to estimate the constant energy *EK* in eq. (5), which is equivalent to an extrapolated energy value of the $K = 2$ band for $I = 0$. This energy difference should be related to the ground band energy $E(2_g)$. In rigid triaxial model [19], one expects $E(3_\gamma) = E(2_\gamma) + E(2_g)$ (in the absence of odd–even staggering), which is also true for axially symmetric rotor. A plot of this energy difference vs. $E(2_g)$ is given in figure 4. The data lie on or near the diagonal line. This further illustrates the validity and usefulness of the present work.

3. Conclusion and discussion

3.1 *Conclusion*

We have illustrated a proper procedure for evaluating the level energies in an excited band with band head energy greater than zero. This yields positive moment of inertia (MoI) θ and positive softness parameter ($\sigma < 1$) for all nuclei, deformed or shape transitional. In the deformed nuclei, the derived θ_0 for the γ-band is almost equal to the derived moment of inertia for the ground band (figure 1), which is also true for the corresponding rotor model values (figure 2).

In the almost spherical nuclei, e.g. $N = 88$ Dy and Gd, the θ_0 in the γ-band differs from the one in the ground band and from the approximate rotor values. In Ba and Xe, also the MoI is positive and varies smoothly with neutron number N. It is small (<10 MeV^{-1}) and is near or slightly less than the ground band MoI (figure 1). So is the case for Ru and Pd. The softness parameter σ is positive in all cases and is less than 1.0 as it should be. Also, the values of σ for the γ-band are correlated to the ground band values (figure 3). These results differ from the ones in ref. [13], wherein negative σ (and $|\sigma| \gg 1$) and negative MoI were obtained.

3.2 *Discussion*

In ref. [5], it was shown that for the ground band, σ is less than 0.5 for most nuclei with $R_{4/2} > 2.5$. For the γ-band of the nuclei dealt with here, our results lead to the same conclusion. In deformed nuclei, the $K = 2$ γ-band is basically a rotational band on the γ-vibration band head and its structure depends on the deformation of the nuclear core, which is almost the same as that of the ground band. In shape transitional nuclei and almost spherical vibrators, the MoI of the two bands can differ. The values of energy ratio $R_{4/2}$, R_γ, moment of inertia θ_0 and softness parameter σ for the two K-bands provide information on the structure of these bands. We have also illustrated the values of odd–even energy staggering index $|S(4)|$, which is small for well-deformed nuclei, so that the odd–even spin members form a single $K = 2$ band. Only in the almost spherical nuclei, the odd spin levels form almost a split separate band [14,16]. In such cases, these have to be fitted separately. Earlier, Gupta and Kavathekar in [12] provided an explanation for the odd–even spin energy staggering in terms of the split of the two-phonon and three-phonon multiplets. As one progresses from the spherical to the deformed region, degenerate states get split, but the $K = 2$, $I^\pi = 2^+$ state remains far from the ($I = 3^+, 4^+$). The partial degeneracy of the latter pair of states persists till the rotor regime is attained. Of course there are other local interactions, which give a variation with N, Z and also with spin. The relevance of the energy constant EK is further demonstrated in figure 4, which relates it to the energy $E(2_g)$.

This aspect of the $K = 2$ γ-band also differs from the treatment in [13] and leads to a proper perspective. The same procedure also applies to the $K = 0$ β-bands and other K-bands. These results for γ-bands of light Ba, Xe, Pd and Ru nuclei provide initial data for further detailed studies of the structure of these nuclei.

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