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Anisotropic cosmological models in $f(R, T)$ theory of gravitation

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Abstract. A class of non-singular bouncing cosmological models of a general class of Bianchi models filled with perfect fluid in the framework of $f(R, T)$ gravity is presented. The model initially accelerates for a certain period of time and decelerates thereafter. The physical behaviour of the model is also studied.

Keywords. Bianchi models; perfect fluid; $f(R, T)$ gravity.

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1. Introduction

The aim of modern cosmology is to determine the large-scale structure of the Universe. The astronomical observations of type-Ia supernovae experiments [1–3] suggest that the observable Universe is undergoing an accelerated expansion. Also, observations such as cosmic microwave background radiation [4] and large-scale structure [5] provide an indirect evidence for the late time accelerated expansion of the Universe. It is generally believed that some sort of 'dark energy' is pervading the whole Universe. This is a hypothetical form of energy that permeates all of the space and tends to increase the rate of expansion of the Universe [6]. The dark energy is a prime candidate for explaining the recent cosmic observations. In view of the late time acceleration of the Universe and the existence of dark energy and dark matter, several modified theories of gravity have been developed and studied. Noteworthy amongst them is the $f(R)$ gravity theory [7,8]. Bertolami *et al* [9] proposed a generalization of $f(R)$ theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Nojiri and Odintsov [10] developed a general scheme for the modified $f(R)$ gravity reconstruction from any realistic FRW cosmology. They have shown that modified $f(R)$ gravity indeed represents a realistic alternative to general relativity, being more consistent in dark epoch. Nojiri *et al* [11] developed a general programme for the unification of matter-dominated era with acceleration epoch for scalar–tensor theory or dark fluid. Shamir [12] proposed a physically viable $f(R)$ gravity model, which showed the unification of early time inflation and late time acceleration.

Harko *et al* [13] developed $f(R, T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy–momentum tensor. It is to be noted that the dependence of T may be induced by exotic imperfect fluid or quantum effects. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion of test particles, which follow from the covariant divergence of the stress–energy tensor. They have derived some particular models corresponding to specific choices of the function $f(R, T)$. They have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of the function $f(R, T)$.

The Friedmann–Robertson–Walker models are the only globally acceptable perfect fluid space-times which are spatially homogeneous and isotropic. The adequacy of isotropic cosmological models for describing the present state of the Universe is no basis for expecting that they are equally suitable for describing the early stages of the evolution of the Universe. At the early stages of the evolution of Universe, it is, in general spatially homogeneous and anisotropic. Bianchi spaces are useful tools for constructing spatially homogeneous and anisotropic cosmological models in general relativity and scalar–tensor theories of gravitation. Adhav [14] obtained exact solutions of the field equations for LRS Bianchi type-I space-time with perfect fluid in the framework of $f(R, T)$ theory of gravity by applying the laws of variation of Hubble's parameter proposed by Berman [15]. Reddy *et al* [16] presented a spatially homogeneous Bianchi type-III cosmological model in the presence of a perfect fluid source in $f(R, T)$ theory with negative constant deceleration parameter. Reddy *et al* [17] also investigated a five-dimensional Kaluza–Klein cosmological model in $f(R, T)$ gravity with a negative constant deceleration parameter with an appropriate choice of the function $f(R, T)$. Shamir *et al* [18] obtained exact solution of Bianchi type-I and type-V cosmological models in $f(R, T)$ gravity. Recently, Chaubey and Shukla [19] obtained a new class of Bianchi cosmological models in $f(R, T)$ gravity by using a special law of variation for the average scale factor when the deceleration parameter is linear with a negative slope proposed by Akarsu and Dereli [20].

In this paper, we reconsider the field equations for Bianchi type spaces in $f(R, T)$ gravity discussed by Chaubey and Shukla [19] and obtain a new class of cosmological models using the special form of the average scale factor derived by Abdussattar and Prajapati [21] by constraining the deceleration parameter. The paper is organized as follows: In §2, we present the explicit field equations in $f(R, T)$ gravity for a general class of Bianchi cosmological models in the presence of a perfect fluid for the particular choice of $f(R, T) = R + 2\lambda T$, where λ is a constant. We obtain a new class of exact solutions of the field equations in §3. In §4, we discuss some physical properties of the cosmological models. Conclusions are given in §5.

2. Metric and field equations

The diagonal form of the metric of the general class of Bianchi cosmological models is given by

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2x} dy^2 - a_3^2 e^{-2mx} dz^2, \tag{2.1}$$

where $a_1(t)$, $a_2(t)$ and $a_3(t)$ are the scale factors. The metric (2.1) corresponds to a Bianchi type-III model for $m = 0$, type-V for $m = 1$ and type-VI_0 for $m = -1$.

The field equations in $f(R, T)$ theory of gravity for the function $f(R, T) = R + f(T)$ when the matter source is a perfect fluid are given by [13]

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f' T_{ij} + [2pf'(T) + f(T)] g_{ij}, \tag{2.2}$$

where

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \tag{2.3}$$

and the prime denotes differentiation with respect to the argument.

We choose the function $f(T)$ of the trace of the energy tensor of the matter so that

$$f(T) = \lambda T, \tag{2.4}$$

where λ is a constant.

Now choosing comoving coordinates the field eq. (2.2), with the help of eqs (2.2) and (2.3) for the metric (2.1), can be written as [19]

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} - \frac{m^2 + m + 1}{a_1^2} = \lambda p - (8\pi + 3\lambda)\rho, \tag{2.5}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{m}{a_1^2} = (8\pi + 3\lambda) p - \lambda\rho, \tag{2.6}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_2 a_3} - \frac{m^2}{a_1} = (8\pi + 3\lambda) p - \lambda\rho, \tag{2.7}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{1}{a^2} = (8\pi + 3\lambda) p - \lambda\rho, \tag{2.8}$$

$$(m + 1)\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} - m\frac{\dot{a}_3}{a_3} = 0. \tag{2.9}$$

Here the overdot denotes ordinary differentiation with respect to time t.

Let us introduce as usual the dynamical shear scalar σ^2, the mean anisotropy parameter A, the spatial volume V, the average scale factor and the Hubble parameter H for the metric (2.1)

$$V = a^3 = a_1 a_2 a_3, \tag{2.10}$$

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right), \tag{2.11}$$

$$\theta = 3H, \tag{2.12}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{a}_1}{a_1}\right)^2 + \left(\frac{\dot{a}_2}{a_2}\right)^2 + \left(\frac{\dot{a}_3}{a_3}\right)^2\right] - \frac{1}{6}\theta^2, \tag{2.13}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{2.14}$$

where

$$\Delta H_i = H_i - H, \quad i = 1, 2, 3$$

and

$$H_1 = \frac{\dot{a}_1}{a_1}, \quad H_2 = \frac{\dot{a}_2}{a_2}, \quad H_3 = \frac{\dot{a}_3}{a_3}.$$

Akarsu and Dereli [20] proposed the linearly varying deceleration parameter q defined as

$$q = -\frac{a\ddot{a}}{a} = -(kt + n - 1) \tag{2.15}$$

and obtained three different forms of the average scale factor a where k and n are positive constants. Chaubey and Shukla [19] obtained exact solutions of the field eqs (2.5)–(2.9) by using different forms of the law of variation for specific values of k and n. Here we obtain exact solutions of eqs (2.5)–(2.9) by using one of the three different forms of the average scale factor derived by Abdussattar and Prajapti [21] which leads to a class of non-singular bouncing FRW models obtained by constraining the deceleration parameter in the presence of an interacting dark energy represented by a time-varying cosmological constant.

3. Cosmological solution

The two observable parameters q and H are related by the relation

$$q = -1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right). \tag{3.1}$$

This equation, on integration, gives the scale factor $a(t)$ as

$$a(t) = \mathrm{e}^{\delta} \exp \int \frac{\mathrm{d}t}{\int (1+q)\mathrm{d}t + r}, \tag{3.2}$$

where r and δ are arbitrary constants. For the complete determination of $a(t)$, Abdussattar and Prajapati [21] proposed the following choice of q as

$$q = -\frac{\alpha}{t^2} + (\beta - 1), \tag{3.3}$$

where $\alpha > 0$ is a parameter having the dimension of square of time and $\beta > 1$ is a dimensionless constant. Different values of α and β give rise to different models. With q given by eq. (3.3) and $\delta = 0$, $r = 0$, eq. (3.2) can be integrated to give the scale factor as

$$a(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{1/2\beta}. \tag{3.4}$$

Abdussattar and Prajapati [21] discussed in detail the non-singular bouncing model with $a(t)$ given by (3.4) and have shown the variation of different cosmological parameters graphically for specific values of parameters of the model.

It is difficult to solve eqs (2.5)–(2.9) for five unknowns a_1, a_2, a_3, ρ and p in the exact form. In order to solve the system completely we assume that $a_3 = V^b$, where b is any constant number. Then from eqs (2.9), (2.10) and (3.4),we obtain the exact expression for the scale factors:

$$a_1(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{(3+3mb-3b)/2\beta(m+2)}, \tag{3.5}$$

$$a_2(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{(3+3m-3b-6mb)/2\beta(m+2)}, \tag{3.6}$$

$$a_3(t) = \left(t^2 + \frac{\alpha}{\beta}\gamma\right)^{3b/2\beta}. \tag{3.7}$$

4. Physical properties

For the model presented by the scale factors (3.5)–(3.7) the dynamical parameters have values as given below:

The directional Hubble parameters and average Hubble parameter are given by

$$H_1 = \frac{3 + 3mb - 3b}{\beta(m+2)} \frac{t}{\left(t^2 + \frac{\alpha}{\beta}\right)}, \tag{4.1}$$

$$H_2 = \frac{3 + 3m - 3b - 6mb}{\beta(m+2)} \frac{t}{\left(t^2 + \frac{\alpha}{\beta}\right)}, \tag{4.2}$$

$$H_3 = \frac{3b}{\beta} \frac{t}{\left(t^2 + \frac{\alpha}{\beta}\right)}, \tag{4.3}$$

$$H = \frac{t}{\beta\left(t^2 + \frac{\alpha}{\beta}\right)}. \tag{4.4}$$

The expansion scalar, shear scalar and mean anisotropic parameter are found as

$$\theta = 3H = \frac{3t}{\beta\left(t^2 + \frac{\alpha}{\beta}\right)}, \tag{4.5}$$

$$\sigma^2 = A_1 \frac{t^2}{\left(t^2 + \frac{\alpha}{\beta}\right)^2}, \tag{4.6}$$

$$\begin{aligned} A &= \frac{2}{3(m+2)^2}[(3+3mb-3b)(6-3mb-6b+3m) \\ &\quad + (3+3m-3b-6mb)(3+3m+3m-3mb)] \\ &\quad + \frac{2}{3(m+2)^2}\left[(3b(m+2))(6mb+3b+3)\right], \end{aligned} \tag{4.7}$$

where

$$A_1 = \frac{(3+3mb-3b)^2 + (3+3m-3b-6mb)^2 + 9(m+2)^2(b^2-2)}{2\beta^2(m+2)^2}.$$

Using eqs (3.5)–(3.7), we obtain the values of pressure and energy density as

$$\begin{aligned} p &= \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{t^2}{(t^2\beta+\alpha)^2(m+2)^2}[3(8\pi+3\lambda)A_3 - 3\lambda A_2]\right] \\ &\quad - \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{3(8\pi+3\lambda)E_1}{(t^2+\frac{\alpha}{\beta})^{(3+3mb-3b)/\beta(m+2)}}\right] \\ &\quad - \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{E_2}{(t^2+\frac{\alpha}{\beta})^{(3+3mb-3b)/\beta(m+2)}}\right], \end{aligned} \tag{4.8}$$

$$\begin{aligned} \rho &= \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{t^2}{[t^2\beta+\alpha]^2(m+2)^2}[3\lambda A_2 - 3(8\pi+3\lambda)A_3]\right] \\ &\quad - \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{3E_1\lambda}{[t^2+\frac{\alpha}{\beta}]^{(3+3mb-2b)/\beta(m+2)}}\right] \\ &\quad - \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{E_3}{(t^2+\frac{\alpha}{\beta})^{(3+3mb-3b)/\beta(m+2)}}\right], \end{aligned} \tag{4.9}$$

where

$$\begin{aligned} A_2 &= (1+mb-b-2mb)(1+mb-b) + b(m+2)(1+m-b-2mb) \\ &\quad + b(1+mb-b)(m+2), \end{aligned}$$

$$\begin{aligned} A_3 &= (3b^2 - b\beta)(m+2) + 3(1+m-b-2mb)(1+m-b-2mb) \\ &\quad - (3(1+m-b-2mb)(m+2) - \beta(m+2))(3+3m-3b-6mb), \end{aligned}$$

$$E_1 = b\alpha(m+2) + \alpha(1+m-b-2mb),$$

$$E_2 = m\lambda - (8\pi + 3\lambda)(m^2+m+1),$$

$$E_3 = m(8\pi + 2\lambda) - (m^2+1)\lambda.$$

For the purpose of reference, we set the origin of the time coordinate at the bounce of this bouncing model and concentrate only on the expanding part of the solutions which represents the observable Universe as in [21].

At the origin of the time coordinate $t = 0$, we find that $a(0) \neq 0$, $\dot{a}(0) = 0$ but $\ddot{a}(0) =$ constant, which indicate that the model is free from initial singularity and start expanding with finite acceleration. From (3.3) we observe that the deceleration parameter $q \to -\infty$ at $t = 0$ and reduces to zero at $t = \sqrt{\alpha/(\beta-1)}$. The period of accelerated expansion also depends on the values of α and β. Afterwards, the model decelerates with deceleration parameter q approaching $\beta - 1$ for sufficiently large values of t. Obviously, we have a constraint on β as $1 \leq \beta \leq 2$. For $\beta = 1$ the Universe has an accelerated expansion throughout the evolution. We see that θ, σ^2, H ultimately tend to zero as $t \to \infty$. The energy density and pressure tend to zero for large values of time provided $b < 1/(1-m)$.

5. Conclusion

In this paper we have obtained a general class of non-singular cosmological model of the early Universe with a perfect fluid as the source of matter in $f(R, T)$ theory of gravity. We have additional classes of Bianchi models from the general class of Bianchi model for different values of m as follows: Bianchi type-III corresponds to $m = 0$, Bianchi type-V corresponds to $m = 1$, Bianchi type-VI_0 corresponds to $m = -1$ and all other values of m give Bianchi type-VI. An important aspect of the presented model is that it starts from rest with a finite volume and finite acceleration which gradually decrease and reduce to zero after some time, thereafter the model decelerates with gradually increasing deceleration approaching a constant value for large values of t. It is also observed that all the physical parameters are decreasing functions of time and they approach zero for large t. The model obtained may throw some light on our understanding of $f(R, T)$ cosmology.

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