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Nonlinear propagation of dust-acoustic solitary waves in a dusty plasma with arbitrarily charged dust and trapped electrons

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Abstract. A theoretical investigation of dust-acoustic solitary waves in three-component unmagnetized dusty plasma consisting of trapped electrons, Maxwellian ions, and arbitrarily charged cold mobile dust was done. It has been found that, owing to the departure from the Maxwellian electron distribution to a vortex-like one, the dynamics of small but finite amplitude dust-acoustic (DA) waves is governed by a nonlinear equation of modified Korteweg–de Vries (mKdV) type (instead of KdV). The reductive perturbation method was employed to study the basic features (amplitude, width, speed, etc.) of DA solitary waves which are significantly modified by the presence of trapped electrons. The implications of our results in space and laboratory plasmas are briefly discussed.

Keywords. Vortex-like distribution; solitary waves; Maxwellian distribution; modified Korteweg–de Vries (mKdV) equation.

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1. Introduction

There has been a great deal of interest in understanding different types of collective processes in dusty plasmas (plasmas with extremely massive and negatively charged dust grains), because of its vital role in the study of astrophysical and space environments, such as cometary tails, asteroid zones, planetary rings, interstellar medium, earth environment, etc. [1–7]. These dust grains are invariably immersed in the ambient plasma and radiative background. The interaction of these dust grains with the other plasma particles (viz. electrons and ions) is due to the charge carried by them. The dust grains are charged by a number of competing processes, depending upon the local conditions, such as photoelectric emission stimulated by the ultraviolet radiation, collisional charging by electrons and ions, disruption and secondary emission due to the Maxwellian stress, etc. [8–12].

It is found that the presence of static charged dust grains modifies the existing plasma wave spectra [13–20], whereas the dust charge dynamics introduces new eigenmodes in

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dusty plasmas. Bliokh and Yarroshenko [13] studied electrostatic waves in dusty plasmas and applied their results in interpreting the spoke-like structures in Saturn's rings. Angelis *et al* [14] investigated the propagation of ion-acoustic waves in a dusty plasma, in which a spatial inhomogeneity is formed by the distribution of immobile dust particles [21]. They applied their results for interpreting the low-frequency noise enhancement observed by the Vega and Giotto space probes in the dusty regions of Halley's comet. Rao *et al* [22], for example, were the first to report theoretically the existence of extremely low phase velocity (in comparison with the electron and ion thermal velocities) DA waves in an unmagnetized dusty plasma whose constituents are inertial charged dust fluid and Boltzmann electrons and ions. Thus, in the DA waves the mass of the dust particle provides the inertia, whereas the restoring force comes from the pressures of inertialess electrons and ions. A laboratory experiment [23] has conclusively verified the theoretical prediction of Rao *et al* [22] and reported some nonlinear features of the DA waves. Mamun *et al* have studied nonlinear DA waves in a two-component dusty plasma consisting of a negatively charged dust fluid and Maxwellian [24] or non-Maxwellian [25] distributed ions. This study has extended our earlier work to a three-component dusty plasma which consists of a negatively charged dust fluid, Boltzman distributed electrons and free as well as trapped ions [26,27] which has been found to exhibit by the numerical simulation studies on linear and nonlinear properties of DA waves [28]. Therefore, in our present work, we consider an unmagnetized dusty plasma system containing vortex-like electrons, Maxwellian ions, and arbitrarily charged cold mobile dust and study the basic properties such as amplitude and width of DA solitary waves.

The manuscript is organized as follows. The basic equations governing the plasma system under consideration is presented in §2. The mKdV equation is derived by employing the reductive perturbation method for trapped electron in §3. The solitary wave solution of this mKdV equation is obtained and the properties of these DA solitary structures are discussed in §4. Finally, a brief discussion is presented in §5.

2. Governing equation

We consider a three-component unmagnetized dusty plasma consisting of Maxwellian ions, trapped electrons, and arbitrarily charged cold mobile dust. At equilibrium, we have $n_{i0} = n_{e0} - jZ_d n_{d0}$, where n_{i0}, n_{d0}, and n_{e0} are the unperturbed ion, dust, and electron number densities, respectively, Z_d is the number of electrons residing on the dust grains, and $j = +1$ (-1) for positive (negative) dust. The dynamics of such DA waves in one-dimensional form whose phase speed is in between dust thermal speed (V_{Td}) and ion thermal speed (V_{Ti}), i.e. $V_{Td} \ll V_p \ll V_{Ti}$ is governed by [24]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = -j\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu_e n_e - \mu_i n_i - jn_d, \tag{3}$$

where $n_{\rm d}$ is the number density of the dust particle normalized to $n_{\rm d0}$, $n_{\rm i}$ is the number density of the ion normalized to $n_{\rm i0}$, $n_{\rm e}$ is the number density of electron normalized to $n_{\rm e0}$, $u_{\rm d}$ is the speed of dust particle normalized to $C_{\rm d} = (Z_{\rm d}k_{\rm B}T_{\rm e}/m_{\rm d})^{1/2}$, and ϕ is the electrostatic wave potential normalized to $k_{\rm B}T_{\rm e}/e$, where $T_{\rm e}$ is the electron temperature, $m_{\rm d}$ is the mass of arbitrarily charged dust particles, and e is the magnitude of the electron charge. $\mu_{\rm e} = n_{\rm e0}/Z_{\rm d}n_{\rm d0}$ and $\mu_{\rm i} = n_{\rm i0}/Z_{\rm d}n_{\rm d0}$. The time and space variables are in units of dust plasma period $\omega_{\rm pd}^{-1} = (m_{\rm d}/4\pi n_{\rm d0}Z_{\rm d}^2e^2)^{1/2}$ and the Debye length $\lambda_{\rm D} = (k_{\rm B}T_{\rm e}/4\pi n_{\rm d0}e^2)^{1/2}$, respectively.

To model an electron distribution with trapped particles, we employ a vortex-like electron distribution function of Schamel [26,27], which solves the electron Vlasov equation. Thus we have

$$\left.\begin{aligned} f_{\rm ef} &= \frac{1}{\sqrt{2\pi}}{\rm e}^{-1/2(v^2-2\phi)}, \qquad |v| > \sqrt{2\phi} \\ f_{\rm et} &= \frac{1}{\sqrt{2\pi}}{\rm e}^{-1/2\beta_{\rm e}(v^2-2\phi)}, \quad |v| \le \sqrt{2\phi} \end{aligned}\right\}, \tag{4}$$

where the subscript f (t) represents the free (trapped) electron contribution. It may be noted here that the distribution function, as presented here, is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution [29]. Here, the velocity v is normalized to the electron thermal velocity $v_{\rm te}$ and $\beta_{\rm e}$, which is the ratio of free-electron temperature ($T_{\rm ef}$) to trapped electron temperature ($T_{\rm et}$), is a parameter determining the number of trapped electrons. It has been assumed that the velocity of nonlinear DA waves is small compared to the electron or ion thermal velocity.

Integrating the electron distribution functions over the velocity space, we readily obtain the electron number density $n_{\rm e}$ as

$$\begin{aligned} n_{\rm e} &= I(\phi) + \frac{{\rm e}^{\beta_{\rm e}\phi}}{\sqrt{|\beta_{\rm e}|}}{\rm erf}(\sqrt{\beta_{\rm e}\phi}), \qquad \beta_{\rm e} \ge 0, \\ n_{\rm e} &= I(\phi) + \frac{2}{\sqrt{\pi|\beta_{\rm e}|}}W(\sqrt{-\beta_{\rm e}\phi}), \quad \beta_{\rm e} < 0, \end{aligned} \tag{5}$$

where

$$\left.\begin{aligned} I(\phi) &= [1-{\rm erf}(\sqrt{\phi})]{\rm e}^{\phi} \\ {\rm erf}(\sqrt{\beta_{\rm e}\phi}) &= \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\beta_{\rm e}\phi}} {\rm e}^{-y^2}{\rm d}y \\ W(\sqrt{-\beta_{\rm e}\phi}) &= {\rm e}^{\beta_{\rm e}\phi}\int_0^{\sqrt{-\beta_{\rm e}\phi}} {\rm e}^{y^2}{\rm d}y \end{aligned}\right\}. \tag{6}$$

If we expand $n_{\rm e}$ for the small-amplitude limit and keep the terms up to ϕ^2, it is found that $n_{\rm e}$ is the same for both $\beta_{\rm e} \ge 0$ and $\beta_{\rm e} < 0$ and is finally given by

$$n_{\rm e} = 1 + \phi - \gamma\phi^{3/2} + \frac{1}{2}\phi^2, \tag{7}$$

where $\gamma = 4(1-\beta_{\rm e})/3\sqrt{\pi}$ and $\beta_{\rm e}$ is a parameter which determines the number of trapped ions [30]. We note that $\beta_{\rm e} = 1$ ($\beta_{\rm e} = 0$) represents a Maxwellian (flat-topped) distribution, whereas $\beta_{\rm e} < 0$ represents a trapped electron distribution.

3. Modified KdV equation for trapped electrons

We now follow the reductive perturbation technique [31] and construct a weakly nonlinear theory for the DA waves with small but finite amplitude, which leads to the scaling of independent variables through the stretched coordinates [26,27] as

$$\left.\begin{aligned} \xi &= \epsilon^{1/4}(x - v_{\mathrm{p}}t) \\ \tau &= \epsilon^{3/4}t \end{aligned}\right\}, \tag{8}$$

where ϵ is the smallness parameter measuring the weakness of the dispersion and v_{p} is the nonlinear wave phase velocity. We can expand the perturbed quantities n_{d}, u_{d}, and ϕ about their equilibrium values in powers of ϵ, including terms $\epsilon^{3/2}$,

$$\left.\begin{aligned} n_{\mathrm{d}} &= 1 + \epsilon n_{\mathrm{d}}^{(1)} + \epsilon^{3/2} n_{\mathrm{d}}^{(2)} + \cdots \\ u_{\mathrm{d}} &= o + \epsilon u_{\mathrm{d}}^{(1)} + \epsilon^{3/2} u^{(2)} + \cdots \\ \phi &= o + \epsilon \phi^{(1)} + \epsilon^{3/2} \phi^{(2)} + \cdots \end{aligned}\right\}. \tag{9}$$

Next, substituting eqs (7)–(9) into eqs (1)–(3) one can obtain the lowest order continuity equation, momentum equation, and Poisson's equation which in turn can be solved as

$$u_{\mathrm{d}}^{(1)} = \frac{j\phi^{(1)}}{v_{\mathrm{p}}}, \tag{10}$$

$$n_{\mathrm{d}}^{(1)} = \frac{j\phi^{(1)}}{v_{\mathrm{p}}^2}, \tag{11}$$

$$v_{\mathrm{p}}^2 = \frac{j^2}{\mu_{\mathrm{e}} + \alpha\mu_{\mathrm{i}}}, \tag{12}$$

where $\alpha = T_{\mathrm{e}}/T_{\mathrm{i}}$ i.e., α is the ratio of the electron temperature (T_{e}) and ion temperature (T_{i}). Therefore, eq. (9) represents the linear dispersion relation for DA waves. It is clear that the phase speed (v_{p}) of SWs is independent of the polarity of dust particles. Putting the values of eqs (7)–(12) into eqs (1)–(3), we obtain the next higher-order equations,

$$\frac{\partial n_{\mathrm{d}}^{(1)}}{\partial \tau} - v_{\mathrm{p}} \frac{\partial n_{\mathrm{d}}^{(2)}}{\partial \xi} + \frac{\partial u_{\mathrm{d}}^{(2)}}{\partial \xi} = 0, \tag{13}$$

$$\frac{\partial u_{\mathrm{d}}^{(1)}}{\partial \tau} - v_{\mathrm{p}} \frac{\partial u_{\mathrm{d}}^{(2)}}{\partial \xi} + j\frac{\partial \phi^{(2)}}{\partial \xi} = 0, \tag{14}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \mu_{\mathrm{e}}\phi^{(2)} - \gamma[\phi^{(1)}]^{3/2} + \mu_{\mathrm{i}}\alpha\phi^{(2)} - jn_{\mathrm{d}}^{(2)}. \tag{15}$$

Now, using eqs (13)–(15) one can easily eliminate $(\partial n_{\rm d}^{(2)}/\partial\xi)$, $(\partial u_{\rm d}^{(2)}/\partial\xi)$, and $(\partial\phi^{(2)}/\partial\xi)$, and obtain

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\sqrt{\phi^{(1)}}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0, \tag{16}$$

where

$$A = \frac{3\gamma}{4}\frac{v_{\rm p}}{(\mu_{\rm e}+\alpha\mu_{\rm i})}, \tag{17}$$

$$B = \frac{v_{\rm p}}{2(\mu_{\rm e}+\alpha\mu_{\rm i})}. \tag{18}$$

Equation (16) is a mKdV equation for trapped electrons, exhibiting stronger nonlinearity, smaller width, and larger propagation velocity of the nonlinear wave.

4. Solution of mKdV equation

The stationary solution of this mKdV equation can be obtained by transforming the independent variables ξ and τ to $x = \xi - u_0\tau$, $\tau = \tau$, where u_0 is the constant solitary wave velocity. Now using the appropriate boundary conditions for localized disturbances, viz. $\phi^{(1)} \to 0$, $({\rm d}\phi^{(1)}/{\rm d}x) \to 0$, $({\rm d}^2\phi^{(1)}/{\rm d}x^2) \to 0$ at $x \to \pm\infty$. Thus, one can express the stationary solution of this mKdV equation as

$$\phi^{(1)} = \phi_m \,{\rm sech}^4\left[\frac{(\xi - u_0\tau)}{\Delta}\right], \tag{19}$$

where $\phi_m = (15u_0/8A)^2$ is the amplitude and $\Delta = \sqrt{16B/u_0}$ is the width of the solitary waves, respectively. It is clear from eq. (19) that the solitary waves will be associated with

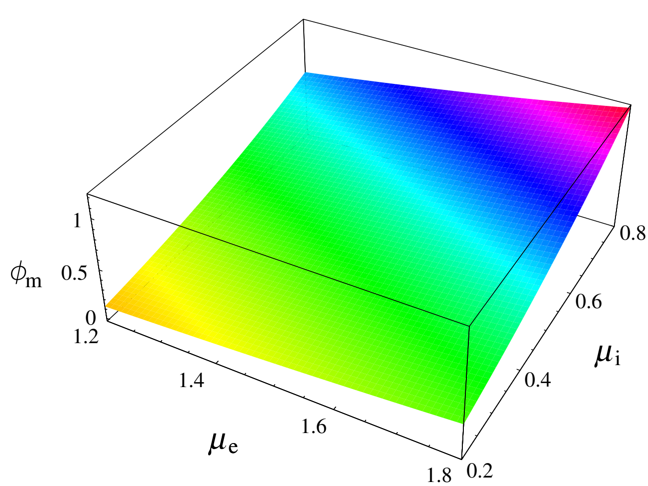

Figure 1. The variation of amplitude (ϕ_m) of the solitary wave with $\mu_{\rm e}$ and $\mu_{\rm i}$ for $u_0 = 0.1$, $j = 1$, $\alpha = 2$, and $\beta_{\rm e} = -0.9$.

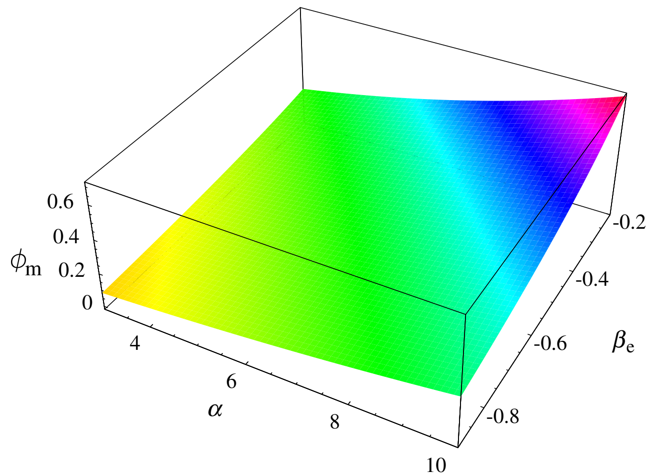

Figure 2. The variation of amplitude (ϕ_m) of the solitary wave with α and β_e for $u_0 = 0.1$, $j = 1$, $\mu_i = 0.1$, and $\mu_e = 1.1$.

positive potential ($\phi_m > 0$), and the width of the solitary waves will have positive value. It should be noted here that the perturbation method, which is only valid for small but finite amplitude limit, is not valid for large amplitude. As $u_0 > 0$, there exist solitary waves with positive potential only, i.e., solitary structures with enhanced density only. It is seen that as u_0 increases, the amplitude increases while the width decreases and, that as $|\beta_e|$ increases, the amplitude decreases for $\beta_e < 0$ (a vortex-like excavated trapped electron distribution)

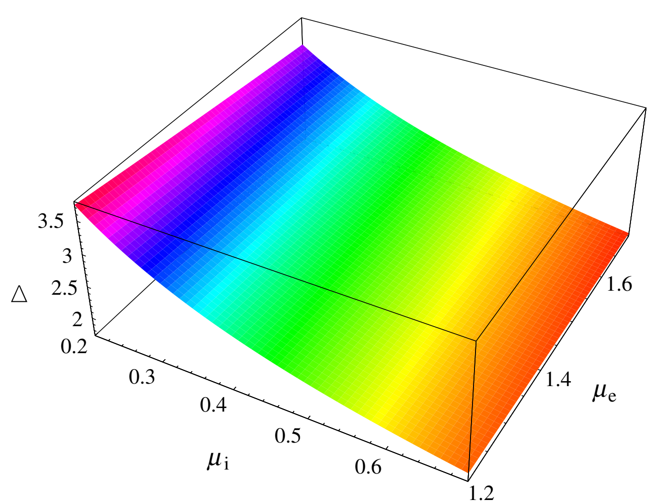

Figure 3. The variation of width (Δ) of the solitary wave with μ_i and μ_e for $u_0 = 0.1$, $j = \pm 1$, and $\alpha = 10$.

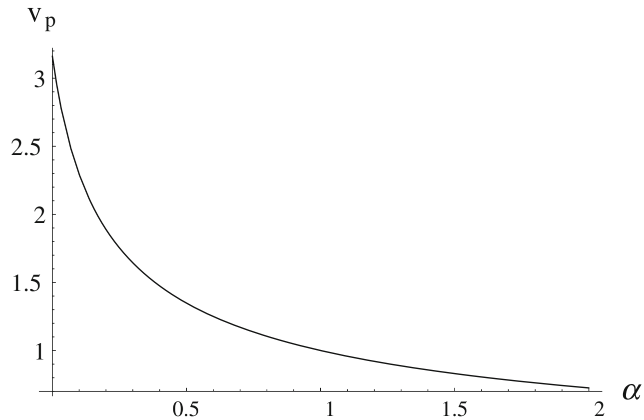

Figure 4. The variation of phase speed ($v_{\rm p}$) of the solitary wave with α for $j = \pm 1$, $\mu_{\rm e} = 0.1$, and $\mu_{\rm i} = 0.9$.

and increases for $\beta_{\rm e} > 0$. It is clear that due to the trapped electrons, we have found soliton-like structures of larger amplitude, smaller width, and higher propagation velocity than that involving isothermal electron. From the dispersion relation we found that the polarity of dust particles has no effect on the phase speed of these solitary waves. Figure 1 shows the variation of the amplitude (ϕ_m) of SWs with $\mu_{\rm e}$ and $\mu_{\rm i}$ for $u_0 = 0.1$, $j = 1$, $\alpha = 4$, and $\beta_{\rm e} = -0.9$. From figure 1 we see that the amplitude (ϕ_m) of the SWs increases by increasing the value of both $\mu_{\rm e}$ (slowly) and $\mu_{\rm i}$ (rapidly), but it has no effect on the polarity of the dust particles. Figure 2 shows the variation of the amplitude (ϕ_m) of SWs with α and $\beta_{\rm e}$ for $u_0 = 0.1$, $j = 1$, $\mu_{\rm i} = 1.5$, and $\mu_{\rm e} = 0.5$. Figure 2 shows that the amplitude (ϕ_m) of the SWs increases by increasing the value of α (slowly) and $\beta_{\rm e}$ (very rapidly). Figure 3 shows the variation of the width (Δ) of SWs with $\mu_{\rm i}$ and $\mu_{\rm e}$ for

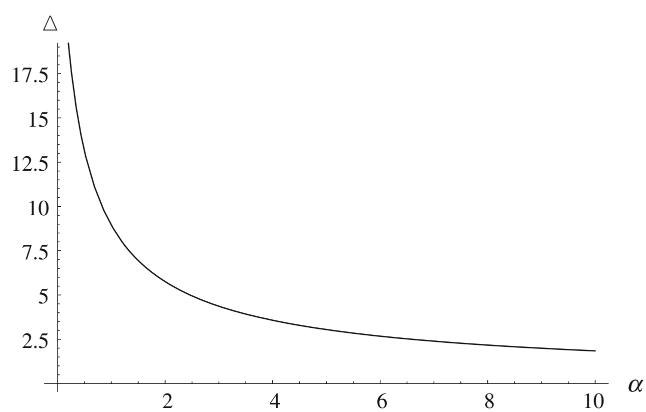

Figure 5. The variation of width (Δ) of the solitary wave with α for $u_0 = 0.1$, $j = \pm 1$, $\mu_{\rm e} = 0.2$, and $\mu_{\rm i} = 0.8$.

$u_0 = 0.1$, $j = 1$, and $\alpha = 10$. Figure 3 shows that the width (Δ) of the SWs decreases rapidly (slowly) by increasing the value of μ_i (μ_e). Figure 4 shows the variation of the phase speed (v_p) of SWs with α for $j = \pm 1$ (i.e., $j = +1$ for positively charged dust and $j = -1$ for negatively charged dust), $\mu_e = 0.1$, and $\mu_i = 0.9$. Figure 4 indicates that the phase of the SWs decreases by increasing the value of α but it does not depend on the polarity of dust particles. Figure 5 shows the variation of the width (Δ) of SWs with α for $u_0 = 0.1$, $\mu_e = 0.2$, $\mu_i = 0.8$, and $j = \pm 1$. Figure 5 also indicates that the width of the SWs decreases by increasing the value of α. It has been found that the width of the solitary waves does not depend on the polarity of the dust particles.

5. Discussion

We have considered an unmagnetized collisionless three-component dusty plasma consisting of extremely massive, micron-sized, arbitrarily charged, cold mobile dust grains, Maxwellian ions, trapped electrons, and have studied the DA solitary waves associated with positive potential only by deriving the mKdV equation. It has been found that the basic features of such DA solitary waves are significantly modified by the presence of trapped electrons. It is also found that the DA solitary waves in our dusty plasma model differ from the usual KdV equation by their polarity, width, speed, and the power of sech. The results, which have been obtained from this investigation, can be summarized as follows:

(1) Dusty plasmas, whose constituents are cold dust particulates, Maxwellian ions, and trapped electrons of different constant temperatures, are found to support solitary waves associated with the non-linear DA waves.
(2) Furthermore, in the presence of vortex-like electron distribution, the dynamics of weakly dispersive non-linear DA waves is governed by the mKdV equation, the stationary solution of which is represented in the form of an inverted secant hyperbolic fourth profile. Thus, the potential polarity of the DA solitary waves in our dusty plasma is different from the usual IA solitary waves in an electron–ion plasma.
(3) It is found that non-isothermal electrons are responsible for DA solitary waves which have smaller width, larger amplitude, and higher propagation velocity than that involving isothermal or Maxwellian electrons, and that they can be represented in the form $\mathrm{sech}^4(z/\Delta)$, instead of $\mathrm{sech}^2(z/\Delta)$ which is the stationary solution of the standard KdV equation.
(4) It is also clear that for a vortex-like excavated trapped electrons distribution, i.e., for $\beta_e < 0$, as $|\beta_e|$ increases the amplitude of the solitary waves decreases.
(5) It is also found that the polarity of dust particles has no effect on the nonlinear propagation of DA solitary waves.
(6) It has also been found that the phase speed of DA solitary waves is independent of the polarity of the dust particles, i.e., either positively charged or negatively charged dust particles.

We hope that our present results should be helpful to understand the basic features of the electrostatic disturbances in space and laboratory devices. The present work can also provide a guideline for interpreting the most recent numerical simulation results, which exhibit the simultaneous presence of non-thermal ion distributions and associated DA localized wave packets.

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