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New travelling wave solutions for nonlinear stochastic evolution equations

HYUNSOO KIM[1] and RATHINASAMY SAKTHIVEL[2,*]

[1]College of Applied Science, Kyung Hee University, Yongin 446-701, South Korea
[2]Department of Mathematics, Sungkyunkwan University, Suwon 440-746, South Korea
[*]Corresponding author. E-mail: krsakthivel@yahoo.com

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Abstract. The nonlinear stochastic evolution equations have a wide range of applications in physics, chemistry, biology, economics and finance from various points of view. In this paper, the (G'/G)-expansion method is implemented for obtaining new travelling wave solutions of the nonlinear (2 + 1)-dimensional stochastic Broer–Kaup equation and stochastic coupled Korteweg–de Vries (KdV) equation. The study highlights the significant features of the method employed and its capability of handling nonlinear stochastic problems.

Keywords. (2 + 1)-dimensional stochastic Broer–Kaup equation; stochastic coupled KdV equation; (G'/G)-expansion method; travelling wave solutions.

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1. Introduction

The investigation of exact solutions of nonlinear evolution equations play an important role in the study of nonlinear physical phenomena. In the past several decades, many powerful methods such as differential transform method [1,2], the extended tanh method [3,4], the exp-function method [5–8], variational iteration method [9], homotopy perturbation method [10,11], Adomian–Pade method [12] and many other techniques were used to obtain wave solutions of nonlinear evolution equations. More precisely, there is no unified method that can be used to handle all types of nonlinear problems. Recently, Wang *et al* [13] introduced a new direct method called the (G'/G)-expansion method to look for travelling wave solutions of nonlinear partial differential equations. It is interesting to mention that, in this method the sign of the parameters can be used to judge the numbers and types of travelling wave solutions. Zhang *et al* [14] proposed a generalized (G'/G)-expansion method to obtain non-travelling wave solutions of the (2 + 1)-dimensional Broer–Kaup equation (for detailed description of the (G'/G)-expansion method and its applications, refer to [15–17]).

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On the other hand, many researchers pay more attention to the study of random waves, which is an important topic of stochastic partial differential equations. It is difficult to solve stochastic partial differential equations compared to deterministic differential equations, because of its additional random terms. As is well known, the motion of long, unidirectional, weakly nonlinear water waves on a channel can be described by the Korteweg–de Vries (KdV) equation [18]. Wadati [19,20] first introduced and studied the stochastic KdV equation analytically and also determined the large time behaviour of one-soliton solutions under Gaussian noise. The stochastic KdV equation arises when modelling the propagation of weakly nonlinear waves in a noisy plasma and dispersive wave turbulence in shallow waters [21,22]. Dai and Chen [23] obtained new exact solutions of $(2+1)$-dimensional stochastic Broer–Kaup equation in the white noise environment by using exp-function method. More recently, Kim and Sakthivel [24] derived new exact travelling wave solutions for the Wick-type stochastic generalized Boussinesq equation and Wick-type stochastic Kadomtsev–Petviashvili equation with variable coefficients.

In this paper, the (G'/G)-expansion method is implemented to search for new travelling wave solutions of some nonlinear stochastic evolution equations such as $(2+1)$-dimensional stochastic Broer–Kaup equation and stochastic coupled KdV equation [25–27]. These equations play important roles in applied scientific fields such as plasma, nonlinear optical fibre and statistical physics [28–31]. In the following section, the (G'/G)-expansion method is described in detail.

2. Description of the (G'/G)-expansion method

Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$P(u, u_t, u_x, u_{xx}, u_{tt}, \ldots) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives. In order to find the solution of u explicitly, we proceed to the following steps:

Step 1. The partial differential equation (PDE) (1) can be converted to ordinary differential equation (ODE)

$$Q(u, u', u'', u''', \ldots) = 0 \tag{2}$$

using a wave variable $\xi = x - \omega t$.

Step 2. Suppose that the solution of ODE (2) can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi) = a_m \left(\frac{G'}{G}\right)^m + a_{m-1}\left(\frac{G'}{G}\right)^{m-1} + \cdots, \tag{3}$$

where $G = G(\xi)$ satisfies the second-order linear ordinary differential equation (LODE) in the form

$$G'' + \lambda G' + \mu G = 0, \tag{4}$$

where $a_m, a_{m-1}, \ldots, \lambda$ and μ are constants to be determined later, $a_m \neq 0$. The unwritten part in (3) is also a polynomial in (G'/G), the degree of which is generally equal to or less than $m-1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (2).

Step 3. By substituting (3) into (2) and using second-order LODE (4), collecting all terms with the same order of (G'/G) together, the left-hand side of (2) is converted into another polynomial in (G'/G). Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $a_m, a_{m-1}, \ldots, \lambda$ and μ.

Step 4. Assuming that the constants $a_m, \ldots, \omega, \lambda$ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second-order LODE (4) have been well known for us, then substituting $a_m, \ldots, \omega$ and the general solutions of (4) into (3) we can obtain the exact solutions of PDE (1).

3. Solutions of (2 + 1)-dimensional stochastic Broer–Kaup equation

In this section, we consider a (2 + 1)-dimensional Wick-type stochastic Broer–Kaup system in the following form:

$$\begin{aligned} &U_{yt} - H(t) \diamond (U_{xxy} - 2\,(U \diamond U_x)_y - 2V_{xx}) = 0, \\ &V_t + H(t) \diamond (V_{xx} + 2\,(U \diamond V)_x) = 0, \end{aligned} \tag{5}$$

where $H(t)$ is the white noise function and $\diamond$ is the Wick product on the Hida distribution space $(S(R^d))$ (for more details, see [32]). Xie [33] obtained stochastic single soliton solution and stochastic multisoliton solutions of Wick-type stochastic KdV equations by using the Hermite transform and the homogeneous balance principle. Along the idea of [29,33], by taking the Hermite transform of (5), we obtain the equations

$$\begin{aligned} &\tilde{U}_{yt} - \tilde{H}(t,z)(\tilde{U}_{xxy} - 2(\tilde{U}\tilde{U}_x)_y - 2\tilde{V}_{xx}) = 0, \\ &\tilde{V}_t + \tilde{H}(t,z)(\tilde{V}_{xx} + 2(\tilde{V}\tilde{U})_x) = 0, \end{aligned} \tag{6}$$

where $z = (z_1, z_2, \ldots) \in (C)_c$ is a vector parameter. For simplicity, we denote $u(t,x,y,z) = \tilde{U}(t,x,y,z)$, $v(t,x,y,z) = \tilde{V}(t,x,y,z)$ and $H(t,z) = \tilde{H}(t,z)$. In order to solve eq. (6), let us consider the following transformation:

$$v = u_y. \tag{7}$$

Substituting the transformation (7) in eq. (6), and integrating the resulting equation with respect to y, yields

$$u_t + Hu_{xx} + 2Huu_x = L(t,x,z). \tag{8}$$

In what follows, we only consider $L(t,x,z) = L(t)$.

To look for the travelling wave solutions, eq. (8) can be converted to the ODE

$$(p_t(t,y,z)+\omega_t(t,y,z))\,u' + Hp^2(t,y,z)u'' + 2Hp(t,y,z)uu' = 0 \tag{9}$$

upon using a wave variable $u(t,x,y,z)=u(\xi)$, $\xi = xp(t,y,z)+\omega(t,y,z)$.

Balancing u'' and uu' in (9), we obtain $m+2=2m+1$ which gives $m=1$. Suppose that the solution of (9) can be expressed by a polynomial in $\left(G'/G\right)$ as follows:

$$u = u(\xi) = a_1(t,y,z)\left(\frac{G'}{G}\right) + a_0(t,y,z), \quad a_1(t,y,z) \neq 0. \tag{10}$$

By substituting eq. (10) in eq. (9) and collecting all terms with the same power of $\left(G'/G\right)$ together, the left-hand side of eq. (9) is converted into a polynomial in $\left(G'/G\right)$. Equating each coefficient of this polynomial to zero, we shall obtain a set of algebraic equations with respect to the unknowns a_1, a_0 and ω.

$$\left(\frac{G'}{G}\right)^0 : -a_1\mu x p_t - a_1\mu\omega_t + Hp(t)^2 a_1\lambda\mu - 2Ha_0a_1\mu - L = 0$$

$$\left(\frac{G'}{G}\right)^1 : -a_1\lambda x p_t - a_1\lambda\omega_t + Hp^2a_1\lambda^2 + 2Hp^2a_1\mu - 2Ha_0a_1\lambda - 2Ha_1^2\mu = 0$$

$$\left(\frac{G'}{G}\right)^2 : -a_1 x p_t - a_1\omega_t + 3Hp^2a_1\lambda - 2Ha_0a_1 - 2Ha_1^2\lambda = 0$$

$$\left(\frac{G'}{G}\right)^3 : 2Hp^2a_1 - 2Ha_1^2 = 0.$$

Solving the system of algebraic equations using *Maple*, we obtain the following sets of nontrivial solutions:

$$\begin{cases} a_0(t,y,z) = a_0(t,y,z),\, a_1(t,y,z) = p^2(t,y,z) \\ \omega(t,y,z) = \int \Big(\lambda H(t,z)p^2(t,y,z)p(t,z) - x\dfrac{\partial p(t,y,z)}{\partial t} \\ \qquad\qquad \times\, 2H(t,z)a_0(t,y,z)\Big)\,\mathrm{d}t, \end{cases} \tag{11}$$

where $H(t,z)$ is an arbitrary function.

Now from eqs (7), (9) and (10), travelling wave solutions can be written as

$$\begin{cases} u(\xi) = a_1(t,y,z)\left(\dfrac{G'}{G}\right) + a_0(t,y,z), \\ v(\xi) = u_y. \end{cases} \tag{12}$$

Substituting eq. (11) in eq. (12), we can obtain many types of travelling wave solutions of (9) as follows. When $\lambda^2 - 4\mu > 0$, we obtain the following hyperbolic function solutions:

$$\begin{cases} u_1(t,x,y,z) = p^2(t,y,z) \\ \qquad \times \left[-\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2-4\mu}}{2} \left(\dfrac{C_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi}{C_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi} \right) \right] \\ \qquad + a_0(t,y,z), \\ v_1(t,x,y,z) = 2p(t,y,z)\dfrac{\partial P(t,y,z)}{\partial y} \\ \qquad \times \left[-\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2-4\mu}}{2} \left(\dfrac{C_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi}{C_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi} \right) \right] \\ \qquad + \dfrac{\lambda^2-4\mu}{4} p^2(t,y,z) \\ \qquad \times \left[1 - \left(\dfrac{C_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi}{C_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi} \right)^2 \right] \dfrac{\partial \xi}{\partial y} \\ \qquad + \dfrac{\partial a_0(t,y,z)}{\partial y}, \end{cases} \tag{13}$$

where

$$\xi = p(t,y,z)x + \omega(t,y,z),$$

$$\omega(t,y,z) = \int \left(\lambda H(t,z) p^2(t,y,z) - x\frac{\partial p(t,y,z)}{\partial t} - 2H(t,z)a_0(t,y,z) \right) \mathrm{d}t$$

and

$$\frac{\partial \xi}{\partial y} = \frac{\partial p(t,y,z)}{\partial y}x + \int 2\lambda H(t,z)p(t,y,z)\frac{\partial p(t,y,z)}{\partial y} - x\frac{\partial^2 p(t,y,z)}{\partial t \partial y} - 2H(t,z)\frac{\partial a_0(t,y,z)}{\partial y}\,\mathrm{d}t.$$

In particular, if $C_1 \neq 0$, $C_2 = 0$, $\lambda > 0$ and $\mu = 0$, then u_1 and v_1 can be written as

$$\begin{cases} u_1(t,x,y,z) = \dfrac{\lambda}{2}\, p^2(t,y,z) \left(\tanh\dfrac{\lambda}{2}\,\xi - 1 \right) + a_0(t,y,z), \\ v_1(t,x,y,z) = \lambda p(t,y,z)\dfrac{\partial p(t,y,z)}{\partial y} \left(\tanh\dfrac{\lambda}{2}\,\xi - 1 \right) \\ \qquad + \dfrac{\lambda^2}{4}\, p^2(t,y,z) \left(1 - \tanh^2\dfrac{\lambda}{2}\,\xi \right) \dfrac{\partial \xi}{\partial y} + \dfrac{\partial a_0(t,y,z)}{\partial y}, \end{cases} \tag{14}$$

where

$$\xi = p(t, y, z)x + \omega(t, y, z),$$

$$\omega(t, y, z) = \int \left(\lambda H(t, z) p^2(t, y, z) - x \frac{\partial p(t, y, z)}{\partial t} - 2H(t, z) a_0(t, y, z) \right) \mathrm{d}t$$

and

$$\frac{\partial \xi}{\partial y} = \frac{\partial p(t, y, z)}{\partial y} x + \int 2\lambda H(t, z) p(t, y, z) \frac{\partial p(t, y, z)}{\partial y} - x \frac{\partial^2 p(t, y, z)}{\partial t \partial y} - 2H(t, z) \frac{\partial a_0(t, y, z)}{\partial y} \mathrm{d}t.$$

Next, we obtain the following trigonometric function solution when: $\lambda^2 - 4\mu < 0$:

$$\begin{cases} u_2(t, x, y, z) = p^2(t, y, z) \\ \qquad \times \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \left(\frac{-C_1 \sin \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + C_2 \cos \frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1 \cos \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + C_2 \sin \frac{\sqrt{4\mu-\lambda^2}}{2}\xi} \right) \right] \\ \qquad + a_0(t, y, z) \\ v_2(t, x, y, z) = 2p(t, y, z) \frac{\partial P(t, y, z)}{\partial y} \\ \qquad \times \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \left(\frac{-C_1 \sin \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + C_2 \cos \frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1 \cos \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + C_2 \sin \frac{\sqrt{4\mu-\lambda^2}}{2}\xi} \right) \right] \\ \qquad - \frac{4\mu - \lambda^2}{4} p^2(t, y, z) \\ \qquad \times \left[1 + \left(\frac{-C_1 \sin \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + C_2 \cos \frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1 \cos \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + C_2 \sin \frac{\sqrt{4\mu-\lambda^2}}{2}\xi} \right)^2 \right] \frac{\partial \xi}{\partial y} \\ \qquad + \frac{\partial a_0(t, y, z)}{\partial y}, \end{cases} \tag{15}$$

where

$$\xi = p(t, y, z)x + \omega(t, y, z),$$

$$\omega(t, y, z) = \int \left(\lambda H(t, z) p^2(t, y, z) - x \frac{\partial p(t, y, z)}{\partial t} - 2H(t, z) a_0(t, y, z) \right) \mathrm{d}t$$

and

$$\frac{\partial \xi}{\partial y} = \frac{\partial p(t,y,z)}{\partial y}x + \int 2\lambda H(t,z)p(t,y,z)\frac{\partial p(t,y,z)}{\partial y} - x\frac{\partial^2 p(t,y,z)}{\partial t \partial y} - 2H(t,z)\frac{\partial a_0(t,y,z)}{\partial y}\mathrm{d}t.$$

Finally, we obtain the following soliton solution when $\lambda^2 - 4\mu = 0$:

$$\begin{cases} u_3(t,x,y,z) = p^2(t,y,z)\left(-\frac{\lambda}{2} + \frac{C_2(1+(\lambda/2+1)\xi)}{C_1+C_2\xi}\right) + a_0(t,y,z), \\ v_3(t,x,y,z) = 2p(t,y,z)\frac{\partial P(t,y,z)}{\partial y}\left(-\frac{\lambda}{2} + \frac{C_2(1+(\lambda/2+1)\xi)}{C_1+C_2\xi}\right) \\ \qquad + p^2(t,y,z)\frac{C_1C_2(\lambda/2+1)-C_2^2}{(C_1+C_2\xi)^2}\frac{\partial \xi}{\partial y} + \frac{\partial a_0(t,y,z)}{\partial y}, \end{cases} \tag{16}$$

where

$$\xi = p(t,y,z)x + \omega(t,y,z),$$

$$\omega(t,y,z) = \int\left(\lambda H(t,z)p^2(t,y,z) - x\frac{\partial p(t,y,z)}{\partial t} - 2H(t,z)a_0(t,y,z)\right)\mathrm{d}t$$

and

$$\frac{\partial \xi}{\partial y} = \frac{\partial p(t,y,z)}{\partial y}x + \int 2\lambda H(t,z)p(t,y,z)\frac{\partial p(t,y,z)}{\partial y} - x\frac{\partial^2 p(t,y,z)}{\partial t \partial y} - 2H(t,z)\frac{\partial a_0(t,y,z)}{\partial y}\mathrm{d}t.$$

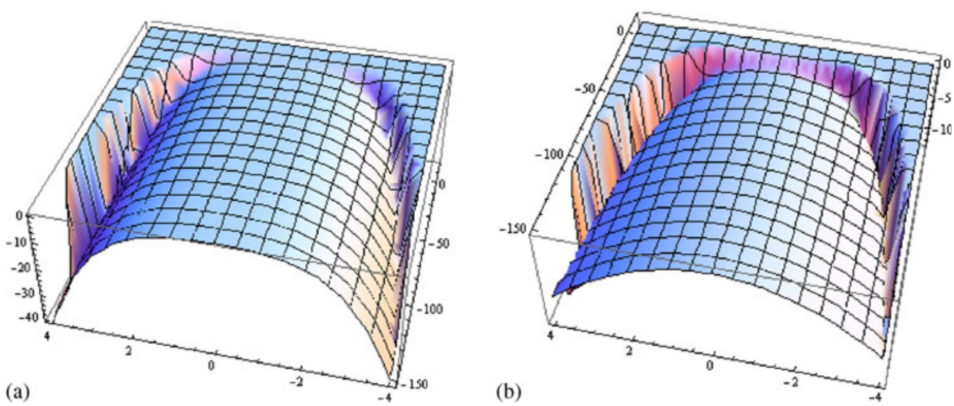


Figure 1. The solutions (**a**) u and (**b**) v of (14) for the stochastic Broer–Kaup equation when $\lambda = 0.5$, $\mu = 0$, $p = 0.5t^2 + y$, $a_0 = 0$, $H = t$ and $y = 1$.

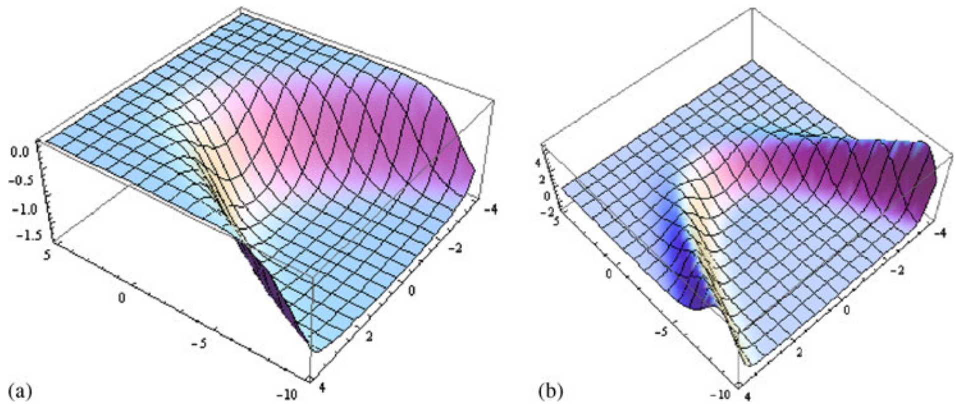


Figure 2. The solutions (**a**) u and (**b**) v of (14) for the stochastic Broer–Kaup equation when $\lambda = 0.8$, $\mu = 0$, $p = \cos(t/10) + y$, $a_0 = 0$, $H = 10\sin(t/10)$, and $y = 0.5$.

The solution (14) contains the arbitrary functions $H(t)$ and $p(t, y, z)$. This implies that the physical quantities u and v may posses rich structures. With different values of λ, μ, y, and a_0, the behaviour of the obtained solution (14) is shown graphically in figures 1 and 2.

4. Solutions of Wick-type stochastic coupled KdV equation

Wadati [19] introduced and investigated stochastic partial differential KdV equation, and gave the diffusion of soliton of the KdV equation under Gaussian noise. Later, Xie [34] derived soliton solutions of Wick-type stochastic KdV equation on white-noise space using F-expansion method with Hermite transform and homogeneous balance principle.

In this section, we consider the Wick-type stochastic coupled KdV equations

$$\begin{cases} U_t + H_1(t)\diamond U\diamond U_x + H_2(t)\diamond V\diamond V_x + H_3(t)\diamond U_{xxx} = 0, \\ V_t + H_4(t)\diamond U\diamond V_x + H_3(t)\diamond V_{xxx} = 0, \end{cases} \tag{17}$$

where $H_1(t) \sim H_2(t)$ are the white-noise functions and $\diamond$ is the Wick production on the Hida distribution $(S(R^d))$ [32]. Along the idea of [29,34], by taking the Hermite transform of (17), we get the equations

$$\begin{cases} \tilde{U}_t(t,x,z) + \tilde{H}_1(t,z)\tilde{U}(t,x,z)\tilde{U}_x(t,x,z) \\ \quad + \tilde{H}_2(t,z)\tilde{V}(t,x,z)\tilde{V}_x(t,x,z) \\ \quad + \tilde{H}_3(t,z)\tilde{U}_{xxx}(t,x,z) = 0, \\ \tilde{V}_t(t,x,z) + \tilde{H}_4(t,z)\tilde{U}(t,x,z)\tilde{V}_x(t,x,z) \\ \quad + \tilde{H}_3(t,z)\tilde{V}_{xxx}(t,x,z) = 0, \end{cases} \tag{18}$$

where $z = (z_1, z_2, \ldots) \in (C)_c$ is a vector parameter.

For simplicity, we take $u(t,x,z) = \tilde{U}(t,x,z), v(t,x,z) = \tilde{V}(t,x,z), H_1(t,z) = \tilde{H}_1(t,z), H_2(t,z) = \tilde{H}_2(t,z), H_3(t,z) = \tilde{H}_3(t,z)$, and $H_4(t,z) = \tilde{H}_4(t,z)$. Then we have the following equations:

$$\begin{cases} u_t + \tilde{H}_1(t,z)uu_x + \tilde{H}_2(t,z)vv_x + \tilde{H}_3(t,z)u_{xxx} = 0, \\ v_t(t,x,z) + \tilde{H}_4(t,z)uv_x + \tilde{H}_3(t,z)v_{xxx} = 0. \end{cases} \tag{19}$$

In order to solve eq. (18), let us make the following transformation:

$$\begin{cases} u(\xi) = u(t,x,z), \\ u_t = \dfrac{\mathrm{d}u}{\mathrm{d}\xi}\dfrac{\partial \xi}{\partial t} = \left(p_t(t,z) + \omega_t(t,z)\right)u', \\ u_x = p(t,z)u', \\ u_{xx} = p^2(t,z)u'',\ u_{xxx} = p^3(t,z)u''', \end{cases} \tag{20}$$

where $\xi = xp(t,z) + \omega(t,z)$.

Substitute the transformation (20) into eq. (19), and then eq. (19) can be converted to the ODE as follows:

$$\begin{cases} (p_t(t,z)x + \omega_t(t,z))u' + H_1(t,z)p(t,z)uu' \\ \quad + H_2(t,z)p(t,z)vv' + H_3(t,z)p^3(t,z)u''' = 0, \\ (p_t(t,z)x + \omega_t(t,z))v' + H_4(t,z)p(t,z)uv' \\ \quad + H_3(t,z)p^3(t,z)v''' = 0. \end{cases} \tag{21}$$

By balancing the highest nonlinear terms with the highest-order partial derivative terms in eq. (21), we can get $m = n = 2$.

We assume that the solution of eq. (21) can be expressed in the following form:

$$\begin{cases} u = u(\xi) = a_2(t,z)\left(\dfrac{G'}{G}\right)^2 + a_1(t,z)\left(\dfrac{G'}{G}\right) + a_0(t,z), \\ v = v(\xi) = b_2(t,z)\left(\dfrac{G'}{G}\right)^2 + b_1(t,z)\left(\dfrac{G'}{G}\right) + b_0(t,z), \end{cases} \tag{22}$$

where $\xi = xp(t,z) + \omega(t,z)$.

By substituting (22) into eq. (21) and collecting all terms with the same power of $\left(G'/G\right)$ together, the left-hand side of eq. (21) is converted into polynomial in $\left(G'/G\right)$. Equating each coefficient of this polynomial to zero, we shall obtain a set of algebraic equations with respect to the unknown variables. Solving the system of algebraic equations using Mathematica, we obtain the following sets of nontrivial solutions:

$$\begin{cases} a_0(t,z) = \pm \dfrac{H_2(t,z) b_0(t,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{p(t,z)(H_1(t,z)-H_4(t,z))\sqrt{H_2(t,z)}}, \\ a_1(t,z) = -\dfrac{12\lambda H_3(t,z) p^2(t,y,z)}{H_4(t,z)}, \\ a_2(t,z) = -\dfrac{12 H_3(t,z) p^2(t,y,z)}{H_4(t,z)}, \\ b_0(t,z) = b_0(t,z), \\ b_1(t,z) = \pm \dfrac{12\lambda H_3(t,z) p^2(t,y,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{H_4(t,z)\sqrt{H_2(t,z)}}, \\ b_2(t,z) = \pm \dfrac{12 H_3(t,z) p^2(t,y,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{H_4(t,z)\sqrt{H_2(t,z)}}, \\ \omega(t,z) = \displaystyle\int (H_4(t,z)-H_1(t,z)) \\ \qquad \times \left\{ -(8\mu+\lambda^2) H_3(t,z) p(t,z) - x\dfrac{\partial p(t,z)}{\partial t} \right\} \\ \qquad \mp \dfrac{H_2(t,z) H_4(t,z) b_0(t,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{\sqrt{H_2(t,z)}} \mathrm{d}t, \end{cases} \tag{23}$$

where $H_i(t,z)$, $i = 1, 2, 3, 4$ are arbitrary functions.

Substituting eq. (23) in eq. (22), we can obtain many types of travelling wave solutions of eq. (17). When $\lambda^2 - 4\mu > 0$, we obtain the following hyperbolic function solution:

$$\begin{cases} u_1(t,x,z) = -\dfrac{\lambda^2-4\mu}{4}\,\dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \\ \qquad \times \left(\dfrac{C_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi}{C_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi} \right)^2 \\ \qquad + \dfrac{\lambda^2}{4}\,\dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \pm \dfrac{H_2(t,z) b_0(t,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{p(t,z)(H_1(t,z)-H_4(t,z))\sqrt{H_2(t,z)}}, \\ v_1(t,x,z) = \pm \dfrac{12 H_3(t,z) p^2(t,y,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{H_4(t,z)\sqrt{H_2(t,z)}} \\ \qquad \times \left(\dfrac{C_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi}{C_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi + C_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\,\xi} \right)^2 \\ \qquad \mp \dfrac{\lambda^2}{4}\,\dfrac{12 H_3(t,z) p^2(t,y,z)\sqrt{(H_4(t,z)-H_1(t,z))p(t,z)}}{H_4(t,z)\sqrt{H_2(t,z)}} + b_0(t,z), \end{cases} \tag{24}$$

where

$$\xi = p(t,z)x + \omega(t,z),$$
$$\omega(t,z) = \int (H_4(t,z) - H_1(t,z)) \left\{ -(8\mu + \lambda^2) H_3(t,z) p(t,z) - x \frac{\partial p(t,z)}{\partial t} \right\} \mp \frac{H_2(t,z) H_4(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{\sqrt{H_2(t,z)}} \, \mathrm{d}t.$$

In particular, if $C_1 \neq 0$, $C_2 = 0$, $\lambda > 0$ and $\mu = 0$, then u_1 and v_1 can be written as

$$\begin{cases} u_1(t,x,z) = -\dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \tanh^2 \dfrac{\lambda}{2} \xi + \dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \\ \qquad \pm \dfrac{H_2(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{p(t,z)(H_1(t,z) - H_4(t,z)) \sqrt{H_2(t,z)}}, \\ v_1(t,x,z) = \pm \dfrac{12 H_3(t,z) p^2(t,y,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{H_4(t,z) \sqrt{H_2(t,z)}} \tanh^2 \dfrac{\lambda}{2} \xi \\ \qquad \mp \dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,y,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{H_4(t,z) \sqrt{H_2(t,z)}} + b_0(t,z), \end{cases} \tag{25}$$

where

$$\xi = p(t,z)x + \omega(t,z),$$
$$\omega(t,z) = \int (H_4(t,z) - H_1(t,z)) \times \left\{ -(8\mu + \lambda^2) H_3(t,z) p(t,z) - x \frac{\partial p(t,z)}{\partial t} \right\} \mp \frac{H_2(t,z) H_4(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{\sqrt{H_2(t,z)}} \, \mathrm{d}t.$$

We obtain the following trigonometric function solution when $\lambda^2 - 4\mu < 0$:

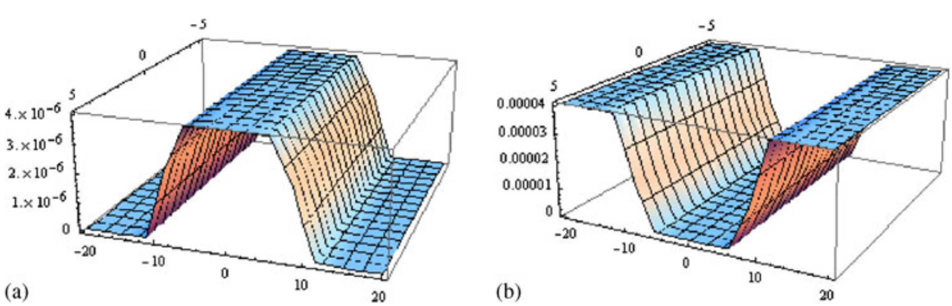
$$\begin{cases} u_2(t,x,z) = -\dfrac{4\mu - \lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \\ \qquad \times \left(\dfrac{-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi} \right)^2 + \dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \\ \qquad \pm \dfrac{H_2(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{p(t,z)(H_1(t,z) - H_4(t,z)) \sqrt{H_2(t,z)}}, \\ v_2(t,x,z) = \pm \dfrac{12 H_3(t,z) p^2(t,y,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{H_4(t,z) \sqrt{H_2(t,z)}} \\ \qquad \times \left(\dfrac{-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi} \right)^2 \\ \qquad \mp \dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,y,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{H_4(t,z) \sqrt{H_2(t,z)}} + b_0(t,z), \end{cases} \tag{26}$$

where

$$\xi = p(t,z)x + \omega(t,z),$$
$$\omega(t,z) = \int (H_4(t,z) - H_1(t,z)) \times \left\{ -(8\mu + \lambda^2) H_3(t,z) p(t,z) - x \frac{\partial p(t,z)}{\partial t} \right\} \mp \frac{H_2(t,z) H_4(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{\sqrt{H_2(t,z)}} \, \mathrm{d}t.$$

Finally, we obtain the following soliton solution when $\lambda^2 - 4\mu = 0$:

$$\begin{cases} u_3(t,x,z) = -\dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \times \left(\dfrac{C_2(1 + (\lambda/2 + 1)\xi)}{C_1 + C_2 \xi} \right)^2 + \dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,z)}{H_4(t,z)} \\ \qquad \pm \dfrac{H_2(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{p(t,z)(H_1(t,z) - H_4(t,z)) \sqrt{H_2(t,z)}}, \\ v_3(t,x,z) = \pm \dfrac{12 H_3(t,z) p^2(t,y,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{H_4(t,z) \sqrt{H_2(t,z)}} \times \left(\dfrac{C_2(1 + (\lambda/2 + 1)\xi)}{C_1 + C_2 \xi} \right)^2 \\ \qquad \mp \dfrac{\lambda^2}{4} \dfrac{12 H_3(t,z) p^2(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{H_4(t,z) \sqrt{H_2(t,z)}} + b_0(t,z), \end{cases} \tag{27}$$



(a) (b)

Figure 3. The solutions (**a**) u and (**b**) v of (25) for the stochastic coupled KdV equation when $\lambda = 0.2$, $\mu = 0$, $p = 0.01$, $b_0 = 0$, $H_1 = t^2$, $H_2 = 2t^2$, $H_3 = t^2$, and $H_4 = 3t^2$.

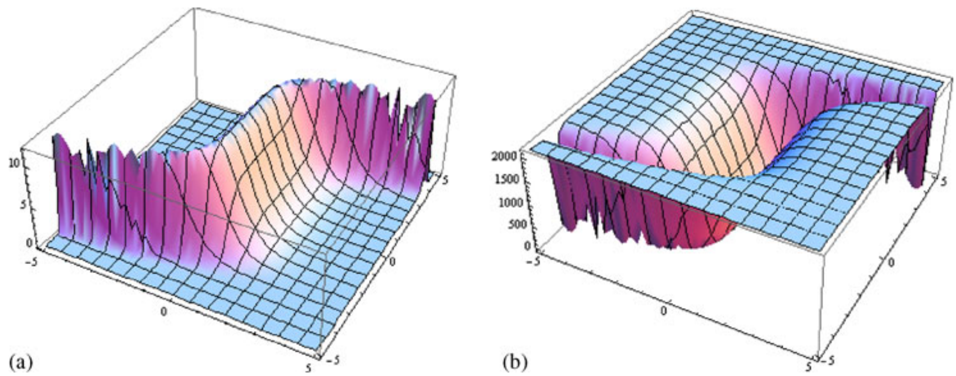


Figure 4. The solutions (**a**) u and (**b**) v of (25) for the stochastic coupled KdV equation when $\lambda = 0.25$, $\mu = 0$, $p = 8$, $b_0 = 0$, and $H_1 = \sinh(t/3) - t^3$, $H_2 = t^3$, $H_3 = H_4 = \sinh(t/3)$.

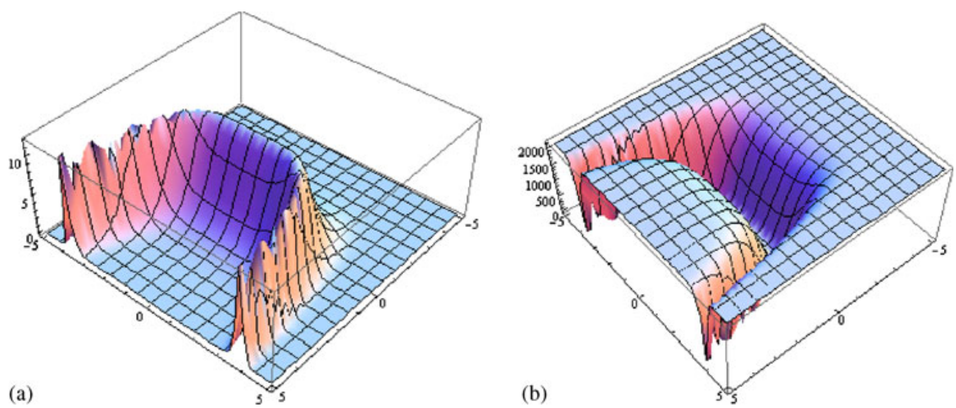


Figure 5. The solutions (**a**) u and (**b**) v of (25) for the stochastic coupled KdV equation when $\lambda = 0.25$, $\mu = 0$, $p = 8$, $b_0 = 0$, and $H_1 = \cosh(t/3) - t^3$, $H_2 = t^3$, $H_3 = H_4 = \cosh(t/3)$.

where

$$\xi = p(t,z)x + \omega(t,z),$$

$$\omega(t,z) = \int (H_4(t,z) - H_1(t,z)) \times \left\{ -(8\mu + \lambda^2) H_3(t,z) p(t,z) - x \frac{\partial p(t,z)}{\partial t} \right\} \mp \frac{H_2(t,z) H_4(t,z) b_0(t,z) \sqrt{(H_4(t,z) - H_1(t,z)) p(t,z)}}{\sqrt{H_2(t,z)}} \mathrm{d}t.$$

Further, the solution (25) contains the arbitrary functions $H(t)$ and $p(t, y, z)$. The behaviour of the obtained solution (25) is shown graphically in figures 3–5 for different values of λ, μ and b_0. Further, from figures 3–5, we can conclude that the state trajectories of the system converges as time approaches infinity, i.e., the wave solutions u and v for large times with the wave speed is asymptotically stable.

5. Conclusion

In this paper, (G'/G)-expansion method is implemented to obtain exact travelling wave solutions of the nonlinear stochastic evolution equations. The travelling wave solutions are expressed in the form of the hyperbolic and the trigonometric function solutions. The key idea of this technique is to take full advantages of a kind of Riccati equation involving two parameters and use its solutions in obtaining the travelling wave solutions. Our results reveal that the (G'/G)-expansion method is concise, direct, easy to apply, yet powerful tool for solving various kinds of nonlinear stochastic problems.

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